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ELASTIC VIBRATIONS IN THE MECHANISM OF CLASS IV

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Abstract. *Was created a dynamic model based spatial topology. Algorithm and a computer program for calculating the elastic vibrations have been developed. To calculate the dynamic model of the following tasks: a) the calculation of the "primary" position of the mechanism and its kinematic parameters, and b) the calculation of the elastic deformation mechanism (realized numerical scheme Newmark). The task of a) is ancillary to the problem b).*

Kinematics of Class IV with external sliding pair spatial topology. In contrast to similar mechanism of class II, the kinematics of this mechanism has no analytic solutions. Thus, kinematics can not be entered directly into the dynamics equations. That is, directly in expression of kinetic energy, potential energy, the generalized forces. Thus, at each step of solving dynamics must first be resolved kinematics.

The system of dynamic equations solved by direct integration using the Newmark numerical scheme. All the components of the elastic linear and angular displacements in the points of the structure are determined.

1 INTRODUCTION

In the paper [1] presented the dynamic equations for the finite element model mechanism

$$M \cdot \ddot{U} + C \cdot \dot{U} + K \cdot U = R \tag{1}$$

where M, C, K - the mass, damping and stiffness matrices, $\ddot{U}, \dot{U}, U$ - vectors of nodal displacements, velocities and accelerations of the model the finite element method. Expression (1) is a system of differential equations of second order. It can be solved by standard methods [2,3]. But they are not effective for a large-order matrix M, C, K. Algorithms used for dynamic finite element analysis can be divided into two groups: the decomposition of the eigenvalues and the direct integration. Direct numerical integration is based on two ideas. First, the equilibrium condition should not be performed at any time T, but only at discrete time interval. Consequently, effective use of numerical analysis becomes possible. Second, changes in the displacement, velocity and acceleration at each time Δt counted in the calculation of the next time slot. The number of operations for the direct integration proportional to the number of time steps, the order of the matrix and the width of the tape matrices. The optimal node numbering may reduce the width of the tape up to certain limits.

The method of expanding on its own forms reduces the number of computational operations. The main idea is to obtain a new matrix $\tilde{M}, \tilde{C}, \tilde{K}$ (with belt width is less than the matrix M, C, K), and the transition from U displacement to generalized coordinates X using a transformation matrix. Effective transformation matrix is determined by solving the general problem of eigenvalues:

$$\mathrm{K}\varphi = \omega^2 \mathrm{M}\varphi, \tag{2}$$

that has "n" solutions (ω_i^2, φ_i), i=1,...n. Here φ_i-th vector of its i-own form, and ω_i - corresponding eigenvalue. Equation (2) is obtained from the equation of free oscillations without damping:

$$M \cdot \ddot{U} + K \cdot U = 0 \tag{3}$$

The solution of equations (3) in the form $U = \varphi \cdot \sin \omega (t - t_0)$, where ω - the natural vibration frequency and t_0 - the initial phase. After solving (2) is made transforming matrix. Thus, the main difference of this method and the method of direct integration - is the transition to the problem (2). Both methods provide similar results. Therefore the choice of method is determined only by considerations of efficiency.

In this paper, we consider the construction high class mechanisms. They are modeled by finite elements of the rod. Equations (1) thus have a relatively low order. Consequently simplification of calculation procedures in this case is more efficient than reduction of number of arithmetical operations. Therefore, the direct integration method is used in this case. It is assumed that in the case of direct integration of the vector displacement U_0, velocity $\dot{U}_0$ and acceleration $\ddot{U}_0$ are known at the time point $t = 0$. Required to find a solution (1) in the interval from 0 to T. Time period T is split into "n" equal $\Delta t = T/(n-1)$ intervals and the solutions shall be at the time points $\Delta t, 2\Delta t, 3\Delta t, ..., T$. And solutions for each next moment shall be calculated using solutions obtained at the previous steps. One effective method of direct integration is method of central difference:

$$\ddot{U}_i = \frac{1}{\Delta t^2}(U_{t-\Delta t} - 2U_t + U_{t+\Delta t}), \dot{U}_i = \frac{1}{2\Delta t}(-U_{t-\Delta t} + U_{t+\Delta t}) \tag{4}$$

Error calculations using formulas (4) is of the order (). This method uses the equilibrium condition for the time:

$$M \cdot \ddot{U}_t + C \cdot \dot{U}_t + K \cdot U_t = R_t \tag{5}$$

and therefore it is called an explicit integration method. By inserting (4) in (5) we will obtain equations expressing $U_{t+\Delta t}$ through U_t *and* $U_{t-\Delta t}$. The peculiarity of the method (4) is the need to perform a special procedure for the initial calculation $U_{-\Delta t}$. The main drawback of the central difference that it is conditionally stable.

