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PARAMETERIZED OPTIMIZATION OF DISC FOR A DAMPED ROTOR MODEL USING LMI APPROACH

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Abstract. *This paper deals with study of dynamics of a viscoelastic rotor shaft system, where Stability Limit of Spin Speed (SLS) and Unbalance Response amplitude (UBR) are two indices. The Rotor internal damping in the system introduces rotary dissipative forces which is tangential to the rotor orbit, well known to cause instability after certain spin speed. The gyroscopic stiffening effect has some influence on the stability. The gyroscopic effect on the disc depends on the disc dimensions and disc positions on the rotor. The dynamic performance of the rotor shaft system is enhanced with the help of gyroscopic stiffening effect by optimizing the various disc parameters (viz. disc position and disc dimension). This optimization problem can be formulated using Linear Matrix Inequalities (LMI) technique. The LMI defines a convex constraint on a variable which makes an optimization problem involving the minimization or maximization of a performance function belong to the class of convex optimization problems and these can incorporate design parameter constraints efficiently. The unbalance response of the system can be treated with H∞ norm together with parameterization of system matrices. The system matrices in the equation of motion here are obtained after discretizing the continuum by beam finite element. The constitutive relationship for the damped beam element is written by assuming a Kelvin – Voigt model and is used to obtain the equation of motion. A numerical example of a viscoelastic rotor is shown to demonstrate the effectiveness of the proposed technique.*

1. INTRODUCTION

Rotating machines represent some of the most common designs in mechanical engineering. Rotating shafts supported by bearings are usually loaded with mechanical components such as gears, pulleys, turbine rotors, etc. In almost all industrial applications rotating machinery may be found. In order to obtain the high specific power output, the aim is to operate the rotor at very high speed. The material damping in the rotor shaft introduces rotary dissipative force which is proportional to spin speed and acts tangential to the rotor orbit, well known to cause instability after certain spin speed Zorzi and Nelson [4]. Thus high speed rotor operation suffers from two problems viz. 1) high transverse response due to resonance and 2) instability of the rotor-shaft system over a spin-speed. Both phenomena occur due to material inherent properties and set limitations on operating speed of a rotor. By using light weight and strong rotor, the rotor operating speed can be enhanced. These two parameters have some practical limitations. In other words, the gyroscopic stiffening effect has some influence on the stability. The gyroscopic effect on the disc depends on the disc dimension and disc position on the rotor. Thus, the proper positioning of the discs and optimized dimensions may ensure high speeds and maximum stability.

A number of researchers have developed numerical methods for optimizing the structural design of a rotor system subject to dynamic performance constraints, in order to obtain high stability with system running at high speeds. Early work by Bhavikatti and Ramakrishnan[5] tackled the problem of minimizing the weight of a rotor subjected to constraints on stresses and eigenvalues of the system, respectively. The design variables considered in these studies included the inner radius of hollow rotor sections, the positions of bearings and rigid disk elements, and the bearing stiffness. Chen and Wang [9] followed similar design optimization problems but used an iterative method to manipulate the eigenvalues of rotor vibration modes. In their study the outer diameter of rotor sections was varied, together with bearing stiffness and damping coefficients. A study by Choi and Yang [7] considered using immune genetic algorithms to minimize rotor weight and transmitted bearing forces. Further work by Shiauand Chang [6] involved a two-stage optimization with a genetic algorithm to find initial values of design variables for further optimization. In their study, various parameter constraints were incorporated in an objective function using a Lagrange multiplier method.

In this paper, a theoretical approach has been presented to study variation in the system's stability with position of discs on a rotor system; as can be seen from the literature survey, a little work is done in this field. This paper also includes the effect of disc dimensions on the stability. Thus the various disc parameters (viz. disc position and disc dimension) of the rotor shaft system are optimized in order to obtain higher stability. Linear Matrix Inequalities (LMI) method is used for obtaining the optimum dimensions of the disc. The LMIs are capable of dealing with non-linear systems and they can include design constraints directly. The LMI defines a convex constraint on a variable which makes an optimization problem involving the minimization or maximization of a performance function belong to the class of convex optimization problems and these can incorporate design parameter constraints efficiently. Multiple LMIs relating to performance, stability, or parameter constraints can be combined to form a single LMI, which can be solved using the same usual algorithms. This flexibility means that LMIs can be used to solve a wide range of optimization problems, Cole et.al. [8].

