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MODELLING SELF-INTERRUPTION IN DRILL-STRING DYNAMICS

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Abstract. *Self-interrupted dynamics of a rotary drill-string system is analyzed in this paper. The degrees of freedom of the drill-string system have been chosen to be co-ordinate corresponding to the first axial vibration mode of the drill bit and first torsional vibration mode of the drill-string. Our model is based on the approach of P. Wahi and A. Chatterjee 2005 for modeling self-interruption and the cutting force model along with the system equations for the axial and torsional vibrations have been adapted from a recent model proposed by K. Nandakumar and M. Wiercigroch 2012. Our current model effectively captures the interruption in the cutting process as a result of the bit-bounce phenomenon which results from large axial vibrations. There is a possibility of interruption due to the stick-slip motion which is a result of excessive torsional vibrations. We have not incorporated the self-interruption due to stick-slip in our model at this moment and future work will incorporate this effect as well. Modeling of the interrupted cutting using a state-dependent delay model as proposed by T. Richard* et al. *2007 or K. Nandakumar and M. Wiercigroch 2012 turns out to be cumbersome as the equation governing the delay to be used in the model changes during different phases of motion. In contrast, we propose a coupled PDE-ODE model where there is no requirement of capturing the time-delay. The boundary conditions for the PDEs are chosen to capture the regenerative and possibly self-interrupted vibrations. Finite-dimensional approximation for our coupled PDE-ODE model has been obtained and the results have been compared against the existing results of K. Nandakumar and M. Wiercigroch 2012. A very decent match has been obtained till the vibrations do not grow to levels that cause the loss of contact between the tool and the workpiece.*

1 INTRODUCTION

Dynamical behavior of a drill-string used in the oil and gas industry is complex. So it is necessary to understand the complex vibrational state experienced by such system in order to control their constructive and destructive potentials [1]. A drilling assembly consist of a series of hollow cylindrical steel pipes connected to form a long flexible drill string to which a short heavier segment containing a cutting device (drill-bit) at the free end is attached. This segment may contain stabilizing fins designed to minimize lateral motion during drilling and together with the drill-bit constitutes the bottom-hole assembly (BHA).

Drill-string can primarily vibrate axially, torsionally, and laterally. In extreme condition axial vibration leads to bit-bounce [2], torsional vibration leads to stick-slip motion [3] and Lateral vibration leads to backward or forward whirl [4, 5], which causes interruption in drilling process.

A. S. Yigit and A. P. Christoforou [6] in 1996 studied the coupled axial and transverse vibration of drill-string considering it as a slender beam and equations of motion have been derived by using the assumed mode method. Later on in 2000 [7]they studied coupled torsional and bending vibration using a lumped parameter model. But they did not consider any interaction forces in their model. In 2007 T. Richard *et.al.* [8] studied coupled axial and torsional vibration using lumped parameter model and incorporated bit-rock interaction which accounts for both frictional contact and the cutting process. Later on this model is modified by K. Nandakumar and M. Wiercigroch [9] by including damping and axial stiffness in the equations of motion. It should be noted that all these studies did not include self-interruption because of excessive vibrations. It is very difficult to include interrupted cutting in the model proposed by T. Richard *et.al.* [8] and K. Nandakumar and M. Wiercigroch [9], because this will require keeping track of the correct equation determining the time-delay in the state dependent delay differential equation model.

In the present work we used the approach developed by P. Wahi and A. Chatterjee [10], in which the cut surface is modeled using a partial differential equation (PDE) with boundary conditions. This PDE together with ordinary differential equations describe the complete dynamics of self interrupted cutting without requiring to explicitly track the delays in the system. For the choice of parameters and initial conditions given in [9], a very decent match has been found with the results obtained by K. Nandakumar and M. Wiercigroch [9]. We note that the current model gets into trouble when the torsional vibrations become large leading to stick-slip motions and possible reverse slipping. Work is in progress to account for these in the model.

