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STABILITY, MULTIPLICITY AND CONTROL PROBLEMS OF A BI-LINEAR MODEL OF THE VALVE IN A STRONG VIBRATION FIELD

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Abstract. *Valves are widely used in many technical applications including different mechanical devices, cosmic bodies, chemical plants, nuclear power stations and some others. In this paper, which is continuation of previous works [2, 3], we would like to demonstrate the abilities of application of the method of complete bifurcation groups to solve such kind of strongly nonlinear problems. Dynamical model of the valve is essentially nonlinear and such systems may have good working modes and parasitic modes. Therefore great importance is attached to the safety of the valves, especially if valve operate in the strong vibration fields. In this presentation we try to explain some dynamical properties of the simplest bilinear systems. Paper deals with complete bifurcation analysis of the bilinear model under consideration on the base of the method of complete bifurcation groups [1]. The base model has one degree of freedom, bilinear elastic force, linear damping and harmonic external excitation which simulates a vibration field. Similar models with bilinear elastic characteristic are used to study forced oscillations in wide variety of applications, for example, switches, suspension bridges, offshore structures, etc. Obtained results are represented in form of one-parameter bifurcation diagrams with amplitude of harmonic excitation. Bifurcation analysis shows that in such model there are multiplicity of different bifurcation groups which have complex topology with periodic and chaotic attractors, some of them are dangerous. In spite of the fact that bilinear valve is a very simple piecewise linear model, behavior of such a system has great complexity. For example, in some parameter region there are up to seven coexisting attractors. Also several previously unknown rare periodic and chaotic attractors are found in the model. Some new qualitative topological results have been obtained, for example, merging of two different bifurcation groups into the one large group. Results of complete bifurcation analysis help to choose not expensive approach for control in order to ensure structure stability. All obtained results may be checked using exact analytical solution for each linear subsystem.*

1 INTRODUCTION

This paper considers the dynamics of a simple bilinear model of the valve system under the conditions of forced oscillations. During the examination of this system, the so-called bifurcation theory of nonlinear dynamical systems and the method of complete bifurcation groups have been applied [4]. The aim of this research is to determine the basic behaviour of bilinear system with “soft” impact, i.e., taking into account end stiffness of the valve saddle. In the present paper, the stiffness of a valve saddle is presented in dimensionless values (c = 100). It has been found by complete bifurcation analysis that the system has a great number of periodic attractors in the case of the same excitation parameters. These data may be used for control, if necessary.

2 VALVE MODEL AND RESULTS OF BIFURCATION ANALYSIS

The bilinear system with linear friction and harmonic excitation is considered below:

$$m\ddot{x} + b\dot{x} + f(x) = h\cos\omega t \tag{1}$$

$$f(x) = \begin{cases} c_1 x, & \text{if } x < d, \\ c_2 x - (c_2 - c_1)\, d, & \text{if } x \geq d, \end{cases}$$

where m – oscillating valve mass; b – linear dissipation coefficient; c_1 and c_2 – elastic restoring force coefficients $c_1 >> c_2$; d – the break point of bilinear characteristic; h, ω – the amplitude and frequency of excitation. Bilinear characteristic, backbone curve and damped free oscillations are shown in Figure 1.

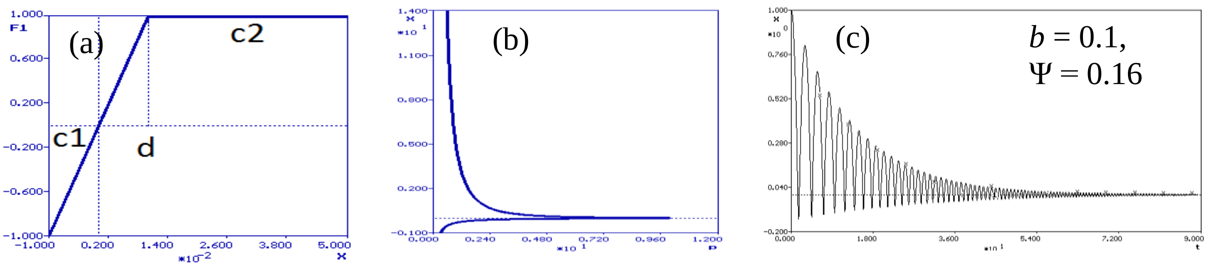


Figure 1: Free oscillations of bilinear system with one potential well. (a) bilinear characteristic; (b) backbone curve; (c) damped free oscillations. Dimensionless parameters: c_1 = 100, c_2 = 0, d = 0.01, b = 0.1

