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MODAL CHARACTERIZATION OF GARTEUR SM-AG19 THROUGH MODAL IDENTIFICATION METHOD GRFP

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Abstract. *The aim of this paper is to achieve the modal parameters of aircraft prototype inter-laboratory GARTEUR (Group for Aeronautical Research and Technology in EURope) model SM-AG19 (Structures and Materials-Action Group) using the modal identification method GRFP (Global Rational Fraction Polynomial) and to compare this results with commercial software. Thus the functionality of the implemented method is verified. The GRFP is an expansion of the method RFP (Rational Fraction Polynomial) and it was implemented through computer language Matlab® to UGE (Unified Graphical Environment). The implementation was performed and a user can control certain parameters during the identification, as for example, to select the frequency range of interest, system order and tolerance. To verify the implementation of GRFP, an EMA (Experimental Modal Analysis) is conducted and the first fourteen natural frequencies and damping factors are achieved using GRFP. These results were compared with those of the LMS© TestLab® with Time MDOF® routine. Comparison was also made with RFP, and the percentage error was taken as evaluation parameter. In accordance with achieved results, it was possible to conclude that the GRFP estimated the natural frequencies and damping factors with success.*

1 INTRODUCTION

The Group for Aeronautical Research and Technology in EURope (GARTEUR) is an organization that was created in 1973 for researching collaboration in aeronautics at Europe. Nowadays, the group is formed by France, Germany, Italy, the Netherlands, Spain, Sweden and the United Kingdom. Together GARTEUR keeps more than 120 collaborative projects, whereabouts, 52 in Aerodynamic (AD), 19 in Flight Mechanic, Systems and Integration (FM), 20 in Helicopters (HC) and 35 in Structural and Material (SM).

This work aims to identify the modal parameters of GARTEUR model SM-AG 19 (Structures and Materials-Action Group), and to verify the functionally of Global Rational Fraction Polynomial (GRFP) method implemented in Matlab® language.

2 BIBLIOGRAPHIC REVISION

The functionally verification of modal identification method is quite a lot used and necessary according to [1], wherever done study of case comparing two modal identification methods. Modal identification methods, also known as “modal parameter extraction methods” are widely used in EMA and aeroelastic analyses, as example, study realized by [2].

In 1986, Ewins in [3] described several modal identification methods in frequency and time domains to procedures Single-Input Single-Output (SISO), Single-Input Multiple-Output (SIMO), Multiple-Input Single-Output (MISO) and Multiple-Input Multiple-Output (MIMO).

Physically, it can be said that the structural vibration mode is characterized by natural frequency and the predominant motion is defined as modal shape. According to [4], this modal shape is the energy manifestation which is imprisoned in boundaries of structures and couldn’t get easily escape. When the structure is excited, it can estimate your linear response through function of combined motions of yours vibration modes. Thereby, the structural general motion can be represented as a linear combination of each mode.

The vibration modes can be highlighted by peaks of this frequency spectrum by means of measuring and decomposing the signal of vibration of a structure in frequency spectrum. Each mode is one global property of structure, and each mode is defined by natural frequency, damping and modal shape. By definition, Frequency Response Function (FRF) is a structural measure normalized [4]. The modal characterization, also known as curve fitting, is a numerical procedure that is typically used to represent one set of points of experimental data by assuming analytical function. The coefficients used to define the analytical function are achieved through curve fitting procedure.