Consider unconditionally stable integration scheme. Consider unconditionally stable integration scheme. Hubolt method uses the expression [2,3]:

$$\ddot{U}_{t+\Delta t} = \frac{1}{\Delta t^2}(U_{t+\Delta t} - 5U_t + 4U_{t-\Delta t} - U_{t-2\Delta t}),$$

$$\dot{U}_{t+\Delta t} = \frac{1}{6\Delta t}(11U_{t+\Delta t} - 18U_t + 9U_{t-\Delta t} - 2U_{t-2\Delta t})$$

These extrapolation formulas have errors of the order of $(\Delta t)^2$. The first step is to calculate $U_{\Delta t}$ and $U_{2\Delta t}$ with a special opening procedure. Wilson θ-method uses linear acceleration change during the period from t before $t + \theta \cdot \Delta t$. If $\theta = 1$, it is - the usual method of linear acceleration. Unconditional stability is achieved at $\theta \geq 1.37$, $\theta = 1.4$ usually. Wilson's method considers equilibrium equation (1) at a time moment $t + \theta \cdot \Delta t$, where the vector R also varies linearly. The velocity and acceleration are approximated as:

$$\ddot{U}_{t+\theta\Delta t} = \frac{6}{\theta^2 \Delta t^2}(U_{t+\theta\Delta t} - U_t) - \frac{6}{\theta\Delta t}\dot{U}_t - 2\ddot{U}_t,\ \dot{U}_{t+\theta\Delta t} = \frac{3}{\theta\Delta t}(U_{t+\theta\Delta t} - U_t) - 2\dot{U}_t - \frac{\theta\Delta t}{2}\ddot{U}_t,$$

no special initial procedure is required.

In this article a method Newmark [2,3] is used to resolve the fundamental equation (1):

$$\dot{U}_{t+\Delta t} = \dot{U}_t + ((1-\delta)\ddot{U}_t + \delta\ddot{U}_{t+\Delta t})\Delta t, \dot{U}_{t+\Delta t} = U_t - \dot{U}_t\Delta t + ((\frac{1}{2} - \alpha)\ddot{U}_t + \alpha\ddot{U}_{t+\Delta t})\Delta t^2 \tag{6}$$

where α and δ the parameters determining the accuracy and stability of integration. When $\delta = 0.5$ and $\alpha = 1/6$ we get method of linear acceleration. Newmark chooses as the an unconditionally stable scheme - a method of constant average acceleration, for which $\delta = 0.5$ and $\alpha = 1/4$. This scheme does not require any special initial procedure. Expressing $\ddot{U}_{t+\Delta t}$ from the second equation (6) and inserting it in the first equation we obtain equations for calculation of $\ddot{U}_{t+\Delta t}$ and $\dot{U}_{t+\Delta t}$ through unknown displacement vector $U_{t+\Delta t}$. The equilibrium equation (1) for the time moment $t + \Delta t$ looks as follows:

$$M \cdot \ddot{U}_{t+\Delta t} + C \cdot \dot{U}_{t+\Delta t} + K \cdot U_{t+\Delta t} = R_{t+\Delta t} \tag{7}$$

Expressions $\ddot{U}_{t+\Delta t}$ and $\dot{U}_{t+\Delta t}$ are inserted in (7) to determine $U_{t+\Delta t}$ and then it is possible to determine $\ddot{U}_{t+\Delta t}$ and $\dot{U}_{t+\Delta t}$ using (6). Below see the step-by-step description of the method.