2 MATHEMATICAL FORMULATION

In this section equations of motion for drill-string vibration and PDE governing the evolution of cutting surface are derived.

A drill-string can be modeled as two degree freedom system for axial and torsional vibration [8, 9] one for axial vibration and one for torsional vibration. For torsional vibration drill-string can be considered as torsional pendulum, with combined rotational inertia of BHA and drill pipes, denoted by I, lumped at the end of torsional spring with stiffness K_t and torsional viscous damper of coefficient C_t. For axial vibration drill-string can be modeled as spring-mass-damper system, with combined inertia and mass of drill-string, denoted by M, lumped at the end of axial spring of stiffness K_a and damper with coefficient C_a. Let the drill-string is moving with penetration velocity V_0, rotating with angular velocity Ω_0 and applied weight on bit W_0

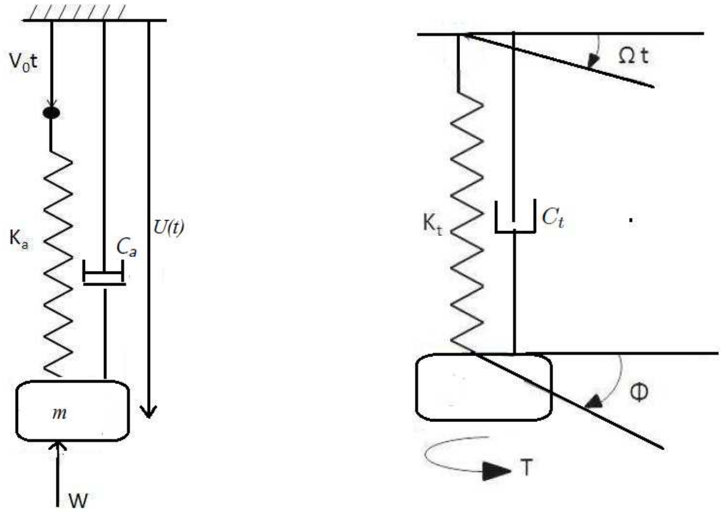

Figure 1: Axial and torsional model of drill string

So the equations of motion for this lumped parameter model of drill-string can be written as (with respect to moving frame of reference),

$$
\begin{aligned}
M\ddot{x} + C_a\left(\dot{x} + V_0\right) + K_a x &= W_0 - F_c - F_f, \\
I\ddot{\theta} + C_t\left(\dot{\theta} + \Omega\right) + K_t\theta &= -T_c - T_f,
\end{aligned} \tag{1}
$$

where, F_c is the cutting force, F_f is the frictional force because of rock-drill bit interaction, T_c is the cutting torque ,T_f is the torque because of frictional force and these quantities are defined as [8],

$$
\begin{aligned}
F_c &= \xi\epsilon a d, \\
F_f &= \sigma a l, \\
T_c &= \frac{\epsilon a^2 d}{2}, \\
T_f &= \frac{\mu a \gamma F_f}{2}.
\end{aligned} \tag{2}
$$

In the above expressions d is the depth of cut per revolution and remaining quantities are defined in Table 1. γ is the bit-geometry number and should be grater than 1, for the present work it is taken as 2. It should be noted that in the equation of motion for axial vibration V_0 represents steady state penetration rate which can be determined as,

$$
V_0 = \frac{\Omega_0 d_0}{2\pi}, \tag{3}
$$

where d_0 represents steady state depth of cut and given by [9]

$$
d_0 = \frac{W_0 - \sigma a l}{\zeta a\epsilon + \frac{C_a\Omega_0}{2\pi}}, \tag{4}
$$

The equilibrium twist of drill-bit is give by [9],

$$\phi_s = -\left(\frac{a^2\epsilon d_0 + \mu\gamma a^2\sigma l + 2C_t\Omega_0}{2K_t}\right) \tag{5}$$

If $c(t)$ is the instantaneous chip thickness cut by one cutter, so d, depth of cut per revolution, can be written as,

$$d = nc(t), \tag{6}$$

where, n is the number of cutters on drill-bit.