The system matrices here are obtained using finite element modelling, since it offers obvious modelling advantages, particularly in modelling large scale systems. The constitutive relationship for the damped beam element is written by assuming a Kelvin – Voigt model and is used to obtain the equation of motion. A numerical problem of a viscoelastic rotor has been included towards the end of this work.

2. CONSTITUTIVE RELATIONS AND EQUATIONS OF MOTION

In this section the mathematical modelling of viscoelastic rotor shaft is represented. The finite element model of the viscoelastic rotor shaft system is based on the Euler-Bernoulli beam theory. The equation of motion is obtained from the constitutive relation where the damped shaft element is assumed to behave as Voigt model [1] i.e., combination of a spring and dashpot in parallel.

Figure (1) shows the displaced position of the shaft cross section. (v, w) indicate the displacement of the shaft centre along Y and Z direction and an element of differential radial thickness dr at a distance r (where r varies from 0 to r_0) subtending an angle $d(\Omega t)$ where Ω is the spin speed in rad/sec and Ωt varies from 0 to 2π at any instant of time $'t'$. Due to transverse vibration the shaft is under two types of rotation simultaneously, i.e., spin and whirl. ω is the whirl speed.

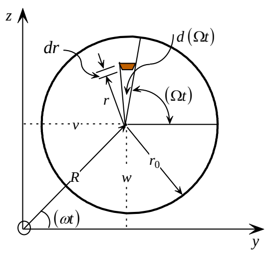


Figure1: Displaced position of the shaft cross-section

The dynamic longitudinal stress and strain induced in the infinitesimal area are σ_x and ε_x respectively. The expression of σ_x and ε_x at an instant of time are written after following Zorzi and Nelson [4].

$$\sigma_x = E(\varepsilon + \eta_v \dot{\varepsilon}); \ \varepsilon_x = -r\cos[(\Omega - \omega)t]\frac{\partial^2 R(x,t)}{\partial x^2} \tag{1}$$

Where, E is the Young's modulus, η_v is viscous damping coefficient. Following Zorzi and Nelson [4] the bending moments at any instant of time about the y and z-axes are expressed as:

$$M_{zz} = \int_0^{2\pi}\int_0^{r_0} -\left(v + r\cos(\Omega t)\right)\sigma_x r dr d(\Omega t)$$
$$M_{yy} = \int_0^{2\pi}\int_0^{r_0} \left(w + r\sin(\Omega t)\right)\sigma_x r dr d(\Omega t) \tag{2}$$

After utilizing the Eqs. (1) and (2), the governing differential equation for one shaft element is given as (Zorzi and Nelson [4]).

$$\left([M_T]+[M_R]\right)\{\ddot{q}\}+\left(\eta_V[K_B]+\Omega[G]\right)\{\dot{q}\}+\left([K_B]+\eta_V\Omega[K_C]\right)\{q\}=[F]f \tag{3a}$$

In the preceding equation $[M_T]_{(8\times8)},[M_R]_{(8\times8)},[G]_{(8\times8)},[K_B]_{(8\times8)}$ and $[K_C]_{(8\times8)}$ are the translational mass matrix, rotary inertia matrix, gyroscopic matrix, bending stiffness matrix and skew symmetric circulatory matrix, respectively and $\{f\}$ is the unbalance force due to disc eccentricity. After following Rao [3], the expressions for those matrices are given below.

$$\left[M_T\right]=\int_0^l \rho A\phi(x)\phi(x)^T\,dx,\ \left[M_R\right]=\int_0^l \rho I\phi'(x)\phi'(x)^T\,dx,\ \left[G\right]=\int_0^l 2\rho I\phi'(x)\begin{bmatrix}0&1\\-1&0\end{bmatrix}\phi'(x)^T\,dx,$$

$$\left[K_B\right]=\int_0^l EI\left[\phi''(x)\right]\left[\phi''(x)\right]^T dx,\ \left[K_B\right]=\int_0^l EI\left[\phi''(x)\right]\left[\phi''(x)\right]^T dx,\ \left[K_C\right]=\int_0^l EI\left[\phi''(x)\right]\begin{bmatrix}0&1\\-1&0\end{bmatrix}\left[\phi''(x)\right]^T dx$$

