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RANKING OF UNCERTAIN MODELS IN PRESENCE OF ERRONEOUS OBJECTIVES IN INVERSE ACOUSTIC PROBLEMS

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Abstract. *The inversion of acoustic problems in unbounded domains rests on measurements of the acoustic pressures radiated (the objective) and a model of propagation which, in general, is an a priori. When dealing with an exact objective, we have shown in various configurations in the automotive world that the ranking of different models can be obtained. Is this possible with erroneous data (objective) due to non-structured uncertainty and when the various models have a structured uncertainty?*

The way to adapt an erroneous model with a structured uncertainty could contribute to answering the question. Indeed, we have noticed recently that the adaptation of such a model behaves like a filter, extracting the quasi-exact acoustic pressure from the perturbed one. After adaptation, a distance remains between the adapted model and the exact pressure. It is expected that the less adequate the model, the greater this distance will be, as the extraction would more difficult since the model is not in the right form (i.e. does not describe the proper physical operator of propagation). The paper will show that this is indeed the case, completing the first step toward the ranking of uncertain models with a perturbed objective. Following that, how can the residual angle be accessed in the real world (where the exact pressure field is unknown) and if it is impossible, how to proceed? The answer brought here will conclude with a possible means of achieving the ranking sought.

The present work describes numerical simulations carried out on simple examples to support the assertion.

1 QUESTIONS TO BE ANSWERED

A large number of inverse problems in acoustics consist in searching for the source driving signals or their vibratory velocities from given acoustic pressures most often measured at an microphone array [1,2] called objective. Transfer functions between sources and microphones, also said radiation model, help in reaching the objective. When the sources the control/velocity of which are sought are at the origin of the objective to be reached (holography [3,4], beam-forming [5]), it is possible to obtain their actual velocities if perfect objective and radiation models are available, and also if the computer precision is infinite since most often the problems under study are ill-conditioned. When the model is erroneous, it cannot radiate the perfect objective and the geometrical interpretation of these inverse problems [6,7] shows that this inability manifests itself by a distance or an angle ψ between the vector of pressures at the microphones (of dimension M, the number of sensors) and the set made up of all the acoustic fields the model can generate. The pressure vector belongs to $\mathbb{C}^M$ and the set in question or hyper-plan is a restriction in $\mathbb{C}^M$ (for general aspects see [8-11]). The greater the angle ψ, the less adequate the radiation model, given an exact objective. This assertion leads the way to rank various models [12].

However, not knowing a model may arise from two different reasons : its forms and the values of its parameters. The first uncertainty, said epistemic, concerns the type of source radiation (simple, dipolar, ...) and its environment [13] (confined medium with n walls, ...); the second uncertainty, said structured, concerns the sound speed, the impedance of boundary conditions, source and microphone locations [14-17]...

In presence of a model of right form but with a structured uncertainty, we have shown how to identify the values of the parameters (a boundary condition in [15], the sound speed in [18,19]) thanks to the exact objective. Indeed the minimization of (positive) angle ψ by letting the parameter value vary will result finally in $\psi_{\min}$, i.e. zero when the parameter has its exact value since it will be then possible for the exact model to generate the exact given objective. Now when there is a choice between various models, all with their structured uncertainty, is it still possible to establish a hierarchy?

And, last step, is this ranking accessible when, added to the uncertain model, the objective is also erroneous for measurement reasons (unstructured uncertainty)? Here the angle between objective and model is $\tilde{\psi}$ and its minimization leading to $\tilde{\psi}_{\min}$ to make the model approach the given erroneous objective - at the end of the process the model is said to be adapted - is accompanied by a residual distance described by an angle ψ_{res} between the exact objective and the adapted model. This latter angle ought to be used for ranking the models but it is not accessible since the exact pressure is unknown. How can this drawback be overcome?

The present work brings answers to these questions through numerical experiments. It can be read as a deepening of [12].

2 ENVISAGED EXACT (NOMINAL) CONFIGURATION AND THE VARIOUS MODELS

The present work is at the exploratory stage and not at that of generalization. For the time being, an elementary, particular and arbitrary configuration is under study. However we should say that it has been chosen influenced by recent works carried out in aero-acoustic and moving source problems [20,21].