Parameter	Symbol	Value	Units
Drill-pipe axial stiffness	K_a	5e5	N/m
Vibrational mass	M	30667	Kg
Drill-pipe torsional stiffness	K_t	700	Nm/rad
Vibrational mass moment of inertia	I	130	Kg/m^2
Radius of bit	a	0.1	m
Wear flat length	l	0.001	m
Rock specific strength	ϵ	50	MPa
Rock contact stress	σ	70	MPa
Coefficient of friction	μ	0.4	-
Cutter inclination coefficient	ζ	0.4	-
Axial damping coefficient	ξ	0.02	-
Torsional damping coefficient	κ	0.03	-
Number of blades	n	6	-

Table 1: Parameter values used in simulation [9].

In order to take care of self interruption because of bit-bounce we have used the approach developed by P. Wahi and A. Chatterjee [10], in which cut surface is modeled using a function L. The function $L(\phi, t)$, $\phi \in [0, \frac{2\pi}{n}]$ defines the perpendicular distance between the points on cutting surface and reference frame. The angle ϕ is measured in fixed co-ordinate system and $\phi = 0$, $\phi = \frac{2\pi}{n}$ represents the position of two successive cutter on drill bit. If $L(0, t)$ represents the point on the surface cut by the cutter at position $\phi = 0$ and $L(\frac{2\pi}{n}, t)$ represents the point on the surface cut by the following cutter at $\phi = \frac{2\pi}{n}$, then at any instant of time chip thickness can be written as,

$$c(t) = L\left(\frac{2\pi}{n}, t\right) - L(0, t), \tag{7}$$

when there is no cutting *i.e.* when the drill-bit leaves the cutting surface because of bit-bounce, then $c(t) = 0$ and

$$L\left(\frac{2\pi}{n}, t\right) = L(0, t) \tag{8}$$

If $L(0, 0)$ represents the position of cutter at $\phi = 0$ and at time $t = 0$, then nominal position of cutter is given by