2.1 The RFP method

The Rational Fraction Polynomial (RFP) is a SISO type modal identification method in frequency domain, as shown by [5] and later discussed in [6]. Since it is a SISO method, the RFP consider only one FRF. In other words, SISO works with data obtained from one selected accelerometer (sensor). This method obtains the modal parameters through resolution of rational polynomial fractions. According to [5], the FRF can be interpreted in form of rational, Eq. (1), or partial fractions, Eq. (2).

$$H(\omega) = \left. \frac{\sum_{k=0}^{m} a_k s^k}{\sum_{k=0}^{n} b_k s^k} \right|_{s=j\omega} \tag{1}$$

$$H(\omega) = \sum_{k=1}^{n/2} \left[\frac{r_k}{s-p_k} + \frac{r_k^*}{s-p_k^*} \right] \Bigg|_{s=j\omega} \tag{2}$$

Where, * denotes complex conjugate.

$$p_k = -\sigma\_k + j\omega_k = k^{th}\ \ pole$$

$$r_k = residue\ to\ k^{th}\ \ pole$$

The rational form, shown in Eq. (1), is basically the coefficient of two polynomials where the order of numerator and denominator are independent each other. The denominator polynomial is known as characteristic polynomial of system. To sum up, the FRF is a transfer function available along of frequency axis and the poles corresponding to value of variable “s” when characteristic polynomial is equal to zero.

2.2 The GRFP method

The GRFP is a modal identification method in frequency domain (type SIMO) that achieves modal parameters through resolution of rational orthogonal polynomial fractions. As it is a SIMO type method, the GRFP considers a set of FRFs. In other words, SIMO method works with data from a set of selected accelerometers. The GRFP is an expansion of RFP and it was implemented through computational language Matlab® in Unified Graphical Environment (UGE) in Aeroelastic Division (ALA-L) at Instituto de Aeronáutica e Espaço (IAE). The implemented routine allows the control of certain parameters during the identification for example, to choose the interest frequency band, system order and tolerance. This implementation represents the continuation of previous works. A Peak Amplitude Method, [7], A Circle Fit Method, [8] were implemented, as well as, an improved graphical interface of RFP [9].

According to the formulation presented by [4], GRFP can be implemented using ordinary or orthogonal polynomials. Equation (3) shows ordinary polynomials form.

$$h_i = \sum_{k=0}^{m} t_{i,k} a_k \quad i = 1, \dots, L \tag{3}$$

Where:

$$t_{i,k} = \frac{(j\omega_i)^k}{\sum_{k=0}^{n} b_k (j\omega_i)^k} = \frac{(j\omega_i)^k}{\sum_{K=1}^{modes} (\omega_k{}^2 - \omega_i{}^2 + j2\sigma_k\omega_i)} \tag{4}$$

In matricial form:

$$\begin{Bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{Bmatrix} = \begin{bmatrix} t_{1,0} & t_{1,m} \\ t_{L,0} & t_{L,m} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{Bmatrix} \ \ ou\ \ \{H\}[T]\{A\} \tag{5}$$

The orthogonal polynomials form is:

$$h_i = \sum_{k=0}^{m} z_{i,k} c_k \quad i = 1, \dots, L \tag{6}$$

Wherever:

$$z_{i,k} = \frac{\phi_{i,k}}{\sum_{K=1}^{modos}(\omega_k{}^2 - \omega_i{}^2 + j2\sigma_k\omega_i)} = \frac{\phi_{i,k}}{g_i} \tag{7}$$

Using Eq (8) it is possible verify the orthogonality of system by Eq (9).

$$\sum_{i=1}^{L} \frac{\phi_{i,k}\phi_{i,j}}{|g_i|^2} = \begin{cases} 0{,}5 & k = j \\ 0 & k \neq j \end{cases} \tag{8}$$

In matricial form:

$$\begin{Bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{Bmatrix} = \begin{bmatrix} z_{1,0} & \cdots & z_{1,m} \\ \vdots & & \vdots \\ z_{L,0} & \cdots & z_{L,m} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{Bmatrix} \; ou \; \{H\}[Z]\{C\} \tag{9}$$

3 METHODOLOGY

An EMA is performed and the results are kept in an input file with Universal File Format (UFF). Tested model is GARTEUR SM-AG 19, as specified in [10]. Dimensions are shown in Fig. (2). However, the viscoelastic material and thermal (shear) treatment of aluminum are disregarded. Therefore, the GARTEUR SM-AG 19 mass is 41 kg instead of 44 kg of the model in reference [10]. The identification of modal parameters of GARTEUR SM-AG 19 is performed using GRFP method. These modal parameters are compared with Time MDOF® results using the same UFF input file. The location and measurements direction of accelerometers are shown in Fig (1).