We will consider the model with the following parameters: c_1 = 100, c_2 = 0, d = 0.01, b = 0.1, ω = 5; the excitation amplitude h is a variable parameter. The results of bifurcation analysis are shown in Figure 2. Five different bifurcation groups: 1T, 3T, 4T, 5T and 6T, have been found within the range of the excitation amplitude under consideration. The stable orbits (coordinates of fixed points), shown in Figure 2 are marked in black, while the unstable ones – in reddish.

In the vicinity of h = 1.1, the system has several stable sub-harmonic regimes of different orders. All bifurcation groups have rare attractors (RA) of a tip type with periodic and chaotic orbits. The topology of bifurcation curves is rather complex with several folds and multiplicity. These results can be used in control problem. Some stable orbits with their passport data are shown in Figures 3-4. In Figure 4 (d-e) forces and sector power for P5 RA are shown.

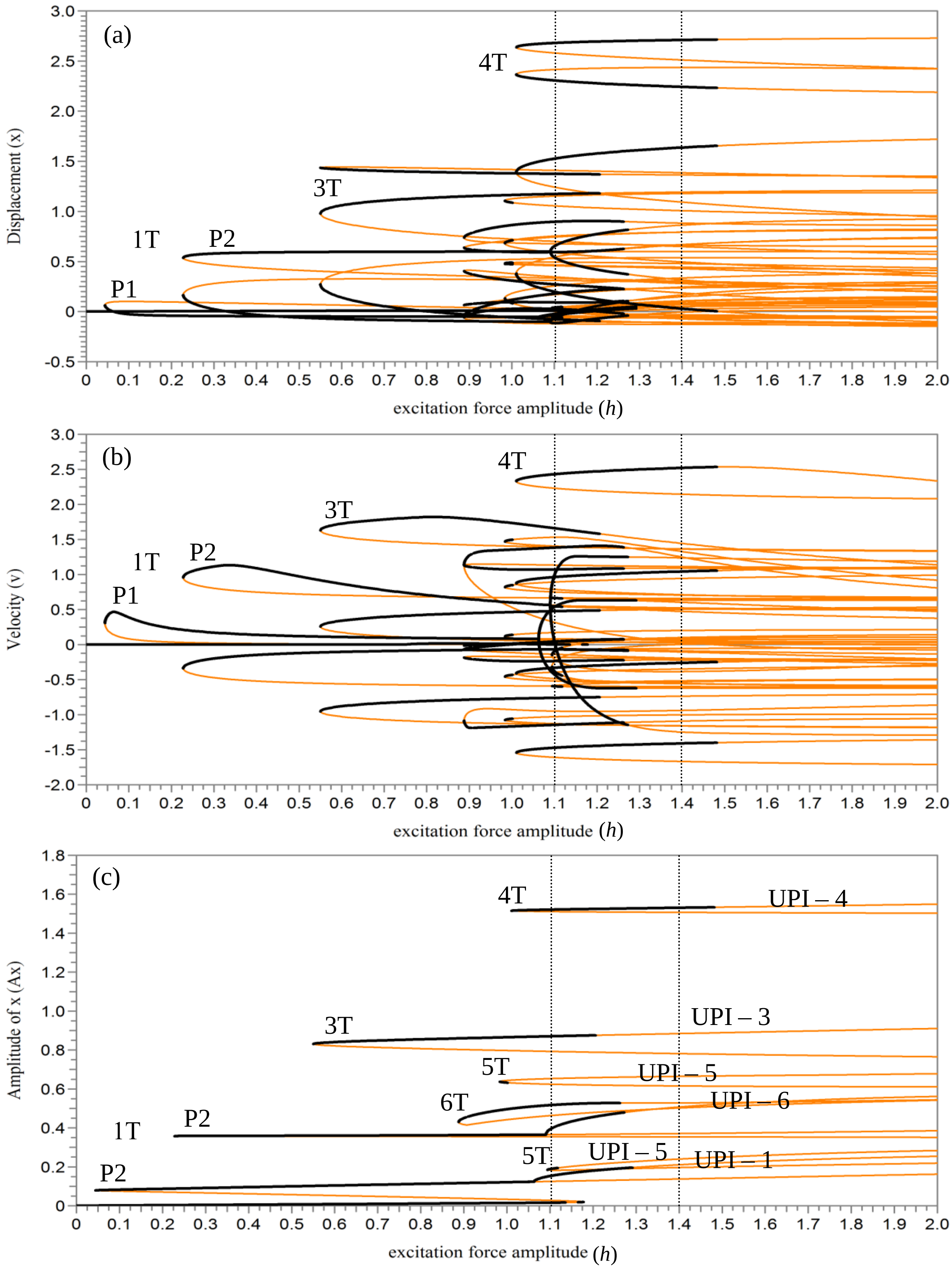
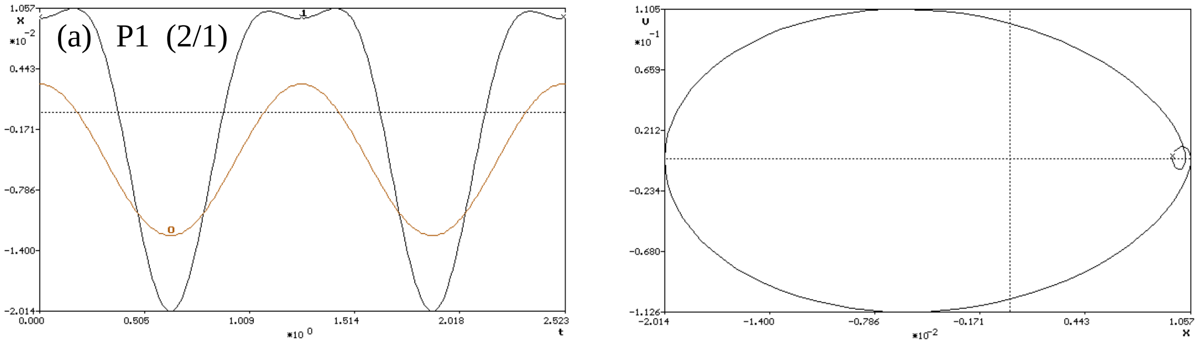
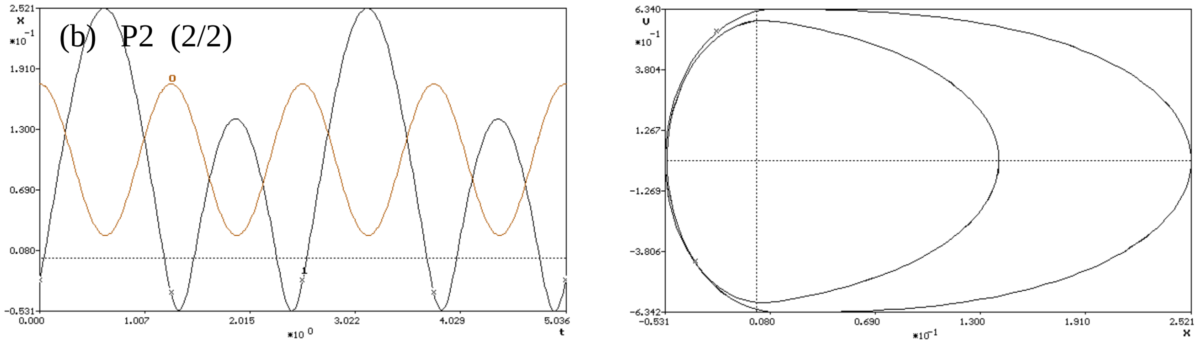