Where, ρ is the mass density, $'I'$ is the area moment of inertia, $\left(I=\int_A y^2dA\right)$. The Hermits shape function matrix, $\phi(x)$, is given by $\left[\phi(x)\right]=\begin{bmatrix}\{\phi_{xy}(x)\} & \{0\}\\ \{0\} & \{\phi_{zx}(x)\}\end{bmatrix}$, where subscripts in the elements show the respective planes.

The equation of motion for whole system is obtained by assembling the element matrix to global matrix and it is rewritten as:

$$[M]\{\ddot{q}\}+[G]\{\dot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=[F]f \tag{3b}$$

Where, $[M],[G],[C],[K]$ are the global mass, gyroscopic, damping and stiffness matrices respectively and are written as:

$$[M]=\left([M_T]+[M_R]\right),\ [G]=\Omega[G],\ [C]=\eta_V[K_B],\ [K]=\left([K_B]+\eta_V\Omega[K_C]\right)$$

The disc mass is incorporated with the global mass matrix at appropriate node. The global gyroscopic matrix contains the gyroscopic effects of shaft and disc andthe global damping matrix contains the effects of rotating and non-rotating damping. The shaft stiffness as well as support stiffness are included in the global stiffness matrix. According to the disc position, the global mass and global gyroscopic matrices are formed. The various disc positions for a single disc rotor are the consecutive nodes. But obtaining the different sets of disc position for a multi disc rotor is not straight forward. It has been done by performing the permutation between the total number of nodes and total number of disc. So the total sets of disc position for a simply supported rotor are given by $N={}^nP_j$.

Where $(n+2)$ is the total number of nodes and j is the number of discs.

3. LMI FORMULATION

The dynamic performance of a rotor system under linear behaviour can be directly assessed from the transfer function. Rotor unbalance vibration response, stability levels, and critical speed locations are commonly used indicators of dynamic performance, and these generally have equivalent transfer function specifications.

The model in the Eq. (3b) can also be represented in state space and transfer function forms as follows:

$$E\dot{x} = Ax + Bf \tag{4}$$

$$T = (sE - A)^{-1} B \tag{5}$$

Where, $E = \begin{bmatrix} I & 0 \\ G & M \end{bmatrix}, A = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}, B = \begin{bmatrix} 0 \\ F \end{bmatrix}, x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix},$ $[F]$ indicates the appropriate nodal location of unbalance force.

The unbalance-induced vibration can be modelled by a vector of external disturbance forces $f = ue^{i\Omega t}$, where the complex components $u = \{u_k\}$, where $k = 1, 2, ..., N$ specify the unbalance force at each rotor section. For the purpose of design optimization, these components are considered to have bounded magnitude.

$$0 \le u_k \le \Omega^2 m$$

Here, m is the upper limit of mass unbalance at rotor sections. Then the vibration magnitude of the n[th] nodal coordinate is given as

$$Y_n = |C_n T u| \tag{6}$$

Where C_n selects the appropriate rows of T.

It can also be written as

$$Y_n = \left| \sum_{k=1}^{N} g_k u_k \right| \le \sum_{k=1}^{N} |g_k| |u_k| \tag{7}$$

Where,

$$g = [T_1, T_2, ..., T_n] = C_n T$$

The worst case occurs when the maximum value of u_k is reached which is given as $u_k = \Omega^2 m$.

Therefore, the worst case performance for a system can be given as

$$Y_n = \Omega^2 \sum_{k=1}^{N} |g_k| m_k \tag{8}$$

Thus, the worst-case vibration amplitude at a particular machine location is given by the absolute row-sum of the corresponding frequency response matrix 'g' with each input scaled by $\Omega^2 m$. For the purpose of system design, a constraint can be specified in the form $Y \le \gamma f(\Omega)$, giving, for all values of Ω and γ is the scaling factor.

$$\sum_{k=1}^{N} |g_k| m_k \le \gamma f(\Omega) / \Omega^2 \tag{9}$$

Where the bounding function $f(\Omega)$ may be chosen to reflect any design constraints concerning critical speed locations and running speed ranges. The objective of the design optimization is to minimize γ.