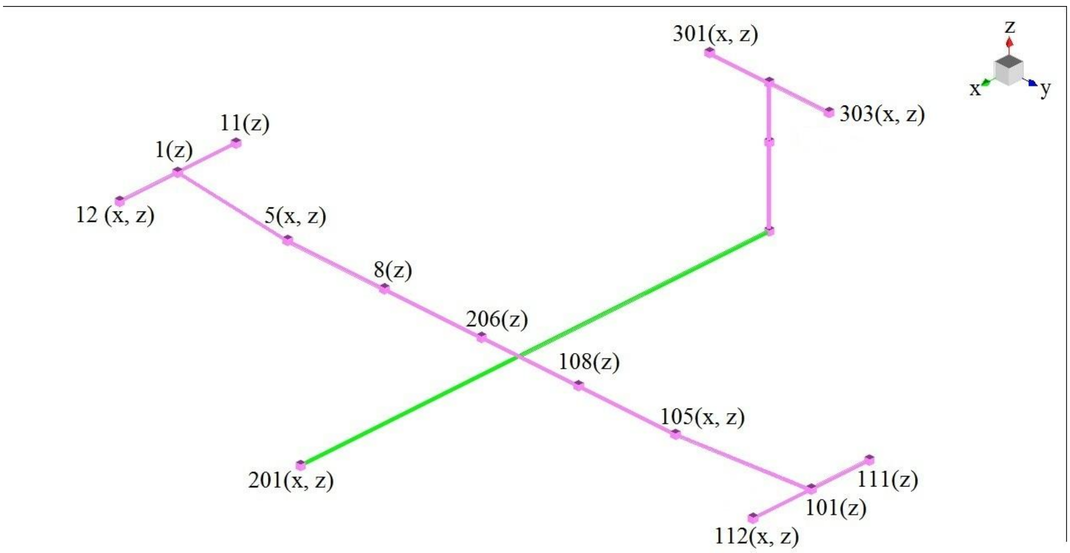


Figure 1: Location and measurement directions of accelerometers

3.1 GARTEUR SM-AG 19

In 1995, the SM-AG 19 started your activities with the aim of compare results and techniques of modal identification applied to common structure. The illustrative image of GARTEUR SM-AG 19 is present by [11] in Fig. (2).