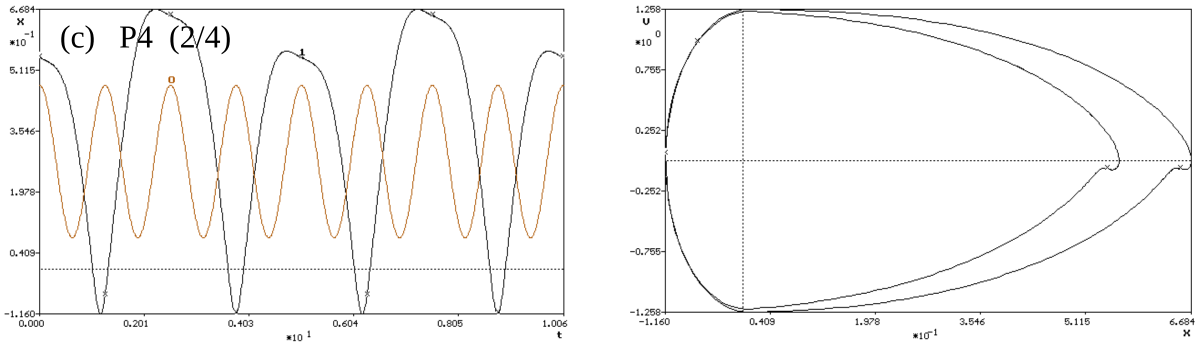
Figure 2: Forced oscillations of bilinear systems with linear friction and periodic harmonic excitation. The system has five bifurcation groups with rare attractors (RA) of tip type. Stable orbits are shown by bold black line and unstable orbits - reddish line. Coordinates of (x) and (v) – is coordinates of fix points. Equation of motion: $\ddot{x}+b\dot{x}+f(x)=h\cos\omega t$, $f(x)=c_1 x$ at $x<d$, $f(x)=c_2 x-(c_2-c_1)\,d$ at $x\geq d$. Parameters: c_1 = 100, c_2 = 0, d = 0.01, b = 0.1, h = var., ω = 5, k = 7



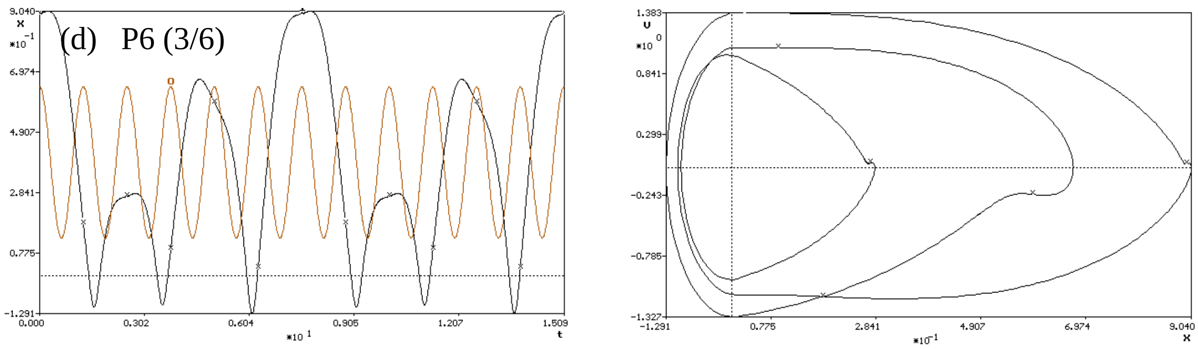
(a) P1(2/1), fixed point (0.009529, 0.001292); $\rho_i = 0.939$, $\alpha = 50.48^o$



(b) P2 (2/2), fixed point (-0.023568, 0.536271); $\rho_i = 0.882$, $\alpha = 54.79^o$

