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RELATING NATURAL FREQUENCIES OF VIBRATION OF TOP PLATE OF A VIOLIN WITH POSITION OF SOUNDPOST

Sankalp Tiwari*[1], Pankaj Wahi[2]

[1]IIT Kanpur
snklptwr@gmail.com

[2]IIT Kanpur
wahi@iitk.ac.in

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Abstract. *Position of the soundpost plays a crucial role as far as the sound output of a violin is concerned. Experimental studies have been done to illustrate the change in sound output as the soundpost is moved around. Stiffness of the top plate gets changed as the soundpost is moved around, and hence the mode shapes and the corresponding natural frequencies also change. Tapping the violin plate to determine the note being produced from the resonating plate is the traditional method of determining the optimum position of the soundpost, but as expected, it is a long and arduous process, and hence we have tried to formulate a relation between soundpost position and corresponding natural frequencies and mode shapes of the top plate using Galerkin Projection Method and the results show good correspondence with those obtained from FEM software, ABAQUS.*

1 INTRODUCTION

Soundpost is a small piece of spruce wood about 6 mm diameter that fits snugly between the top and the bottom plate of the violin. Generally it is placed just inside the treble foot of the bridge, but it can be shifted around to give different tones and textures to sound output. For example, moving it towards the center of the violin shifts the tone more towards the bass side [1].

The top plate of violin is generally made from spruce wood. It has a complicated shape and a curved surface. Position of the soundpost significantly affects its mode shapes and the corresponding natural frequencies. Mode shapes and natural frequencies of top plate affects parameters like radiation efficiency and quality factor [2] and [3]. For some modes, the top plate vibrates and the back plate remains almost stationary [4]. It has also been shown that the top and the back plate can also be decoupled so as to independently examine the vibration characteristics of the top plate [5].

Final sound output of the violin is controlled by numerous factors which are related to each other in a complicated manner. Hence, predicting the response of the individual components and then predicting the response of the assembled instrument from these individual components could save us a lot of time, effort and money that the manual tuning consumes. We have tried to tackle one such individual component: vibration of top plate as the soundpost is moved around, using a simplified model wherein we've considered a rectangular plate pinned in between.

2 PLATE PARAMETERS AND MAJOR ASSUMPTIONS

As stated earlier, we are considering a rectangular plate simply-supported at the edges and pinned in between, where the soudpost is located. Its physical and material properties are:

Dimensions - 0.345 m* 0.21 m *0 .003 m [a =34.5 cm, b =21 cm, h (thickness) =3 mm]

Material – Spruce Wood→ Elastic, Isotropic

Density, ρ - 460 kg/m3

Young's Modulus, E - 1.3 * 10^10

Poisson's Ratio, ν - 0.3

Note that the values of a and b are the corresponding maximum dimension values of a generally available violin.

Several other assumptions are made as stated below:

- Point of contact of the soundpost with the top plate is considered to be a pinned joint
- Curvature of the top plate is ignored and it is considered to be planar
- Top plate is considered to be isotropic and to be having uniform thickness
- Damping is neglected everywhere while formulating equations

3 RELATING NATURAL FREQUENCY AND SOUNDPOST POSITION USING GALERKIN PROJECTION METHOD

An unforced rectangular Kirchoff plate is governed by the equation

$$\rho h w_{,tt} + D\nabla^4 w = 0, \tag{1}$$

where

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4},$$

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$

We also know that the solution for w, for a simply supported case is:

$$w_{mn}(x, y) = A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{2}$$

We assume that the plate is pinned (i.e. the soundpost is located) at a point (a_1, b_1). Now, to use Galerkin Projection Method, we must choose our displacement field, w, such that it satisfies the condition of zero displacement at the point (a_1, b_1).

Ofcourse, one such w could be:

$$w_1(x, y) = \sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b})\sin(\frac{2\pi a_1}{a})\sin(\frac{\pi b_1}{b}) - \sin(\frac{2\pi x}{a})\sin(\frac{\pi y}{b})\sin(\frac{\pi a_1}{a})\sin(\frac{\pi b_1}{b}) \tag{3}$$

which has been formed using $w_{11}(x, y)$ and $w_{21}(x, y)$ functions.

Substituting $x = a_1$ and $y = b_1$, one can easily see that w_1 equates to zero.

