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# NONLINEAR DAMPING IN FIBER REINFORCED COMPOSITES

**Pramod Kumar[1], Rakesh Chandra[2], S P Singh[3*]**

[1] Department of Mechanical Engineering,
Dr B R Ambedkar National Institute of Technology, Jalandhar
kushwahapramod@yahoo.com

[2] Department of Mechanical Engineering,
Dr B R Ambedkar National Institute of Technology, Jalandhar
rakesh_iit@hotmail.com

[3] Department of Mechanical Engineering, Indian Institute of Technology, New Delhi
singhsp@mech.iitd.ernet.in

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**Abstract:** *The work presents the damping studies carried out on composite materials. The frequency dependence as well as strain dependence of the loss factor is captured.. Suitable model to represent the nonlinear damping have been made and the parameters of the model have been optimized to give the best representation of the actual damping behavior of composite material. The simultaneous variation of strain and frequency is used in the experimental set ups for some of the commonly used composite configurations. Hysteretic loop method is used to obtain the damping ratio of the materials. The studies conclude that representation of actual damping behavior with a four parameter model This model is used for prediction of loss factors for different fiber volume fraction of the fiber-reinforced composite.*

## 1 INTRODUCTION

Damping capacity is a measure of a material's ability to dissipate energy during mechanical vibration under cyclic loading [1]. The damping of conventional materials does not generally provide sufficient energy dissipation for many types of engineering structures. Composite materials are used for improved tailoring of properties. Polymers generally display linear behavior in the region of low and intermediate stress. Hysteretic loops remain elliptical at low strain. However significant nonlinearity had been shown by most materials at high strain. Nonlinear behavior in structural dynamics is somewhat difficult to define in a general manner. From an engineering standpoint, a system is considered to be nonlinear if its dynamic properties such as stiffness and/or damping depend on displacement, velocity, acceleration, or any combination of these variables. The general problem of studying the dynamics of a nonlinear system is further compounded by the fact that nonlinear behavior cannot be isolated from the operating range and conditions that give rise to it in the first place. Considering that such conditions may encompass a wide range of situations, a complete analysis, theoretical or experimental, may become very expensive, if not impracticable. Consequently, nonlinear effects are usually ignored in spite of the fact that most practical engineering structures behave in a nonlinear fashion, under certain operating conditions. The detection, identification, and quantification of structural nonlinearities have been the subject of many research papers. Earlier model developed by Nelson and Hancock [2] determines the amount of energy dissipated by fiber-matrix interface due to sliding and the viscoelastic matrix for short fiber. Adams and Maheri [3] predicted moduli and flexural damping of anisotropic CFRP and GFRP beams with respect to fiber orientation using basic elastic relationships for unidirectional composites together with Adams-Bacon criterion. The effect of aspect ratio and stress level on damping is also considered. Orth et al., [4] proposed a new method for measuring hysteretic damping during the dynamic testing which allows four different properties: stress, extension, stiffness, and mechanical energies to be measured simultaneously. Chandra et at., [5] developed pseudo dynamic model based on FEM/strain energy approach to study the contribution of energy dissipation due to sliding at the fiber matrix interface to evaluate its effect on $\eta_{11}$, $\eta_{22}$, $\eta_{12}$ and $\eta_{23}$ in fiber-reinforced composite having damage in the form of hairline debonding. Yi et al., [6] investigated thermally induced stresses that occur during the cool-down process after curing on the basis of nonlinear thermo-viscoelastic finite element methods. Spathis et al. [7] studied the non-linear behavior of polymeric fiber composite materials theoretically in terms of a model developed for elastic plastic materials, and generally valid for elastic–plastic response. Cyclic tensile tests have been carried out by Paepegem [8] to assess the amount of permanent shear strain and the residual shear modulus on $[+45/45]_{2s}$ laminates and off-axis $[10]_{8s}$ composites. Ellyin and Xia [9] presented a nonlinear viscoelastic constitutive model, in differential form, based on the deformation characteristics of thermoset polymers under complex loadings. Micromechanical models for a study of nonlinear viscoelastic (NVE) response of composite laminae are developed by Sridharan [10] and Elastic moduli for composite have been predicted and variation of strain with time has been obtained. Han et al. [11] presented the formulation of a nonlinear composite nine-nodded modified first-order shear deformable element-based Lagrangian shell element for the solution of geometrically nonlinear analysis of laminated composite thin plates and shells.