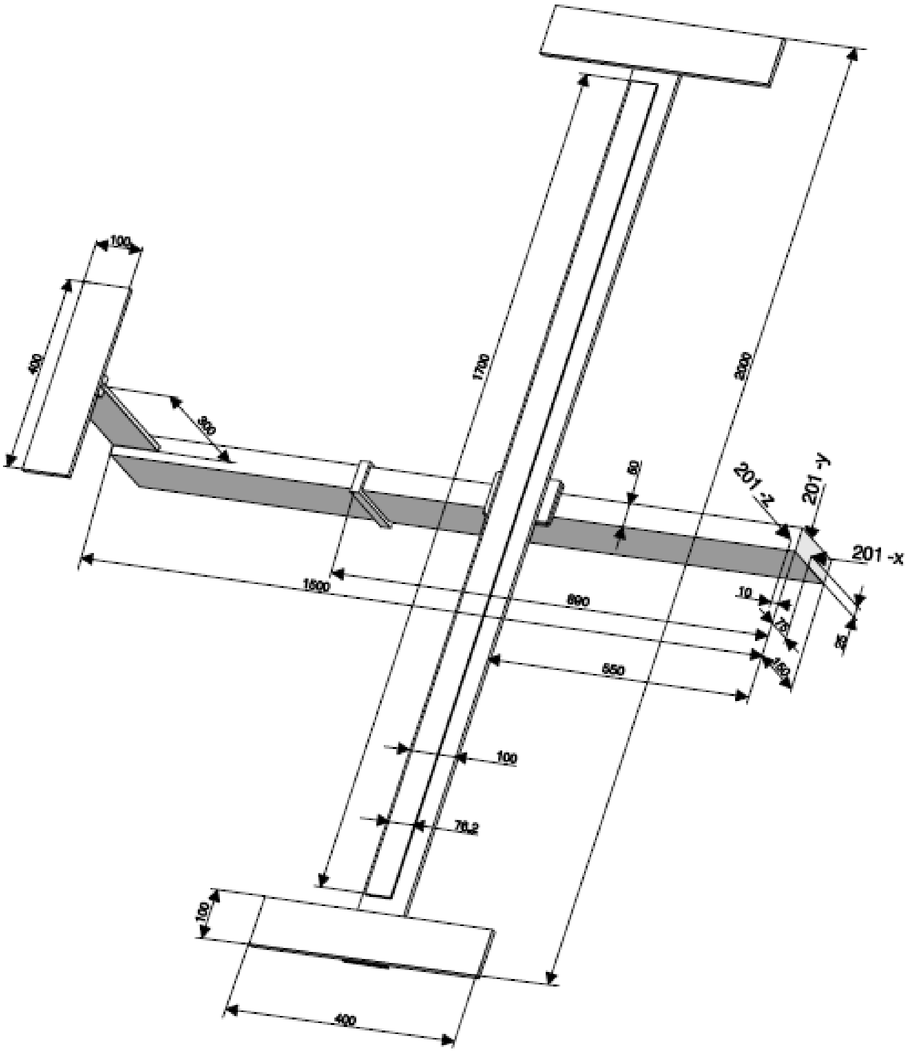


Figure 2: GARTEUR SM-AG 19 (units in millimeters)

4 RESULTS

FRFs (see Fig. (3)) were obtained from all accelerometers in $\boldsymbol{x}$ and $\boldsymbol{z}$ (Fig. 1). Natural frequencies and damping factor are, later, calculated using the GRFP method. The curve fitting performed by GRFP was successful, as show in Fig (4.a). This can be highlighted with a zoom the peak on $126.3\ Hz$, as shown in Fig (4.b).