We also know that for simply supported plate, natural frequencies are given by:

$$\nu_{mn} = \frac{\pi}{2}(\frac{m^2}{a^2} + \frac{n^2}{b^2})\sqrt{\frac{D}{\rho h}} \tag{4}$$

Plugging in the appropriate values, we can determine the first few frequencies and form combinations like above. Finally, we can write an expression for w as:

$$w = A_1(t)w_1(x, y) + A_2(t)w_2(x, y) + A_3(t)w_3(x, y) + ... + A_i(t)w_i(x, y) \tag{5}$$

Using Galerkin Projection Method, we can determine the mass and stiffness matrices as:

$$M[p, q] = \int_{x=0}^{a}\int_{y=0}^{b} w_p w_q dxdy \quad \& \quad K[p, q] = \int_{x=0}^{a}\int_{y=0}^{b} (\frac{\partial^4 w_p}{\partial x^4} + 2\frac{\partial^4 w_p}{\partial^2 x \partial^2 y} + \frac{\partial^4 w_p}{\partial y^4}) w_q dxdy \tag{6}$$

where both M and K are i x i matrices

Once M and K are obtained, eigenvalues (which correspond to the natural frequencies) and eigenvectors (which correspond to the mode shapes) of $M^{-1}K$ matrix can be easily obtained. We have used MAPLE software for obtaining expressions for them, but they are too long to reproduce here.

We have considered a grid of 9 points where the soundpost is to be pinned, as shown in Fig. 1. We will obtain natural frequencies and mode shapes for these positions of soundpost. Since the plate is symmetric, the points have been considered in one quarter of a plate only. If we consider $a_1 = \alpha a$ and $b_1 = \beta b$, and substitute, further simplified expressions can be obtained, but still, they are too long to be written here. The dimensions given below are in terms of α and β.

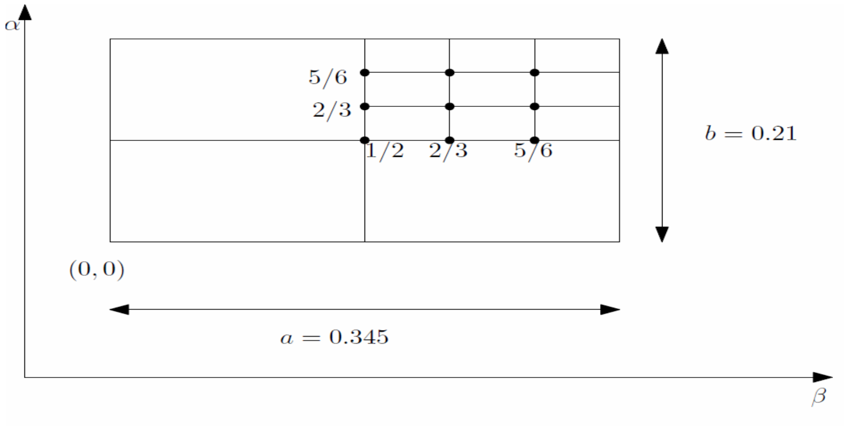

Figure 1: Rectangular Plate and Grid of Pinned Points.

4 RESULTS AND DISCUSSIONS

4.1 Comparison of ABAQUS and Galerkin Method

We can vary the value of i in Eq. (5) to see that as we increase i, the natural frequencies that we get from Galerkin Projection Method tend to reach the corresponding ABAQUS values. As a particular case for instance, let us consider $(\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{2}\right)$ for i=5 and i=10 (note that all the frequencies given in this paper are in Hz):

	ABAQUS	Galerkin (*i*=5)	Galerkin (*i=10)*
Mode 1	426	425.05	426.83
Mode 2	502	540.44	530.29
Mode 3	750	751.59	751.51
Mode 4	940	942.75	942.74
Mode 5	1105	1358.88	1191.43

Table 1: Comparison of ABAQUS and Galerkin Results for $(\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

We can see from Table 1 that for Mode 1, 3 and 4, results are in very good agreement. For Mode 2 and 5, results are closer for $i = 10$, so its intuitive that as the value of i is increased, we will get close to the desired value.

In Figs. 2–6, mode shapes obtained from both Galerkin method (at the left), and ABAQUS (at the right) are shown. It can be clearly seen that Modes 1-4 are in good agreement.

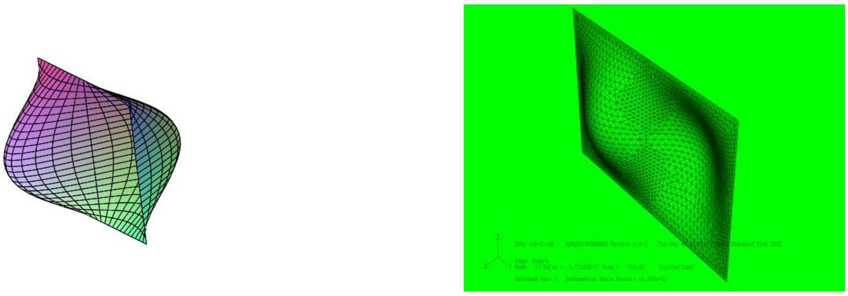

Figure 2. Comparison of Mode 1 shape for $(\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{2}\right)$. Left: Galerkin, Right: ABAQUS.