Most of the work on nonlinear damping is related to the stress and strain evaluation considering the effect of temperature and strain amplitude. Here a nonlinear modified four parameter model for viscoelastic materials has been proposed and results are obtained for damping using hysteretic loop method. This model is used for prediction of damping of the fiber-reinforced composite including the variation of strain with respect to time has been evaluated.

## 2 MATHEMATICAL MODEL

Mathematical model development for rheological behavior of solid is to permit the realistic results to be obtained for complicated structures under various loading conditions such as sinusoidal, random and transient loading. In 1784, Coulumb recognized that the mechanism of damping operative at low stresses may be different than those at high stresses. Even today major emphasis is placed on linear models of damping because of its accuracy at low stress regime and simpliciy of computations than nonlinear ones**.** The linear model for viscoelastic material is Maxwell model which consist of a spring and dashpot in mechanical series as shown in Figure 1. Further Kelvin Voigt model which comprises of a spring in parallel with dashpot is shown in Figure 2. After two parameter, three parameter and four parameters models have been developed which predict, more accurately, the real behavior of viscoelastic solid at small amplitude of strain or stress polymers, elastomers and metals generally display linear rate dependent damping that can be simulated by four parameter viscoelastic model. However, significant nonlinearity is displayed by materials at high strain. To simulate the nonlinear behavior of materials, four parameter must include the nonlinear component. The differential equations derived for nonlinear modeling are discussed below.

Four parameter viscoelastic model consisting of two springs and two dashpots is formed by combining the Maxwell model and Voigt model model in series as shown in Figure 1 and differential equation is derived as below:

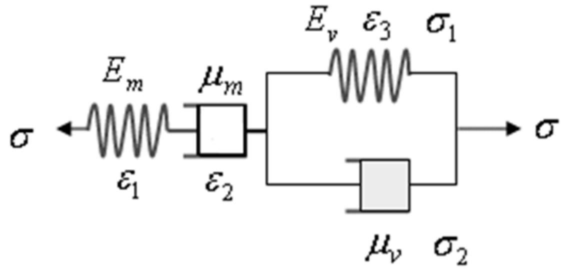


Figure 1 Four parameter viscoelastic model.

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{1}$$

$$\sigma = E_m \varepsilon_1, \quad \sigma = \mu_m \dot{\varepsilon}_2 \tag{2}$$

$$\sigma = \sigma_1 + \sigma_2 = (E_v \varepsilon_3 + \mu_v \dot{\varepsilon}_3) \tag{3}$$

Taking Laplace transformation of the Eq. (2) and Eq. (3)

$$\sigma = \mu_m s \, \varepsilon_2 \tag{4}$$

$$\sigma = (E_v + \mu_v s) \, \varepsilon_3 \tag{5}$$

Multiplying each of these equations with suitable constant and adding we get

$$\sigma E_m(\mu_m s + E_v + \mu_m s) + \sigma(E_v \mu_m s + \mu_v \mu_m s^2)$$
$$= (E_v \mu_m s + \mu_v \mu_m s^2) E_m \varepsilon \tag{6}$$

Eq (6) is of the form

$$\sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} = q_1 \dot{\varepsilon} + q_2 \ddot{\varepsilon} \quad (7)$$

where $p_1 = \dfrac{\mu_m}{E_m} + \dfrac{\mu_v}{E_v} + \dfrac{\mu_m}{E_v}, \quad p_2 = \dfrac{\mu_m \mu_v}{E_m E_v}, \quad q_1 = \mu_m, \quad q_2 = \dfrac{\mu_m \mu_v}{E_v}$

$E_m$, $E_v$ are Maxwell and Voigt elastic moduli and $\mu_m$ , $\mu_v$ are Maxwell and Voigt viscosity of assumed four parameter model for viscoelastic material.

For incorporating the nonlinear phenomena, one dashpot is replaced with nonlinear dashpot which considers the ageing effect on materials. The viscosity coefficient of this dashpot is time dependent and is given by Eq.(8) [1]

$$\sigma = \mu\, t^m (\dot{\varepsilon}) \quad (8)$$

Where *m* is the aging exponent. If *m* is positive material displays time softening. If m is negative, material displays time hardening viscosity [1]. In Eq. (7) $q_1$ is replaced with $q_1\, t^m$. Hence nonlinear four parameter model is given by Eq. (9).