$$L(0, t) = L(0, 0) - V_0 t, \tag{9}$$

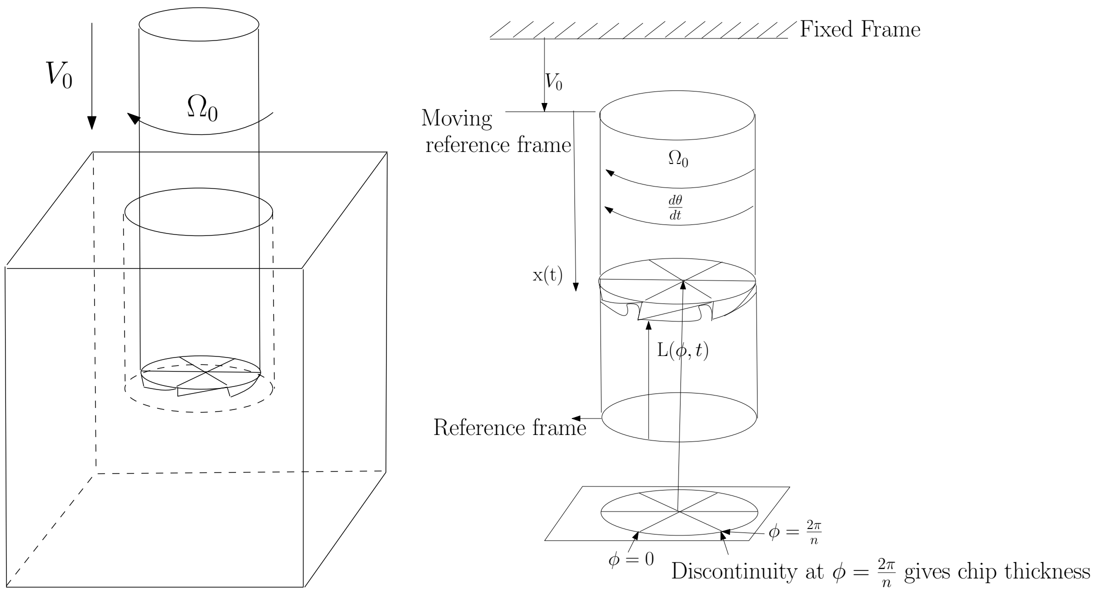

Figure 2: A Schematic of Drilling Process

but because of the vibrations of drill string, the actual position of cutter is

$$L(0,t) = L(0,0) - V_0 t - x(t). \tag{10}$$

If $L(0,0)$ is taken as zero then $c(t)$ from Eq.(7) can be written as,

$$c(t) = max\left\{L\left(\frac{2\pi}{n}, t\right) + V_0 t + x(t), 0\right\}. \tag{11}$$

Let's consider a material point $\phi \in \left(0, \frac{2\pi}{n}\right)$ on the cut surface. As the drill-string is having torsional vibration also, over a small time duration $\triangle t$, with respect to drill string, cutting surface rotates by an angle $\triangle\phi = \Omega_0 \triangle t + \frac{d\theta}{dt} \triangle t$. So,

$$L(\phi + \Omega_0 \triangle t + \frac{d\theta}{dt} \triangle t, t + \triangle t) = L(\phi, t) \tag{12}$$

and it follows,

$$\frac{\partial L}{\partial t} + \frac{\partial L}{\partial \phi}\left(\Omega_0 + \frac{d\theta}{dt}\right) = 0 \tag{13}$$

The above PDE governs evolution of cutting surface for $\phi \in \left(0, \frac{2\pi}{n}\right)$ and the boundary condition is given as,

$$L(0,t) = L\left(\frac{2\pi}{n}, t\right) - c(t). \tag{14}$$

On defining

$$\bar{L} = L + V_0 t, \tag{15}$$

$c(t)$ from Eq.(11) can be written as,

$$c(t) = max\left\{\bar{L}\left(\frac{2\pi}{n}, t\right) + x(t), 0\right\}. \tag{16}$$

Eq.(13) can be written as,

$$\frac{\partial \bar{L}}{\partial t} + \frac{\partial \bar{L}}{\partial \phi}\left(\Omega_0 + \frac{d\theta}{dt}\right) - V_0 = 0, \tag{17}$$

and the boundary condition (Eq.(14)) modifies to,

$$\bar{L}(0,t) = \bar{L}\left(\frac{2\pi}{n}, t\right) - c(t). \tag{18}$$

From Eq.(16), above boundary condition can be written as,

$$\bar{L}(0,t) = max\left\{\bar{L}\left(\frac{2\pi}{n}, t\right), -x(t)\right\} \tag{19}$$

Eq.(16), Eq.(17), Eq.(19) together with Eq.(1) describe the complete dynamics of drilling process.

3 SOLUTION USING GALERKIN PROJECTION

In this section we use Galerkin projection to solve Eq.(17). We can approximate function $\bar{L}(\phi,t)$ as,

$$\bar{L}(\phi,t) = a_0(t)\left(1 - \frac{n\phi}{2\pi}\right) + a_1(t)\frac{n\phi}{2\pi} + \sum_{k=1}^{N-1} a_{k+1}(t)\sin\left(\frac{nk\phi}{2}\right) \tag{20}$$

From above equation we can write $\bar{L}(0,t) = a_0(t)$ and $\bar{L}(2\pi/n,t) = a_1(t)$. So the chip thickness from Eq.(16) can be written as,

$$c(t) = max\{a_1(t) + x(t), 0\} \tag{21}$$

and from Eq.(19),

$$a_0(t) = min\{a_1(t), -x(t)\}. \tag{22}$$

So $a_0(t)$ acts as a dummy variable, during drilling $a_0(t) = -x(t)$ and during interruption $a_0(t) = a_1(t)$.