2.1 Exact (or nominal or of reference) configuration

Let us consider an exact (or nominal or of reference) configuration with two sources - a monopole and a dipole - and five microphones in the half-space $y > 0$ bounded by a rigid wall (Figure 1). The reflecting boundary induces image sources. Care must be taken regarding the phase of the dipole image with respect to the true dipole according to its orientation.

The source are located at $(0.,0.8,0.4)m$ and $(0.,1.3,0.7)m$; and microphones at $(0.,2.5,0.1)m$, $(0.,2.5,0.3)m$, $(0.,2.5,0.7)m$, $(0.,2.5,1.2)m$, $(0.,3.1,0.2)m$.

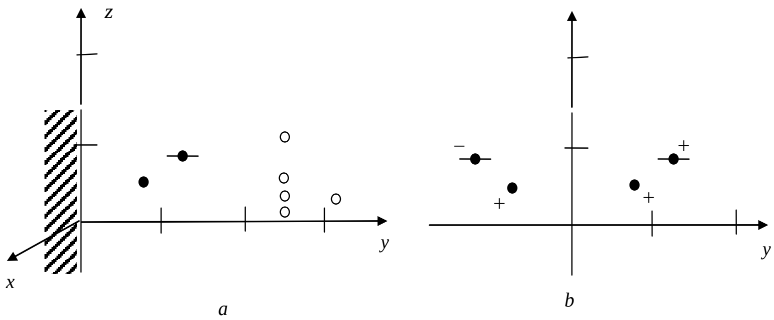

Figure 1: a) a monopole and a dipole oriented along Oy axis (black circles) radiate an acoustic pressure at the five microphones (white circles) in the half-space $y > 0$; b) the sources, their images and relative phases.

The radiated pressure at point $\mathbf{x}$ is :

$$p(\mathbf{x}) = -i\rho\omega\left(\frac{e^{-ikr_m}}{4\pi r_m} + \frac{e^{-ikr_m^{'}}}{4\pi r_m^{'}}\right)q_m - k^2\rho c d q_d\left(\cos\vartheta\left(1 - \frac{i}{kr_d}\right)\frac{e^{-ikr_d}}{4\pi r_d} - \cos\vartheta^{'}\left(1 - \frac{i}{kr_d^{'}}\right)\frac{e^{-ikr_d^{'}}}{4\pi r_d^{'}}\right) \quad (1)$$

with ρ the air density ($1.2kg.m^{-3}$); ω the circular frequency (corresponding to $500Hz$); k the wavenumber; c the sound speed ($343.0ms^{-1}$ for its nominal value); d the distance between the poles of the dipole ($5cm$); r_m the distance between the monopole and point $\mathbf{x}$; $r_m^{'}$ that between the monopole image and point $\mathbf{x}$; r_d that between the dipole and point $\mathbf{x}$; $r_d^{'}$ that between the dipole image and point $\mathbf{x}$; θ the angle between the dipole axis and vector distance $\boldsymbol{r}_d$; $\theta^{'}$ the angle between the dipole image and vector distance $\boldsymbol{r}_d^{'}$; q_m the monopole source strength; q_d the dipole source strength [22].

Presently angles θ are small (and *a fortiori* angles $\theta^{'}$) resulting in cosine almost unity and, as a consequence, with a dipole radiation of similar form as that of a monopole seen from the microphones. Moreover distances $r_d^{'}$ being larger than distances r_d, the dipole and its image will radiate almost like a simple monopole. To a lesser extent the same could be said of the monopole and its image.

2.2 Various models to be ranked

Often the nominal model (i.e., the true model in the real world) is not known by those in charge of the inverse problem, and what is sought is to invent some possible models and to rank them in order to choose the best. Here 5 models are envisaged: 2 monopoles without boundary, 2 monopoles with boundary, 2 dipoles without and with boundary, 1 monopole and 1 dipole without boundary. Figure 2 shows the various configurations without of with the image sources according to the boundary not or taken into consideration.