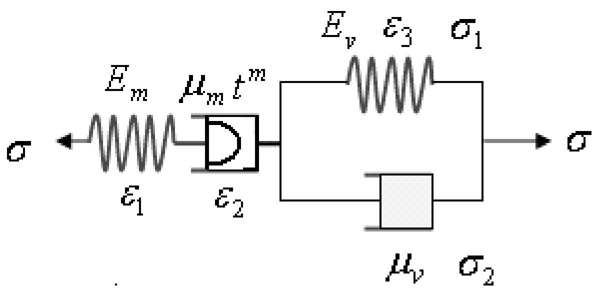


Figure 2 Four parameter nonlinear viscoelastic model.

$$\sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} = q_1 t^m \dot{\varepsilon} + q_2 \ddot{\varepsilon} \quad (9)$$

For obtaining a theoretical prediction of the damping of composites, the matrix material is first analysed in terms of its parameters which can be used in Eq. (9). This can be obtained by plots of stress and strain for cyclic loading obtained for matrix material.

For moving from matrix to composites, use is made of Bridging model [12] given in Eq. (10). Thus we can get the stress level in the fiber. Further Eqs. (11-12) are used for prediction of stress-strain variation in the composite.

$$\{d\sigma_i^m\} = [A_{ij}^{fm}]\{d\sigma_j^f\} \quad (10)$$

where $\{d\sigma_i\} = \{d\sigma_{11}, d\sigma_{22,} d\sigma_{33,} d\sigma_{13,} d\sigma_{12,} d\sigma_{23}\}^T$

$[A_{ij}]$ = bridging matrix

$$\{d\sigma\} = V_f \{d\sigma^f\} + V_m \{d\sigma^m\} \quad (11)$$

$$\{d\varepsilon\} = V_f \{d\varepsilon^f\} + V_m \{d\varepsilon^m\} \quad (12)$$

Suffix *f* and *m* refer fiber and matrix respectively, whereas quantity without suffix represents the composite.

$$\left[A_{ij}^{fm}\right]=\begin{bmatrix} a_{11}^{fm} & a_{12}^{fm} & a_{13}^{fm} & 0 & 0 & 0 \\ a_{21}^{fm} & a_{22}^{fm} & a_{23}^{fm} & 0 & 0 & 0 \\ a_{31}^{fm} & a_{32}^{fm} & a_{33}^{fm} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44}^{fm} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55}^{fm} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66}^{fm} \end{bmatrix}$$

$$a_{33} = a_{22} \tag{13}$$

$$a_{13}^{fm} = a_{12}^{fm} = \frac{\left(S_{12}^{f} - S_{12}^{m}\right)\left(a_{11}^{fm} - a_{22}^{fm}\right)}{\left(S_{11}^{f} - S_{11}^{m}\right)} \tag{14}$$

$$a_{11}^{fm} = E^{m} / E^{f} \tag{15}$$

$$a_{22}^{fm} = \beta + (1-\beta)\,E^{m} / E^{f} \tag{16}$$

$$a_{44}^{fm} = \beta + (1-\beta)E^{m} / E^{f} \tag{17}$$

$$a_{55}^{fm} = a_{66}^{fm} = \alpha + (1-\alpha)\,G^{m} \big/ G^{f} \tag{18}$$

where $\alpha$ and $\beta$ are fiber packing factors.

Using this model polymer composite is then modeled. This model is analyzed under cyclic loading and the results obtained are plotted in form of hysteresis loop. Hysteresis loop is further used to measure the damping in fiber reinforced composite as discussed below.

## 3 HYSTERESIS LOOPS FOR VIBRATING SYSTEMS

In study of vibrations, hysteresis loop is very often used as measure of energy dissipation. A hysteresis loop is obtained by recording the magnitude of a force verses the displacement brought about by its action. The area enclosed by the loop is proportional to the amount of energy dissipated in the system within one cycle as given by Eq. (21).