A direct calculation of the worst-case vibration components $t_k = |g_k|$ can be used to select $d_k = \sqrt{t_k / m_k}$, the input scaling factor. Then, a norm bound can be applied on the system as follows:

$$\sum_{k=1}^{N} \frac{|g_k|^2 m_k}{t_k} \le \gamma_{min} \tag{10}$$

The time domain equivalent of Eq. (9), is the peak RMS bound

$$\int_0^T y_n^2 dt \le \gamma \int_0^T f^T D^2 f dt \tag{11}$$

For all $f(t)$, where $D = diag\{d_1, d_2, d_3, ..., d_k\}$, is a diagonal scaling matrix. Quadratic stability of the system can be proved by the existence of a Lyapunov function of the form.

$$V(t) = x(t)^T Px(t) \tag{12}$$

Where P is a positive definite matrix such that $\dot{V} < 0$ for all possible state variables with $f = [0]$, from Eq. (12)

$$\dot{V} = (Ax + Bf)^T QEx + x^T E^T P(Ax + Bf) \tag{13}$$

Combining Eqs. (11) and (13) we get

$$\dot{V} + y_n^2 - \gamma \mathrm{f}^T D^2 \mathrm{f} < 0 \tag{14}$$

With $y_n = C_n x$, the above equation becomes

$$(Ax + Bf)^T QEx + x^T E^T P(Ax + Bf) + x^T C_n^T C_n x - \gamma \mathrm{f}^T D^2 \mathrm{f} < 0 \tag{15}$$

Therefore,

$$\begin{bmatrix} x^T \\ \mathrm{f}^T \end{bmatrix} \begin{bmatrix} A^T PE + E^T PA + C_n^T C_n & E^T PB \\ B^T PE & -\gamma D^2 \end{bmatrix} \begin{bmatrix} x \\ \mathrm{f} \end{bmatrix} < 0 \tag{16}$$

For all $\begin{bmatrix} x^T & \mathrm{f}^T \end{bmatrix}^T \ne 0$ Thus, the design criterion is equivalent to the existence of a symmetric matrix $P > 0$ for which the following symmetric matrix is negative definite:

$$\begin{bmatrix} A^T PE + E^T PA + C_n^T C_n & E^T PB \\ B^T PE & -\gamma D^2 \end{bmatrix} < 0 \tag{17}$$

The state space matrices can be represented as affine parameter depending on the design variable $U(\theta)$ according to

$$E = E_0 + B_u U C_u \tag{18}$$

The design variable matrix $U(\theta)$ can have an arbitrary structure and may be a nonlinear function of the physical design variable. In this case,

$E_0 = \begin{bmatrix} I & 0 \\ G_0 & M_0 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ I \end{bmatrix}, U = [\Delta G \quad \Delta M] \,\&\, C_u = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$ Where $\Delta G \,\&\, \Delta M$ are sparse matrices and can be given as:

$$\Delta G = \begin{bmatrix} \ddots & & & & & \\ & 0 & 0 & 0 & 0 & \\ & 0 & 0 & 0 & 0 & \\ & 0 & 0 & 0 & -\pi\rho r^4 l/2 & \\ & 0 & 0 & -\pi\rho r^4 l/2 & 0 & \\ & & & & & \ddots \end{bmatrix}, \& \ \Delta M = \begin{bmatrix} \ddots & & & & & \\ & \pi\rho r^2 l & 0 & 0 & 0 & \\ & 0 & \pi\rho r^2 l & 0 & 0 & \\ & 0 & 0 & \pi\rho r^4 l/4 & & \\ & 0 & 0 & & \pi\rho r^4 l/4 & \\ & & & & & \ddots \end{bmatrix}$$

Where, l and r taken as thickness and radius of solid rigid disc and these form the variable to be optimized.