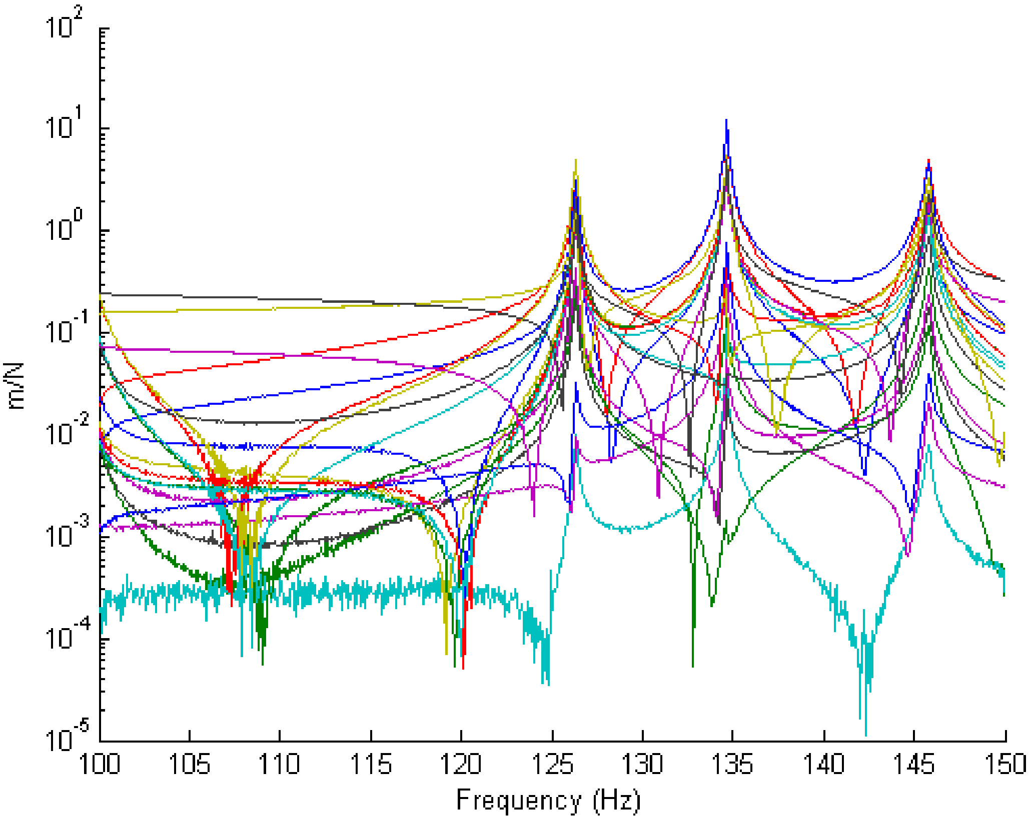
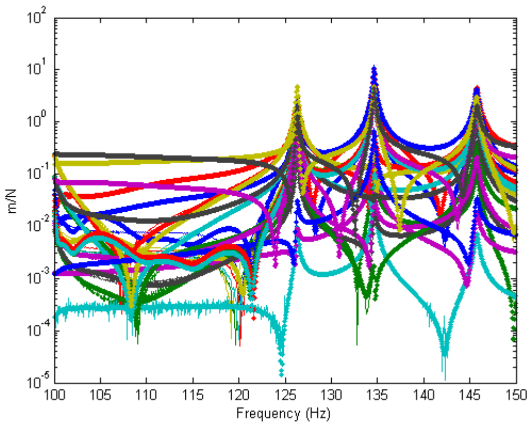


Figure 3: FRF by 21 accelerometers in x and z directions



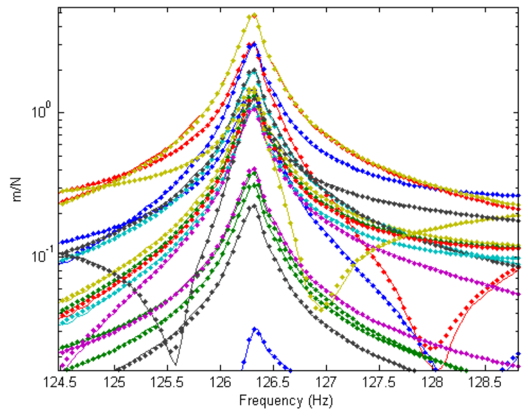


Figure (4.a): Curve fitting by GRFP.

(4.b):zoom at peak 126,30 Hz

The natural frequencies are achieved considering the repetition, according to the implemented stabilization diagram using the formulation present by [12], as shown in Fig. (5). Chosen system order is enough to identify the frequencies in the interest band. If a larger system order is selected, the stabilization diagram will identify stable poles outside the natural frequency similarly to what occurred in Time MDOF® Fig. (6).