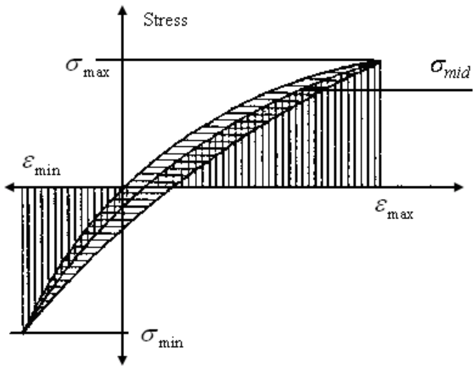


Figure 3 Hysteresis loop for damping measurement.

$$\text{Dynamic Modulus} = \frac{\sigma_{\max} - \sigma_{\min}}{\varepsilon_{\max} - \varepsilon_{\min}} \tag{19}$$

$$\text{Strain energy} = \int_{\varepsilon=0}^{\varepsilon_a} \sigma \, d\varepsilon \tag{20}$$

$$\text{Dissipative Energy} = \oint \sigma \, d\varepsilon \tag{21}$$

Solving nonlinear Eq. (9) the stress and strain are obtained. Loss factor has been obtained by hysteretic loop plotted by using the Eq. (22) [1].

$$\eta = \frac{D}{2\pi U} = \left[\int_0^{2\pi / w} \sigma(d\varepsilon / dt)dt\right]\left[2\pi \int_{\varepsilon=0}^{\varepsilon} \sigma_{mid} d\varepsilon\right]^{-1} \tag{22}$$

Where $\sigma_{mid}$ = mid stress
$U$= Strain Energy, $D$ = Dissipative Energy

## 4 RESULTS AND DISCUSSIONS

Using the material properties given in Table 1, stresses and strains are evaluated for different fiber volume fractions from $V_f$ = 0.4 to $V_f$ = 0.8. Strain varies linearly for very low values of stress and it udergoes sharp variation at higher stresses.

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| $E_f$ (GPa) | $E_m$ (GPa) | $E_v$ (GPa) | $\mu_m$ (GPa-h) | $\mu_v$ (GPa-h) |
|---|---|---|---|---|
| 72.4 | 3 | 3 | 21000 | 3 |

Table 1 Material properties used for simulation

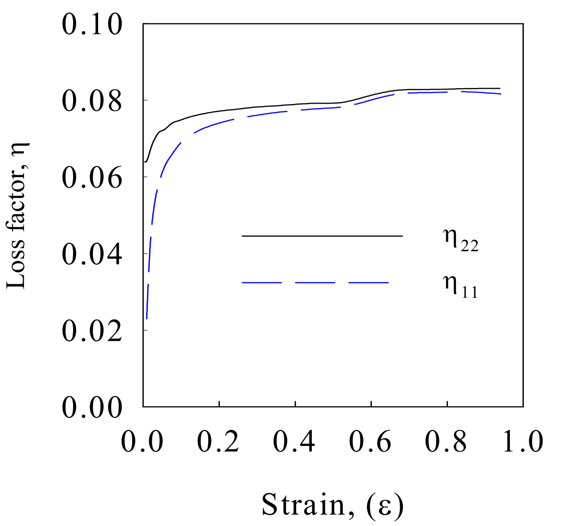


Figure 4 Variation of loss factors with respect to strain

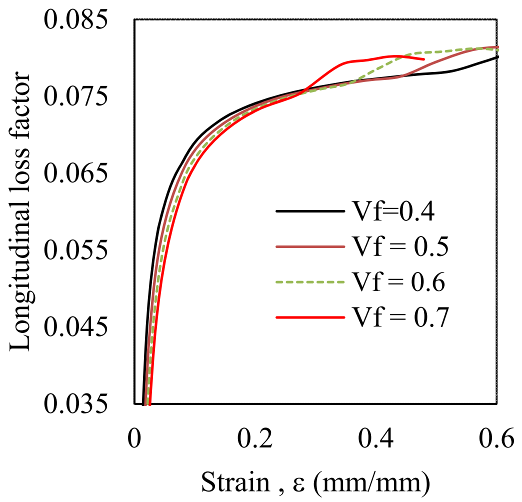


Figure 5 Variation of longitudinal loss factor with respect to strain for different volume fractions

Table 2 shows the variation of longitudinal, transverse and longitudinal shear loss factors with respect to fiber volume fraction. It can be seen that the percentage decrease in loss factors $\eta_{11}$ is of the order of 27% as the volume fraction increases from 0.4 to 0.8. Similarly reduction in loss factor $\eta_{22}$ is of the order of 2.5% for the change in fiber volume fraction from 0.4 to 0.8. Effect of fiber volume fraction on longitudinal loss factor is more because properties in longitudinal direction are dominated by fiber. Whereas properties in transverse direction are dependent on matrix properties, therefore decrease in loss factor is very less.