The optimization problem takes a new shape as can be seen below:

$$\begin{bmatrix} A^T P(E_0 + B_u U C_u) + (E_0 + B_u U C_u)^T PA + C_n^T C_n & (E_0 + B_u U C_u) C_u^T PB \\ B^T P(E_0 + B_u U C_u) & -\gamma D^2 \end{bmatrix} < 0 \tag{19}$$

And it becomes a design problem to find P and U. With some approximations and use of Schur complement to remove the bi-linearity, the above equation becomes
Minimize γ subject to

$$\begin{bmatrix} A^T PE + E^T PA + C_n^T C_n - A^T PB_u B_u^T PA & A^T PB & A^T PB_u + UC_n \\ B^T PA & -\gamma D^2 & 0 \\ B_u^T PA + C_u^T U & 0 & -I \end{bmatrix} < 0 \qquad (20)$$

This inequality is linear in P and U and therefore finding a solution for minimal γ [2] is a generalized eigenvalues problem that can be solved by developing MATLAB code.

4. NUMERICAL PROBLEM

This section involves a design of a solid rotor disc mounted on a rotor shaft as shown in the Figure (2). The rotating shaft is supported by bearings at both ends and assumed to be as damped support. The stiffness and the damping effects of the bearing supports are simulated by springs and viscous dampers (k_{yy} = 70 MN/m, k_{zz} = 50MN/m, d_{yy} = 700 Ns/m and d_{zz} = 500 Ns/m) in the two transverse directions. Following Lalanne and Ferraris [1], the material properties of the steel rotor are shown in Table (1). The purpose is to design the rotor shaft system in order to ensure low unbalance response amplitude (UBR) and high stability limit of spin speed (SLS). The design variables chosen here are the diameter and thickness of a disc and its position on the system. The initial diameter and thickness and the unbalance on the disc are shown in Table (2). The problem involves proper placement of various discs on the rotor shaft system and at the same time to represent the techniques of optimization of various design parameters of the disc for achieving the better gyroscopic stiffening effect. The sole purpose of this study is to represent techniques of optimisation of various parameters of a rotor-shaft-disc system and therefore, to obtain high stability and no feasibility study has been done on the results so obtained.

Material	Density (kg/m^3)	Young's Modulus (GPa)	Length (m)	Diameter (m)	Damping Coefficient (N-s/m)
Mild Steel	7800	200	1.3	0.2	0.0002

Table 1: Rotor Material and its Properties.

Diameter(m)	0.20
Thickness(m)	0.05
Mass Unbalance(kg-m)	2e-3

Table 2: Disc parameters

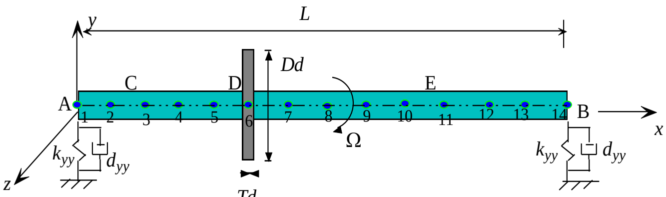


Figure 2: Schematic Diagram of the Rotor shaft system

4.1 LMI Optimizations

The initial dimensions of the disc in the rotor shaft system are shown in Table (2) is optimized here by using LMI technique. The speed-dependent bound on the worst-case vibration response $\gamma f(\Omega)$ is selected for $\gamma = 1$ as shown in Figure (3). The subsequent design optimizations will consider selection of the disc dimensions to minimize the vibration bound γ. The radius to thickness ratio for disc is taken to be 4. The Figure (4) shows the optimization of γ and the final value occurs after 100 iterations. Accordingly, Figure (5) shows the worst case response for optimized and unoptimized disc parameters. The optimized or final bound $\gamma f(\Omega)$ is also shown. Figure (6), shows the decay rate plot, as can be seen for initial dimensions the slope of the curve is steep whereas for optimized dimensions the slope is less steep showing more stability. The optimised γ obtained is 9e-9 and the maximum amplitude obtained in working frequency range is of the order 1e-15. The optimized dimensions of the disc obtained from LMI technique are shown in Table (3).