2 SCHEME OF ALGORITHM NEWMARK

A. Initial calculations.
1. Formation of mass, damping and stiffness matrixes.
2. Preset of initial values of $U_0, \dot{U}_0, \ddot{U}_0$.
3. Selection of the steps $\Delta t, \alpha, \beta$ and calculation of constants: $\delta \geq 0.5$; $\alpha \geq 0.25 \cdot (0.5+\delta)^2$;

$$a_o = \frac{1}{\alpha \Delta t^2}; \quad a_1 = \frac{\delta}{\alpha \Delta t}; \quad a_2 = \frac{1}{\alpha \Delta t}; \quad a_3 = \frac{1}{2\alpha} - 1; \quad a_4 = \frac{\delta}{\alpha} - 1;$$
$$a_5 = \frac{\Delta t}{2}(\frac{\delta}{\alpha} - 2); \quad a_6 = \Delta t(1-\delta); \quad a_7 = \delta \Delta t.$$

4. Formation of the efficient stiffness matrix $\hat{K}$: $\hat{K} = K + a_0 \cdot M + a_1 \cdot C$
5. The matrix $\hat{K}$ is reduced to the triangular form: $\hat{K} = L \cdot D \cdot L^T$

B. For each time step is calculated the following parameters:
1. Useful load for the time $t + \Delta t$:

$$\hat{R}_{t+\Delta t} = R_{t+\Delta t} + M \cdot (a_0 U_t + a_2 \dot{U}_t + a_3 \ddot{U}_t) + C \cdot (a_1 U_t + a_4 \dot{U}_t + a_5 \ddot{U}_t)$$

2. Displacements in time $t + \Delta t$: $L \cdot D \cdot L^T \cdot U_{t+\Delta t} = \hat{R}_{t+\Delta t}$
3. Accelerations and velocities for the time moment $t + \Delta t$:

$$\ddot{U}_{t+\Delta t} = a_0(U_{t+\Delta t} - U_t) - a_2 \dot{U}_t - a_3 \ddot{U}_t, \; \dot{U}_{t+\Delta t} = \dot{U}_t + a_6 \ddot{U}_t + a_7 \ddot{U}_{t+\Delta t}$$

In [1] there is given a computer program Newmark schemes for dynamic problem. Solution of the problem is given by the example of the mechanism of high-end - class IV mechanism with external prismatic pair.

3 FINITE ELEMENT MODEL OF CLASS IV MECHANISM

Class IV spatial mechanism with external prismatic pair (Fig. 1) are modeled using the finite element. They are connected with each other rotational and translational kinematic pairs so as to allow to take into account any changes in the geometry of the mechanism. They are connected with each other rotational and translational kinematic pair. This is taken into account in the model of the mechanism. Each link mechanisms modeled as finite elements having a the same cross-section along its length.

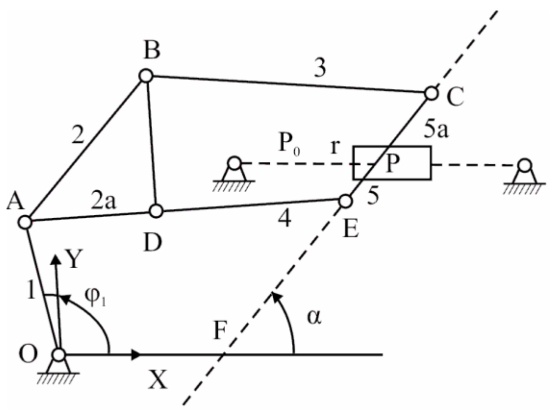

Figure 1: Planar scheme of Class IV mechanism with external prismatic pair.

Kinematics of a Class IV mechanism as opposed to kinematics of the mechanism of class II does not have analytical solutions. Therefore, the kinematics of the mechanism of class 4 can not be directly used to solve the system of differential equations (these equations model the elastic vibrations of the mechanism). Therefore, position, displacement, velocity and acceleration mechanism must be found in advance for each time step [1].

Angular speed and acceleration of the input link, for example, ω_1=100c^{-1} and ε_1 =0c^{-2}. Coordinates rack of crank mechanism: X_o=0.0, Y_o=0.0, Z_o=0.0. The lengths of the links: L_{1-3}=0.17m, L_{5-8} =0.17m, L_{5-9}=0.20m, L_{8-9} =0.258m, L_{13-17} =0.39m, L_{14-18}=0.50m, L_{21-23} =0.10m, L_{22-23}=0.10m, L_{23-24} =0.05m, L_{24-25} =0.05m. All links the mechanism are in planes parallel to the plane of the OXZ. The lengths of axes connecting mechanism links: L_{3-5} =0.02m, L_{8-13} =0.02m, L_{9-14} =0.02m, L_{17-21} =0.02m, L_{18-22} =0.02m. The rotational kinematic pairs parallel to the axis OY (these axes are shown in Figure 2 more to make the model more illustrative). Designation L_{i-j} - is an element or an axis or a part of their length starting at the end of the node "i" i and a node "j". Kinematic analysis showed that when the initial crank angle is zero, the mechanism of four assemblies. For further analysis the 2-nd assembly of the mechanism will be used. To do this assembly kinematic analysis and determine the displacement, velocity and acceleration of nodes in the model [1].