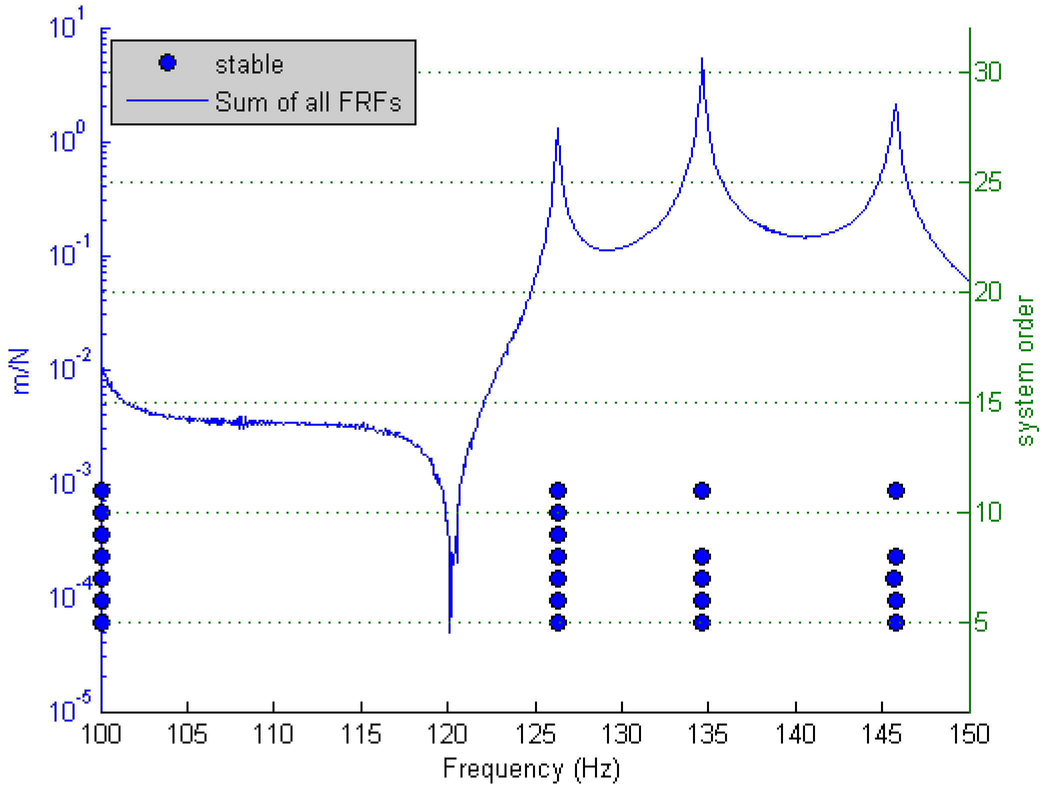


Figure 5: Stabilization diagram by GRFP

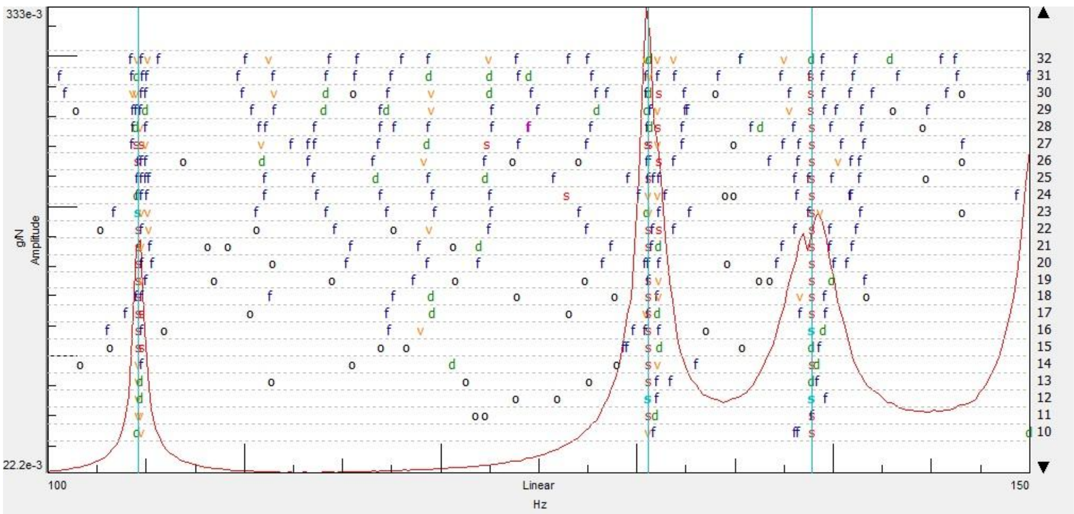


Figure 6. Stabilization diagram by Time MDOF®

The natural frequencies were compared each other. Taking the frequencies achieved by Time MDOF® as reference, a comparison is performed with the ones identified by GRFP and RFP. Percentage error, Eqs. (10-11), are used as evaluation parameters. They can be observed in Tab. (1).