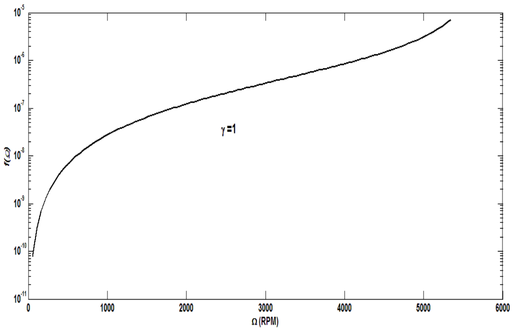


Figure 3: Unbalance Respond Bound.

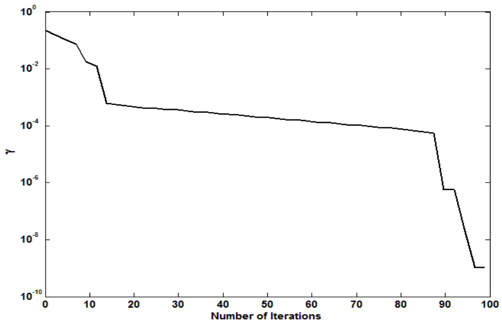


Figure 4: Optimization of γ .

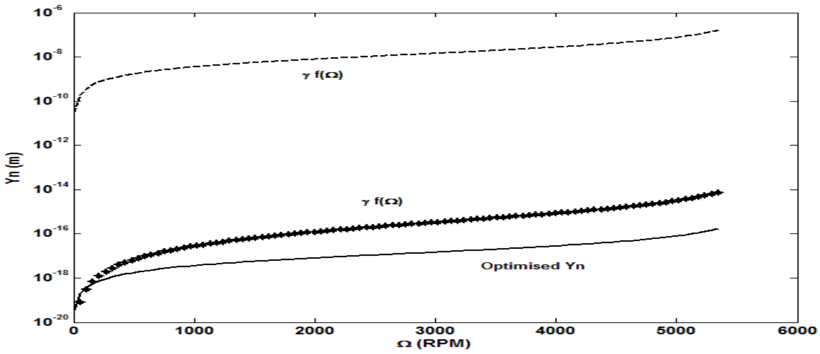


Figure 5: Unbalance Response for Optimized and Initial dimensions of disc

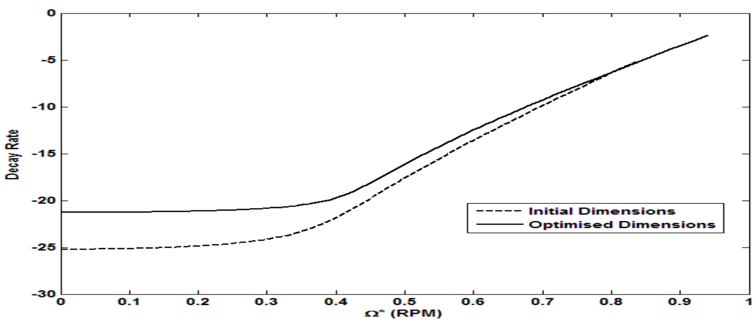


Figure 6: Decay rate plot for initial and optimized dimension of disc.

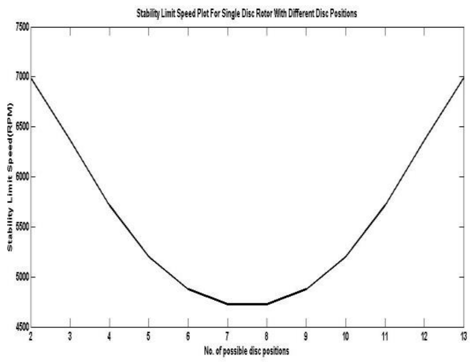
Diameter (m)	0.3163
Thickness (m)	0.0395

Table 3: Optimized Results.