The forces of inertia link mechanism, committing plane-parallel motion can be reduced to the inertia force P_u, applied at the center of mass and the moment of inertia M_u. $P_u = -m \cdot w_s$, $M_u = -J_s \cdot \varepsilon$, where m [kg] – mass link mechanism, w_s [m/sec^{-2}] - acceleration vector of the center of mass, J_s [kgm^2] - the moment of inertia of the link mechanism, ε [c^{-2}] - angular acceleration. With this in mind, each link mechanism must be regarded as one or more finite elements with the same characteristics in order to take into account the inertia motion. Then, in the finite-element model (Fig. 2) the forces and moments of inertia are attached to the nodes - 2 for L_{1-3}, 6 to L_{5-8}, 7 to L_{5-9}, 10 to L_{8-9}, 15 to L_{13-17}, 16 to L_{14-18}, 23 - slider (L21-23, L22-23, L23-24, L24-25) and 4, 11, 12, 19, 20 for the axes L3-5, L8-13, L9-14, L17-21, L18-22. Cross-section of all links and axles - rectangular: the width of 0.01m and 0.02m in height. The nodes of the model is also act weight.

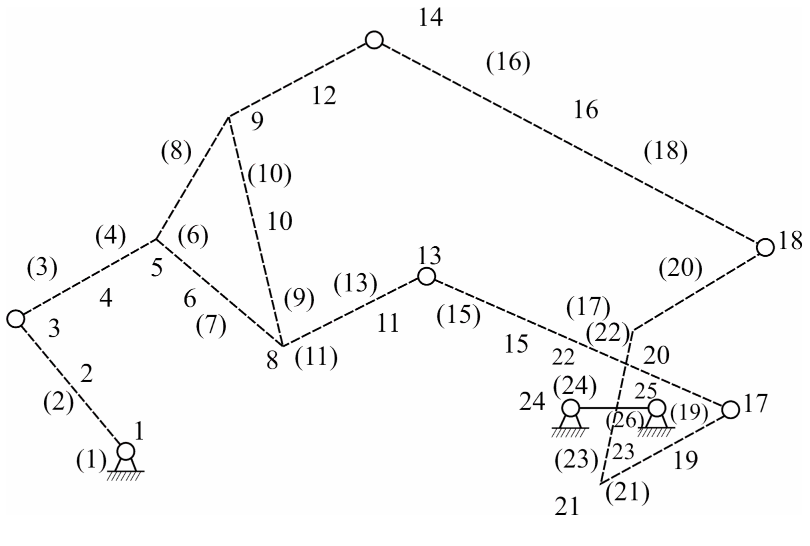

Figure 2: Finite Element Model of Class IV spatial mechanism with external prismatic pair.

The finite element model consists of twenty-six finite elements, connected by twenty-five knots. Elasticity characteristics are taken as equal for all links and axes: elasticity modulus is

$E = 2 \cdot 10^{11}$ [Pa], the Poisson ratio is $\mu = 0.3$, specific weight of material is $\rho = 7850$ [kg/m^3] and corresponds to elasticity characteristics of steel. Time step is equal to $\Delta t = 0.0005235$ second. Finite element model and the corresponding node numbers: 1: nodes 1 and 2, 2: nodes 2 and 3, 3: nodes 3 and 4, 4: nodes 4 and 5, 5: nodes 5 and 6, 6: nodes 5 and 7, 7: nodes 6 and 8, 8: nodes 7 and 9, 9: nodes 8 and 10, 10: nodes 9 and 10, 11: nodes 8 and 11, 12: nodes 9 and 12, 13: nodes 11 and 13, 14: nodes 12 and 14, 15: nodes 13 and 15,BFE 16: nodes 14 and 16, 17: nodes 15 and 17, 18: nodes 16 and 18, 19: nodes 17 and 19, 20: nodes 18 and 20, 21: nodes 19 and 21, 22: nodes 20 and 22, 23: nodes 21 and 23, 24: nodes 22 and 23, 25: nodes 23 and 24, 26: nodes 23 and 25.