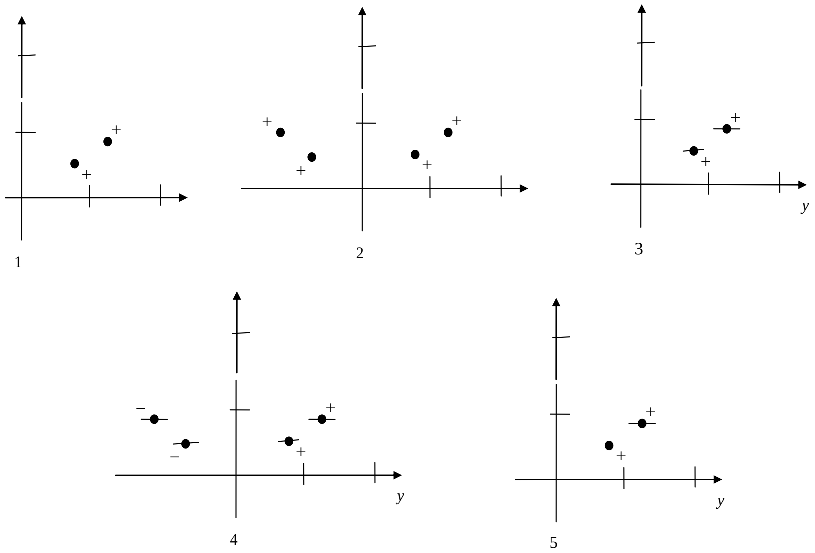

Figure 2: the five radiation models to be ranked.

The pressure fields arising from these configurations are easily deduced from Eq. (1) by paying attention to the relative phases of the dipoles and their images versus their orientation. For "horizontal dipoles", they are out of phase.

3 RANKING OF MODELS OF VARIOUS FORMS IN PRESENCE OF EXACT OBJECTIVES

The solution of over-determined inverse problems results from the algorithm $\min_{\boldsymbol{q}} \|\boldsymbol{Eq}-\boldsymbol{p}\|_2$ where $\boldsymbol{q}$ is the vector made up of the source strengths and called control vector, $\boldsymbol{p}$ the vector made up of the pressures at the microphone array, $\boldsymbol{E}$ the model matrix built from the transfer functions. The solution so obtained is $\boldsymbol{q}^{opt}=\boldsymbol{E}^{\dagger}\boldsymbol{p}$ with $\boldsymbol{E}^{\dagger}$ the pseudo-inverse matrix of $\boldsymbol{E}$. The projection $\hat{\boldsymbol{p}}$ of $\boldsymbol{p}$ on the hyper-plane of the pressures, the model E can radiate - hyperplane called model plane and named also $\boldsymbol{E}$ by taking some liberty in the vocabulary - is $\hat{\boldsymbol{p}}=\boldsymbol{Eq}^{opt}=\boldsymbol{EE}^{\dagger}\boldsymbol{p}$ and therefore angle ψ between the given pressure vector and the model plane is

$$\psi = ArcCos\frac{\|\boldsymbol{EE}^{\dagger}\boldsymbol{p}\|}{\|\boldsymbol{p}\|}, \tag{2}$$

The norms are taken in the L_2 sense. If ψ were not defined between $\boldsymbol{p}$ and its projection $\hat{\boldsymbol{p}}$, it would be obtained through the generalized cosine between two complex vectors.

When tackling the direct problem with various source controls calculated in a certain vicinity of a given control vector, and with the exact model called E_0, various objectives i.e., the pressure fields to be reached, are obtained. Now looking for the angle ψ between various models and the various objectives originating from E_0, it appears the graphs of Figure 3 describing $\psi_j(itest) = ArcCos\dfrac{\left\|\boldsymbol{E}_j\boldsymbol{E}_j^{\dagger}\boldsymbol{p}_{itest}\right\|}{\left\|\boldsymbol{p}_{itest}\right\|}$ with the models indexed by j and the objectives indexed by $itest$.