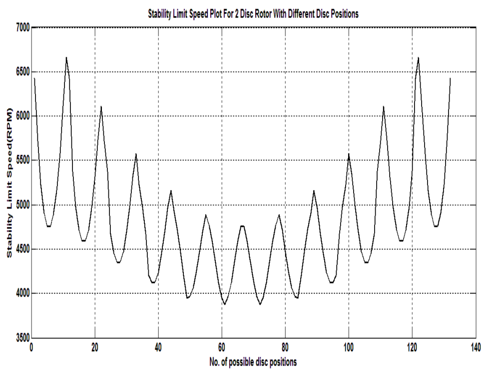
4.2 Disc Positions

The positions of discs play an important role in the stability of a rotor shaft system. Therefore, optimum positioning of a disc in the system is of prime importance and is considered here. The proper positioning helps ensure high SLS. SLS of the rotor–shaft system has been found out from the maximum real part of all eigenvalues. The system becomes unstable when the real part touches the zero line. The SLS plot for different positions so obtained has been shown in Figure (7). The method is extrapolated to two discs and three discs cases. In case of multidisc rotors, the discs are taken to be of different dimensions and there can be different ways to put discs on the rotor and therefore, permutation approach has been used to find out a set of positions for discs as they can be of different dimensions and unbalance. For example, if there are 14 nodes and three discs, the discs can be placed on any of these nodes. However, the first and last nodes are taken away by the bearing supports; the number of nodes remaining for the discs is 12. So total sets of disc positions are ${}^{12}P_3$=1320 and the discs are located as follows: $DN = [i \quad j \quad k]$; '*DN*' is the disc nodes, *i* , *j*, *k* vary from 2 to 13 and when *i*= *j* = *k DN* will be the empty array.

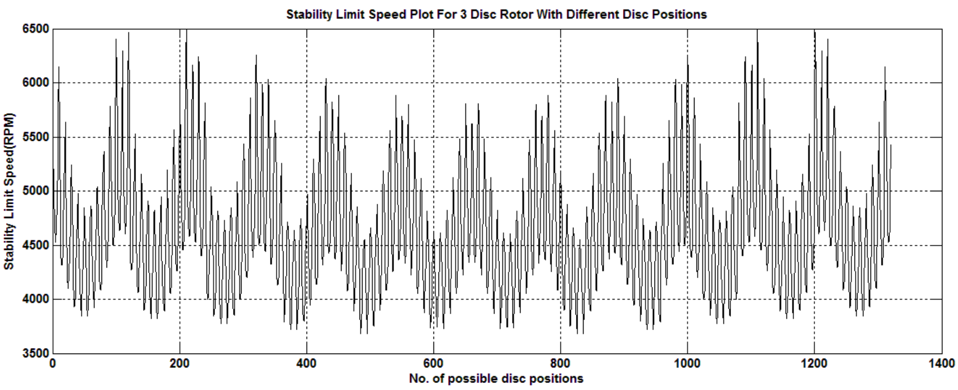
The plots for SLS are found to be symmetric about the horizontal axis and it can be seen that the maximum SLS is obtained when the discs are towards the ends of the rotor and if the discs are more towards the centre of the rotor then minimum SLS is obtained. In other words, the rotor will be more stable if the discs are placed towards the ends of the rotor and will be less stable if the discs are placed more towards the centre of the rotor. It is due to the gyroscopic stiffening of the rotor. This stiffening effect will be less when the discs are towards the centre of the rotor.



(a)



(b)



(c)

Figure 7: Effect on stability (SLS) of the system with position in case of one disc, two disc and three disc rotors

5. CONCLUSIONS

This paper has given the equations of motion of a viscoelastic rotor-shaft system. The linear viscoelastic rotor-material behaviour is represented in the time domain where the damped shaft element is assumed to behave as Voigt model. The finite element model is used to discretize the continuum which is based on Euler-Bernoulli beam theory. Use of LMI technique has been shown here to optimize disc dimension for high dynamic performance of the rotor shaft system. The advantage of the proposed method is the flexibility offered by the LMI formulation, which can be used to create design specifications concerning vibration amplitudes, stability, critical speeds, modal damping levels, and parameter constraints. This work also includes the effects of disc positions in a rotor system. Results are obtained for different sets of disc positions to study the dynamic characteristics, where stability limit of spin speed and unbalance response amplitude are two indices. The rotor will be more stable if the discs are placed towards the ends of the rotor and will be less stable if the discs are placed more towards the centre of the rotor. Thus, proper placement of disc together with optimized dimensions will ensure high stability and less response amplitude.

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