Figure 4 shows the variation of loss factor with respect to strain for constant maximum stress by increasing the strain for a particular volume fraction ( $v_f$ = 0.5). With the increase of strain, loss factor is increasing in general but it also tends to stabilize at a value of around 0.08. Material is getting softer with time [13]. Figure 4 depicts the variation of longitudinal loss factor with respect to strain. Longitudinal shear loss factor increases rapidly initially with strain and then its increase in loss factor is steady with the strain. It was be observed that transverse loss factor don't change much with the increase of strain because transverse loss factor is dominated by matrix property. Figure 5 shows the variation of longitudinal loss factor with respect to strain for different fiber volume fraction. There is marginal increasing effect of fiber volume fraction on loss factor at low strain.

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| Fiber volume fraction, $v_f$ | Loss factor, $\eta_{11}$ | Loss factor, $\eta_{22}$ |
|---|---|---|
| 0.3000 | 0.0587 | 0.0931 |
| 0.4000 | 0.0480 | 0.0909 |
| 0.5000 | 0.0382 | 0.0879 |
| 0.6000 | 0.0289 | 0.0837 |
| 0.7000 | 0.0210 | 0.0776 |
| 0.8000 | 0.0131 | 0.0671 |

Table 2 Loss factor variation with fiber volume fraction

## 5. EXPERIMENTAL PROCEDURE

The tensile cylclic tests were conducted in universal testing machine Zwick UTM (10 kN). The machine is equipped with necessary hardware and software for controlling and data acquisition system. Automatic calibration of testing machine is carried out. The specimen together with its attachment is than positioned in the grip of the hydraulic press. Afterwards, various dimensions of the test samples and velocities of the ram are set as the input to the software. The test experimental results are available as load displacement curve and stress strain curve. Experiments are carried out at engineering strain rate of 0.00119 mm/s and 0.000133 mm/s for $0^0$ and $90^0$ GFRE composite. The results obtained by experiment and model are shown in Table 3. Longitudinal loss factor is obtained at strain amplitude of 0.0116 for zero degree glass fiber epoxy composite for 0.44 fiber volume fraction. Further transverse loss factor is obtained at strain amplitude of 0.00445 for 35% fiber volume fraction. Experimental value is more than the theoretical value because of some defects available in specimen.

| $0^o$ Degree fiber orientation, $V_f$ = 0.44 | | |
|---|---|---|
| Strain amplitude, ε (mm/mm) | Loss factor, $\eta_{11}$ | |
| | Experimental | Analytical |
| 0.0116 | 0.0439 | 0.0401 |
| | | |
| $0^o$ Degree fiber orientation, $V_f$ = 0.72 | | |
| Strain amplitude, ε (mm/mm) | Loss factor, $\eta_{11}$ | |
| | Experimental | Analytical |
| 0.0079 | 0.0133 | 0.0112 |
| | | |
| $90^o$ Degree fiber orientation, $V_f$ = 0.35 | | |
| Strain, ε (mm/mm) | Loss factor, $\eta_{22}$ | |
| | Experimental | Analytical |
| 0.00455 | 0.0162 | 0.0876 |

**Table 3 Comparison of loss factor obtained by experiment and developed model**

## 5 CONCLUSIONS

A nonlinear four parameter model has been developed for viscoelastic materials. For evaluation of damping properties of the composite, the model model has been applied in conjuction with Bridging model in order to evaluate longitudinal and transverse loss factors for unidirectional glass fiber-reinforced epoxy composites using the hysteresis loop method. Longitudinal, transverse, and longitudinal shear loss factor decreases with the increase of fiber volume fraction whereas it increases with increase of strain. Loss factors i.e., longitudinal and transverse are quite higher for higher strains and show nonlinearity.

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