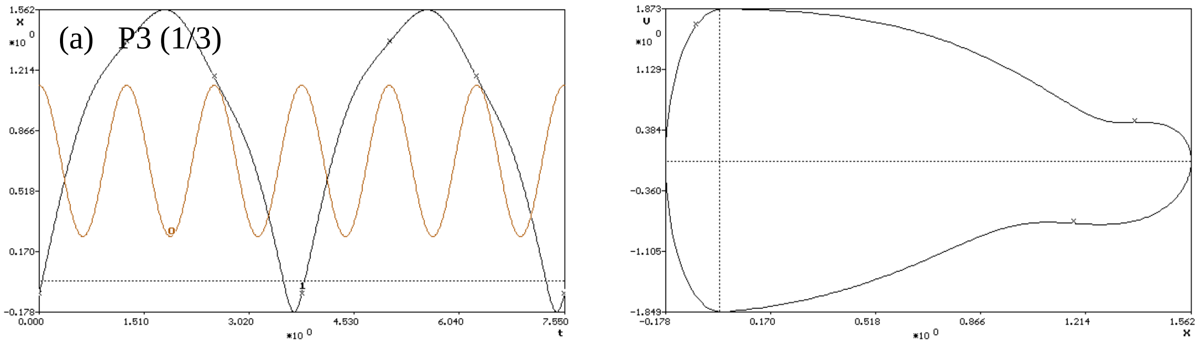


(c) P4(2/4), fixed point (0.651745, -0.068055); $\rho_i = 0.778$, $\alpha = 25.18^o$

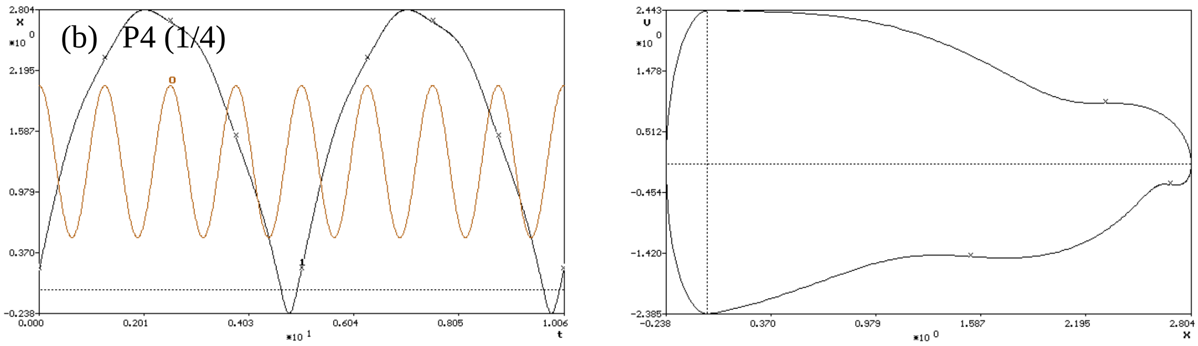


(d) P6 (3/6), fixed point (0.895560, 0.038699); $\rho_i = 0.686$, $\alpha = 123.79^o$