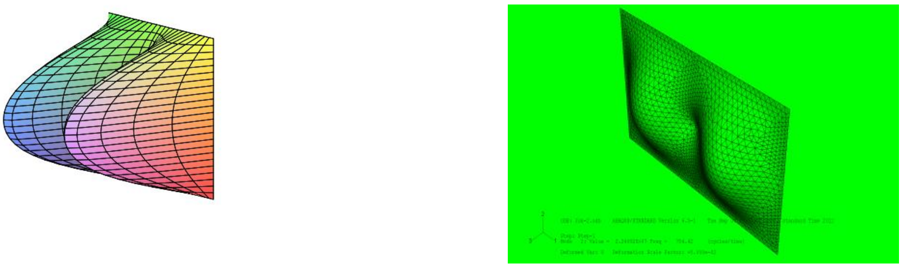

Figure 3. Comparison of Mode 2 shape for $(\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{2}\right)$. Left: Galerkin, Right: ABAQUS.

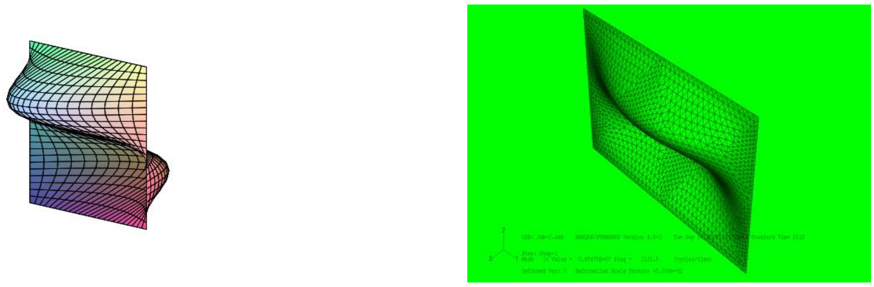

Figure 4. Comparison of Mode 3 shape for (α , β) = $\left(\frac{1}{2},\frac{1}{2}\right)$. Left: Galerkin, Right: ABAQUS.

Figure 5. Comparison of Mode 4 shape for (α , β) = $\left(\frac{1}{2},\frac{1}{2}\right)$. Left: Galerkin, Right: ABAQUS.

Figure 6. Comparison of Mode 5 shape for (α , β) = $\left(\frac{1}{2},\frac{1}{2}\right)$. Left: Galerkin, Right: ABAQUS.

Mode 5 shapes are not in agreement, as can be seen from Fig. 6. That is probably because of the limited value of i that we have chosen.

As another example, let us take (α , β **)** = $\left(\frac{2}{3},\frac{1}{2}\right)$:

	ABAQUS	Galerkin(*i*=5)	Galerkin (*i=10)*
Mode 1	335.78	343.77	339.45
Mode 2	743.93	745.79	745.4
Mode 3	750.09	752.1	751.78
Mode 4	808.08	-	890.84
Mode 5	940.47	943	942.74

Table 2: Comparison of ABAQUS and Galerkin Results for (α , β) = $\left(\frac{2}{3},\frac{1}{2}\right)$.

It can be seen from Table 2, that, apart from Mode 4, all other frequencies are in good agreement. Mode 4 is not at all obtained when we take $i=5$, whereas for $i=10$, result doesn't really agree, but again its intuitive that as i is increased, we will get close to the desired value.

The same exercise is done for all the 9 points and results obtained are in good agreement for all of them.

The agreement of results can also be seen through Figs. 7-9. Note that blue curve denotes the Mode 1 frequencies obtained from Galerkin Method as α is varied from ½ to 1. In Fig. 7, $\beta = 1/2$, in Fig. 8, $\beta = 2/3$ and in Fig. 9, $\beta = 5/6$. Red dots are the ones obtained from ABAQUS for $\alpha = ½$, $\alpha = 2/3$ and $\alpha = 5/6$.

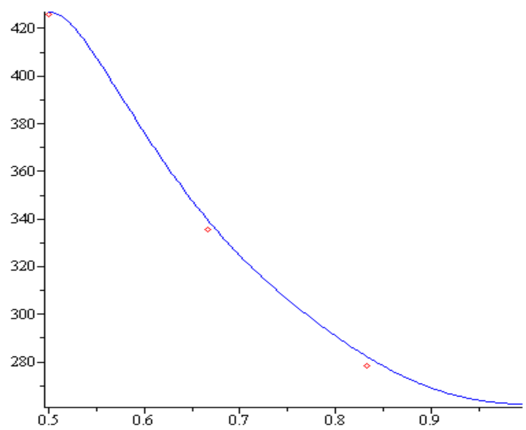

Figure 7. Mode 1 Natural Frequency vs. α for $\beta = 1/2$.