On doing Galerkin projection of PDE (Eq.(17)), using approximation defined by Eq.(20) we will get a set of N ODEs. These N ODEs with Eq.(1)describe the finite dimensional ODE model for drilling process. Numerical simulation results for parameter values defined in Table1 and initial conditions corresponding to K. Nandakumar and M. Wiercigroch [9] are presented in Figure 3 and Figure 4. It should be noted that there is very decent match between the results till the drill bit looses the contact.

4 CONCLUSION

In this work, self-interruption because of bit-bounce is incorporated in drilling dynamics which exactly follows the approach developed by P. Wahi and A. Chatterjee [10]. For the choice of parameters given in K. Nandakumar and M. Wiercigroch [9], interruption because of stick-slip happens prior to bit-bounce and which would not be captured by our model. Work is in progress to include this also in our model. But if choose system parameters in such a way that bit-bounce happens prior to stick-slip then our model will explain the behavior of drill-string vibration during bit-bounce. Exploration of such parameters is in progress.

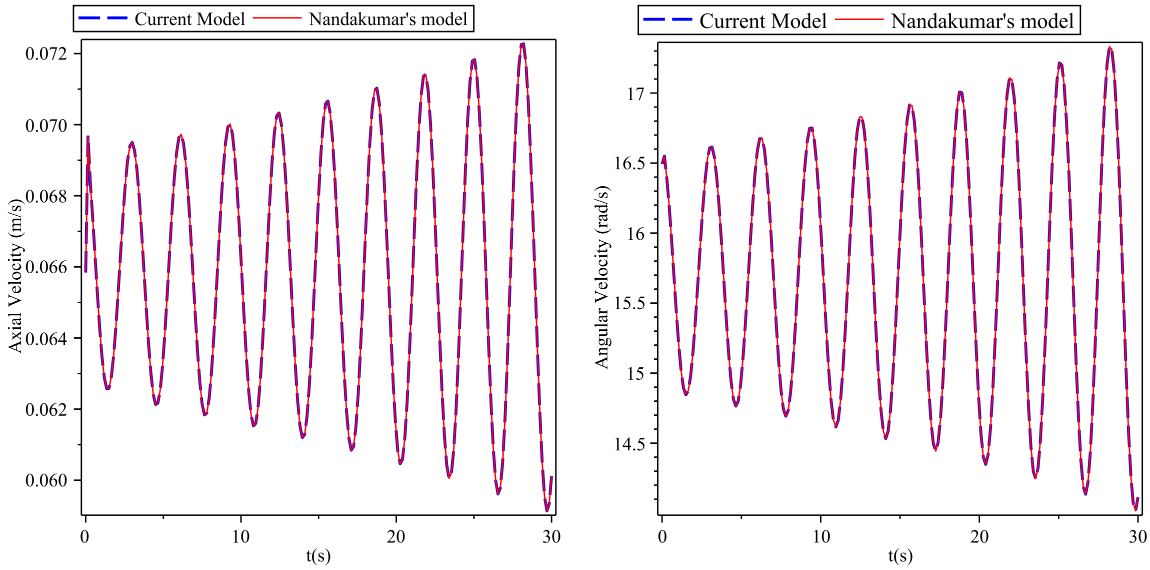

Figure 3: Comparison between results for $W_0 = 30kN$ and Ω_0 =9.425 rad/s

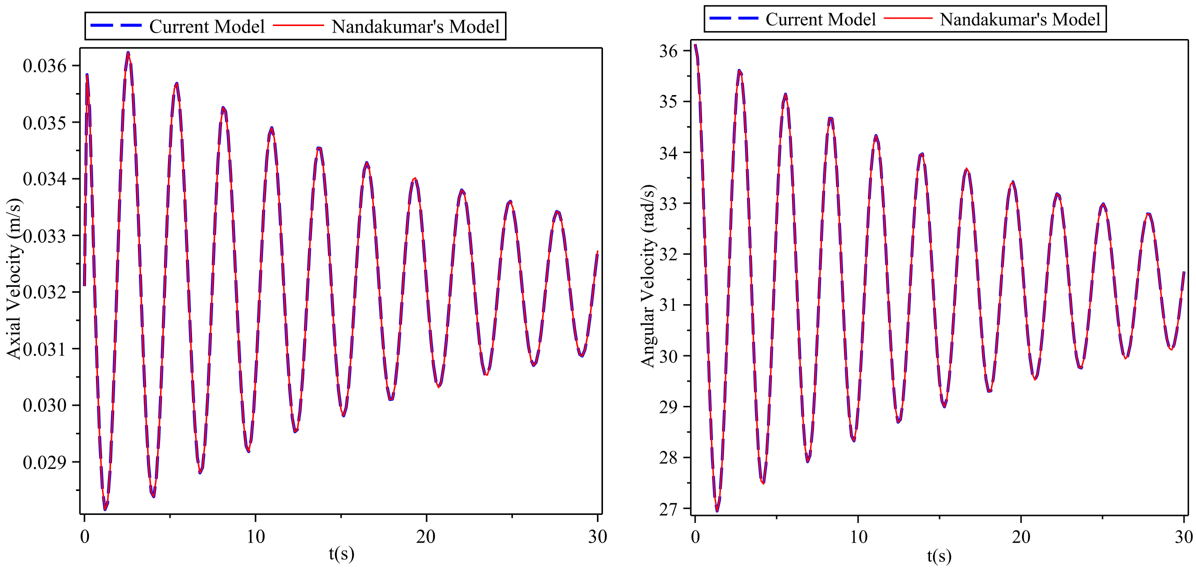

Figure 4: Comparison between results for $W_0 = 20kN$ and Ω_0 =31.416 rad/s

References

[1] P. D. Spanos, A. M. Chevallier, N. P. Politis, and M. L. Payne. Oil and gas well drilling: A vibrations perspective. *The Shock and Vibration Digest*, 32, 2003.