$$\boldsymbol{Error_{GRFP}} = \frac{\boldsymbol{TimeMDOF - GRFP}}{\boldsymbol{TimeMDOF}} \cdot \mathbf{100}\% \quad (10)$$

$$\boldsymbol{Error_{RFP}} = \frac{\boldsymbol{TimeMDOF - RFP}}{\boldsymbol{TimeMDOF}} \cdot \mathbf{100}\% \quad (11)$$

Maia [6] states that the GRFP produces very good results, even for very close complex modes. This can be observed in identified 4th and 5th modes. The GRFP was able to identify two natural frequencies separated by only $0.58\ Hz$.

Table 1. Comparison between natural frequencies estimation

Mode	Time.MDOF® (Hz)	GRFP (Hz)	$Error_{GRFP}$ (%)	RFP (Hz)	$Error_{RFP}$ (%)
1st	5.87	5.87	0	5.86	0.170
2nd	15.41	15.41	0	15.40	0.065
3rd	34.63	34.58	0.144	34.63	0
4th	36.26	36.26	0	36.26	0
5th	36.84	36.84	0	36.86	0.054
6th	46.03	46.03	0	46.04	0.022
7th	49.97	49.97	0	49.99	0.040
8th	54.40	54.40	0	54.40	0
9th	60.81	60.81	0	60.80	0.016
10th	62.03	62.03	0	62.01	0.032
11th	99.50	99.50	0	99.50	0
12th	126.31	126.30	0.008	126.32	0.008
13th	134.64	134.63	0.007	134.67	0.022
14th	145.78	145.74	0.027	145.81	0.020
		Average	0.013	Average	0.074

The damping factors were compared using the quadratic error, Eq. (12), according to Tab. (2). The estimation of damping factors fits very well to the test lab damping factors.

$$Error_{quad} = \left(TimeMDOF_{damp} - GRFP_{damp}\right)^2 \quad (12)$$

Table 2. Damping factor comparison

Mode	$TimeMDOF_{damp}$ (%)	$GRFP_{damp}$ (%)	$Error_{quad}$ $(\%)^2$
1st	0.341	0.371	$9.00e^{-4}$
2nd	0.730	0.731	$1.00e^{-6}$
3rd	0.333	0.212	$1.46e^{-2}$
4th	0.141	0.153	$1.44e^{-4}$
5th	0.106	0.105	$1.00e^{-6}$
6th	0.102	0.103	$1.00e^{-6}$
7th	0.116	0.188	$5.18e^{-3}$
8th	0.078	0.077	$1.00e^{-6}$
9th	0.258	0.258	0
10th	0.178	0.171	$4.90e^{-5}$
11th	0.112	0.113	$1.00e^{-6}$
12th	0.079	0.079	0
13th	0.073	0.073	0
14th	0.101	0.121	$4.00e^{-4}$
		Average	$1.52e^{-3}$

5 CONCLUSIONS

It can be concluded that GRFP is able of performing an accurate curve fitting taken as reference the Fig. (4.a). Fig (5) shows that it is possible to observe that stabilization diagram is able of identify the stable roots (repeated), and thereby, to achieve the natural frequencies. According to results shown in Tab (1), GRFP method presents slightly smaller errors than RFP. Thus, one can conclude that the GRFP method, implemented in AGU, has successfully estimated modal parameters of GARTEUR SM-AG 19 and it is operational.

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