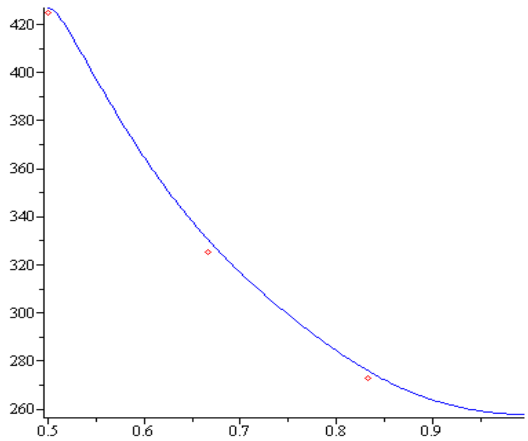

Figure 8. Mode 1 Natural Frequency vs. α for $\beta = 2/3$.

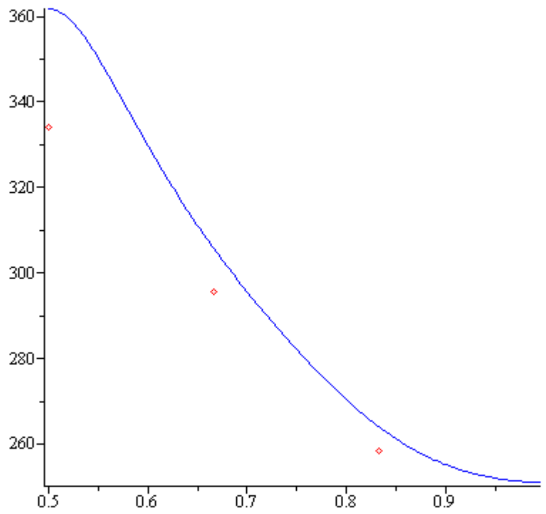

Figure 9. Mode 1 Natural Frequency vs. α for β =5/6.

4.2 ABAQUS Inferences

Some interesting observations have also been made by application of different boundary conditions in ABAQUS:

- If the edges of the plate are considered to be fixed-fixed, natural frequencies have a higher value as compared to corresponding values for simply supported edges. That is to be expected, because the stiffness of the system increases as we fix the edges. This can be seen in the data tabulated below. Note that ‘F’ refers to Fixed-Fixed BC & ‘S’ refers to Simply Supported BC.

(α,β)	(1/2,1/2)		(2/3,1/2)		(5/6,1/2)	
Mode	F	S	F	S	F	S
M1	657.14	426	543.74	335.78	480	278.62
M2	755.22	502	987.66	743.93	762.64	564.72
M3	1133.3	750	1132.8	750.09	1132.8	750.09
M4	1325.1	940	1181.3	808.08	1223	940.47
M5	1471.6	1105	1324.6	940.47	13246	995.41

(α,β)	(1/2,2/3)		(2/3,2/3)		(5/6,2/3)	
Mode	F	S	F	S	F	S
M1	425.24	659.8	513.75	325.54	529.19	272.78
M2	656.96	761.13	656.96	584.58	844.77	529.55
M3	1058.2	1141.9	1026.3	743.93	1014	748.84
M4	1324.6	1333.1	1267.3	845.63	1217	855.67
M5	1432	1500.8	1324.6	1081.9	1451.5	1057.9

(α,β)	(1/2,5/6)		(2/3,5/6)		(5/6,5/6)	
Mode	F	S	F	S	F	S
M1	472.86	334.08	486.07	295.68	458.94	258.41
M2	728.25	426.28	701.33	483.71	677.72	475.3
M3	1084.8	745.45	1009.7	743.93	1032.6	747.33
M4	1171.4	940.47	1189.3	828.42	1148.4	805.31
M5	1382.7	952.84	1432	1081.9	1370	1035.2

Table 3. ABAQUS Results for Natural Frequencies.

- As the pin joint is moved farther away from the center, corresponding natural frequency decreases, unless the mode shape remains same, in which case it remains almost constant. This trend can be observed form the above tabulated data.

5 CONCLUSIONS

We tried to formulate a relation between natural frequencies of vibration of top plate, which has been assumed to be rectangular, and position of soundpost, which has been assumed to be a pinned joint. Galerkin Projection Method was used and expressions were obtained, which could not be reproduced here due to their large length. But the results obtained from them upon plugging in appropriate values showed good correspondence with those obtained from FEM software, ABAQUS.

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