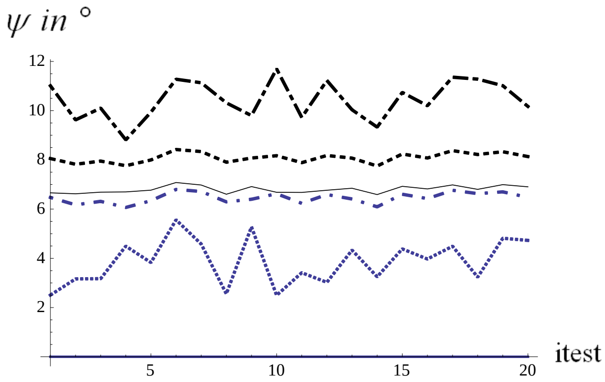

Figure 3: ranking of five models without errors in their parameters; E_0 continuous line merged with the itest-axis; E_1 dot/ dash/dot; E_2 dotted line; E_3 dashed line; E_4 short line/long line/short line; E_5 thin continuous line.

Of course, the nominal model E_0 having radiated the objectives in the direct problem, they belong to plane $\boldsymbol{E}_0$. In the inverse problem, the optimal source strengths are the exact ones (presently the problems are well-conditioned) and the objectives are perfectly reached leading to angle ψ_0 null whatever the test. Models E_1 and E_5 have similar behavior (similar angles) because the dipoles with their orientation behave, seen from the microphones, almost like monopoles (cf. remark in subsection 2.1). The worst of the models is E_4 due to the dipole in the place of the monopole the image of which is out of phase. Model E_3 suffers from the lack of the rigid boundary. And model E_2 is the better model because the dipole is similar here to a monopole and because the opposite phase of one of the image of the monopoles has little influence taking into account the distance of the image to the microphones. Figure 3 provides a clear hierarchy $E_0 > E_2 > E_1 > E_5 > E_3 > E_4$ when understanding sign > as "better than". By a game of logic deductions based on the physics of the radiations, it is possible in certain cases to finally discover what is the nominal model E_0 that was unknown.

Most often, some parameters of the models are of values not well-known. In these conditions, how can a hierarchy of models of various forms be made?

4 RANKING OF MODELS OF VARIOUS FORMS IN PRESENCE OF EXACT OBJECTIVES

Let us consider the nominal model E_0 erroneous due to the sound speed that is not well known. We are therefore in presence of model $\tilde{E}_0(\tilde{c})$ (i.e., erroneous model due to erroneous sound speed). As an objective $\boldsymbol{p}$ arises from the exact model E_0 with the exact sound speed, this pressure-objective does not belong to the set of the pressures $\tilde{E}_0(\tilde{c})$ can generate and therefore angle ψ (more precisely written ψ_0 since associated with model indexed 0) becomes $\psi(\tilde{c})$. Adapting the model or identifying of the value of its parameter c consists in finding the solution of the algorithm

$$\min_{\tilde{c}} \psi\left(\tilde{c}\right) = \min_{\tilde{c}} ArcCos \frac{\left\| \tilde{\boldsymbol{E}}_0\left(\tilde{c}\right) \tilde{\boldsymbol{E}}_0^{\dagger}\left(\tilde{c}\right) \boldsymbol{p} \right\|}{\left\| \boldsymbol{p} \right\|} \tag{3}$$

that will lead to value $\tilde{c}_m$, to angle $\psi\left(\tilde{c}_m\right) = \psi_{\min}$ and to the adapted model $\tilde{E}_{0m}$. However, when dealing with model $\tilde{E}_0(\tilde{c})$, the result will be $\tilde{c}_m = c_{exact}$, $\psi_{\min} = 0$ and $\tilde{E}_{0m} = E_0$. But this will not be the case for the other models. Indeed Figure 4 shows graphs $\psi_{\min,j}(itest) = ArcCos \frac{\left\| \tilde{\boldsymbol{E}}_{jm,itest} \tilde{\boldsymbol{E}}_{jm,itest}{}^{\dagger} \boldsymbol{p}_{itest} \right\|}{\left\| \boldsymbol{p}_{itest} \right\|}$ after having adapted the various models.