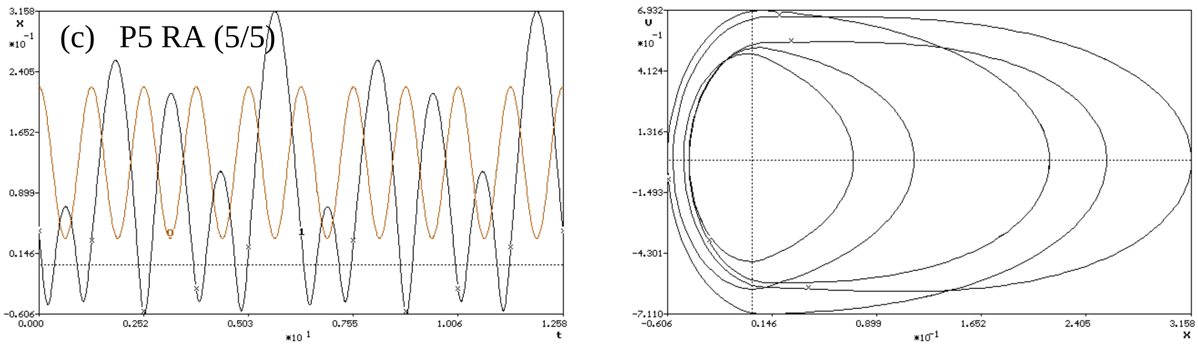
Figure 3: The time history, phase portraits and pasport's date of subharmonic stable regimes for the bifurcation groups: 1T (a, b, c) and 6T (d). Equation of motion: $\ddot{x}+b\dot{x}+f(x)=h\cos\omega t$, $f(x)=c_1x$ at $x < d$, $f(x)=c_2x-(c_2-c_1)\,d$ at $x \geq d$. Parameters: $c_1 = 100$, $c_2 = 0$, $d = 0.01$, $b = 0.1$, $h = 1.1$, $\omega = 5$, $k = 7$



(a) P3 (1/3), fixed point (1.373559, 0.475706); $\rho_i = 0.828$, $\alpha = 143.58^{o}$



(b) P4 (1/4), fixed point (2.679613, -0.331486); $\rho_i = 0.778$, $\alpha = 80.06^{o}$



(c) P5 RA (5/5), fixed point (0.040346, -0.595542); $\rho_i = 0.730$, $\alpha = 79.83^{o}$

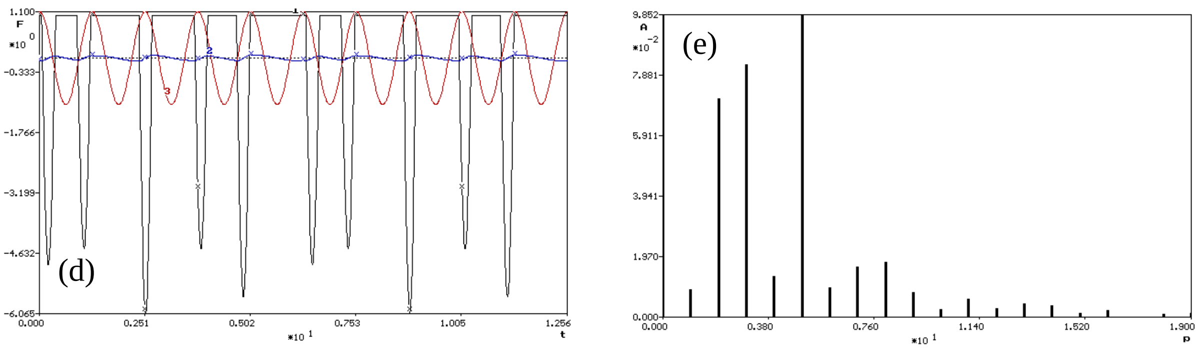


Figure 4: The time history, phase portraits and pasport's data of subharmonic stable regimes for the bifurcation groups: 3T – (a), 4T – (b), 5T – (c). (d) forces and (e) spectral analysis for rare attractor P5. Equation of motion: $\ddot{x} + b\dot{x} + f(x) = h\cos\omega t$, $f(x) = c_1 x$ at $x < d$, $f(x) = c_2 x - (c_2 - c_1)\,d$ at $x \geq d$. Parameters: $c_1 = 100$, $c_2 = 0$, $d = 0.01$, $b = 0.1$, $h = 1.1$, $\omega = 5$, $k = 7$

3 CONCLUSIONS

A simple bilinear damped model of valve (with the soft impact c_1 = 100, c_2 = 0) in strong vibration field is investigated in the paper. Using the method of complete bifurcation groups one-parameter bifurcation diagrams have been build for variable parameter of amplitude of harmonic exaltation h. It is shown that this simple model has complex behaviour with multiplicity of several periodic and chaotic attractors. Obtained results may be used for control problem and for quenching of dangerous vibrations with great amplitudes.

4 ACKNOWLEDGMENT

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