[2] P. D. Spanos, A. K. Sengupta, R. A. Cunningham, and P. R. Paslay. Modeling of

roller cone bit lift-off dynamics in rotary drilling. *ASME Journal of Energy Resources Technology*, 117, 1995.

[3] J. D. Jansen and L. van den Steen. Active damping of self-excited torsional vibrations in oil well drillstrings. *Journal of Sound and Vibration*, 179, 1995.

[4] J. D. Jansen. Non-linear rotordynamics as applied to oilwell drillstring vibrations. *Journal of Sound and Vibration*, 147, 1991.

[5] A. Théron, E. de Langre, and C. Putot. The effect of dynamical parameters on precession in rotary drilling. *ASME Journal of Energy Resources Technology*, 123, 1995.

[6] A. S. Yigit and A. P. Christoforou. Coupled axial and transverse vibrations of oilwell drillstrings. *Journal of Sound and Vibration*, 195, 1996.

[7] A. S. Yigit and A. P. Christoforou. Coupled torsional and bending vibrations of actively controlled drillstrings. *Journal of Sound and Vibration*, 234, 2000.

[8] T. Richard, C. Germay, and E. Detournay. A simplified model to explain the root cause of stick-slip vibrations in drilling systems with drag bits. *Journal of Sound and Vibration*, 305, 2007.

[9] K. Nandakumar and M. Wiercigroch. Galerkin projections for state-dependent delay differential equations with applications to drilling. *Applied Mathematical Modelling*, 47, 2013.

[10] P. Wahi and A. Chatterjee. Self-interrupted regenerative metal cutting in turning. *International Journal of Non-linear Mechanics*, 43, 2008.