Figure 3 shows the graphs of linear and angular elastic vibration nodes 17,18,19 for the model of the mechanism relative to the positions that they would occupy, if we consider the model of the mechanism with rigid links.

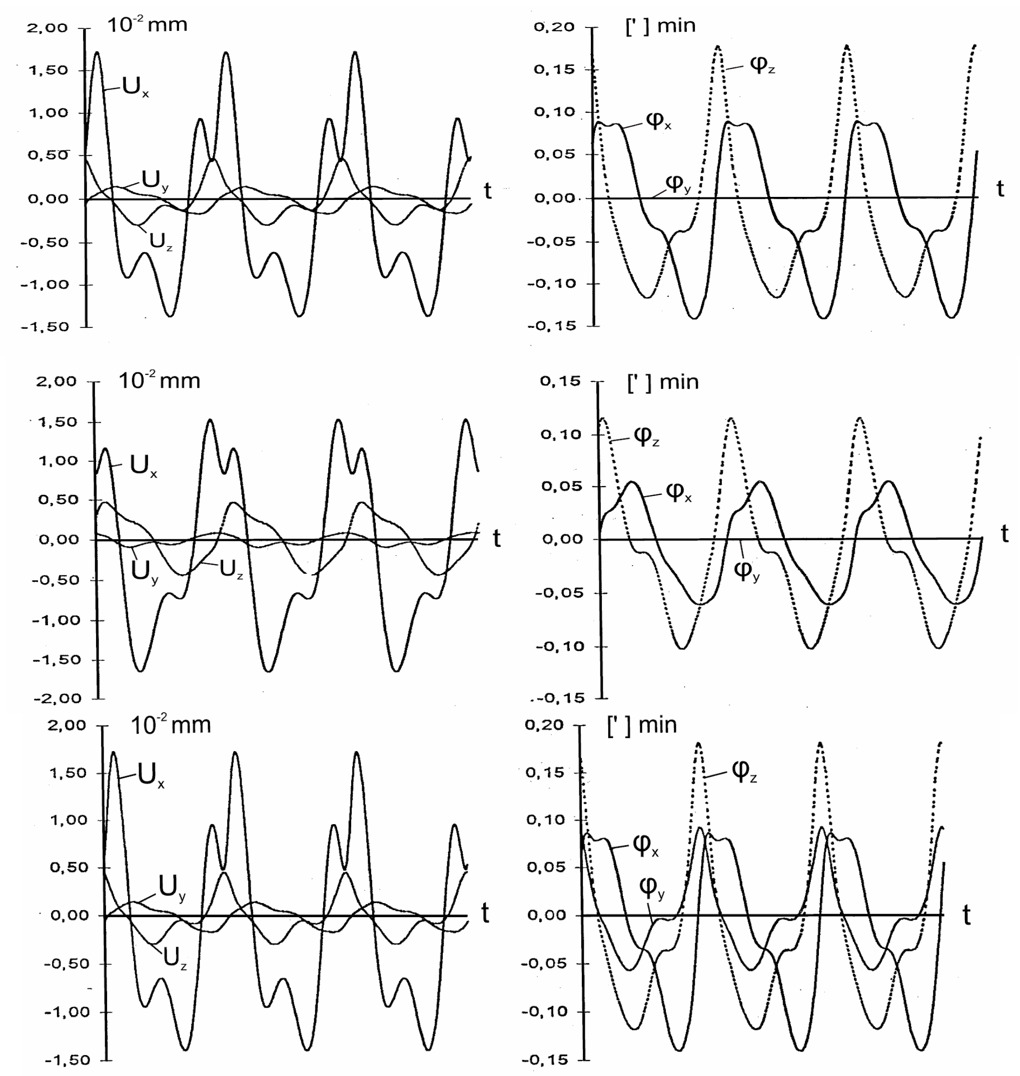

Figure 3: Global linear and angular elastic vibrations in 17,18,19 nodes in the model.

4 CONCLUSIONS

Was created a dynamic model based spatial topology. Algorithm and a computer program for calculating the elastic vibrations have been developed. To calculate the dynamic model of the following tasks: a) the calculation of the "primary" position of the mechanism and its kinematic parameters, and b) the calculation of the elastic deformation mechanism (realized numerical scheme Newmark). The task of a) is ancillary to the problem b).

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