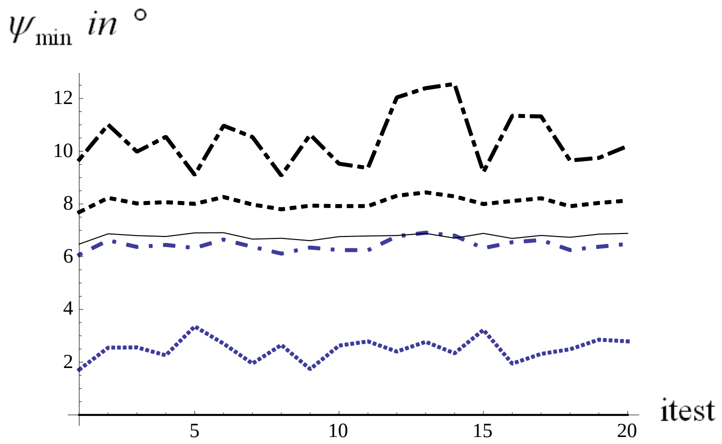

Figure 4: ranking of five adapted models; case of exact objectives; $\tilde{E}_0$ continuous line merged with the itest-axis; $\tilde{E}_1$ dot/dash/dot; $\tilde{E}_2$ dotted line; $\tilde{E}_3$ dashed line; $\tilde{E}_4$ short line/long line/short line; $\tilde{E}_5$ thin continuous line.

It is clear that the hierarchy $E_0 > E_2 > E_1 > E_5 > E_3 > E_4$ is kept leading us to observe that not knowing the parameter value does not prevent establishing a hierarchy between models of various forms. We are not at the stage where this conclusion can be taken for granted whatever the configuration, but at least the result is encouraging for our end in view.

Just to recall, the various source strengths at the origin of the various objectives are taken randomly around a given source strength vector.

The last stage of today's work considers not only the case where the various models have a structured uncertainty but also where the objectives are perturbed by error of measurements, an inherent unstructured error.

5 RANKING OF MODELS OF VARIOUS FORMS IN PRESENCE OF ERRONEOUS OBJECTIVES

In presence of various models with uncertain parameters and in presence of erroneous pressures, the model adaptation can be carried out only through the minimization of the angle $\tilde{\psi}$ between the perturbed objective and the model plane to obtain finally $\tilde{\psi}_{\min}$.

Recently [18,19], the work on a simple configuration with only simple sources showed that the hyper-plane of the adapted nominal model was very near the exact pressure vector and stayed away from the perturbed pressure vector. In other words, if the residual angle between the exact objective and the adapted model is written ψ_{res}, then the inequality $\psi_{res} < \tilde{\psi}_{\min}$ always holds true. Moreover this angle, not accessible since the exact pressure field is not known, can be approximated in a statistical sense [19]. What will occur with a model of wrong form and with dipoles? Will the angle ψ_{res} allow us to rank the models? If yes, how is it possible to find an approximation of ψ_{res}? It so happens that the observed results will simplify these questions.

Dealing with nominal model $\tilde{E}_0$, the angle between an erroneous objective and the model is $\tilde{\psi}_0(\tilde{c})$, the minimal value of which results from

$$\min_{\tilde{c}} \psi_0\left(\tilde{c}\right) = \min_{\tilde{c}} ArcCos \frac{\left\|\tilde{\boldsymbol{E}}_0\left(\tilde{c}\right)\tilde{\boldsymbol{E}}_0^{\dagger}\left(\tilde{c}\right)\tilde{\boldsymbol{p}}\right\|}{\left\|\tilde{\boldsymbol{p}}\right\|} \tag{4}$$

leading to $\tilde{c}_m$ and to the adapted model $\tilde{E}_{0m}$ whose distance from the exact objective $\boldsymbol{p}$ is given by $\psi_{0,res}$ such that

$$\psi_{0,res} = ArcCos \frac{\left\|\tilde{\boldsymbol{E}}_{0m}\left(\tilde{c}_m\right)\tilde{\boldsymbol{E}}_{0m}^{\dagger}\left(\tilde{c}_m\right)\boldsymbol{p}\right\|}{\left\|\boldsymbol{p}\right\|} \tag{5}$$

In this section, the exact pressure originating from a given source strength vector is calculated, and perturbed pressure fields are obtained around this vector in a random way (with a maximum relative pressure field error of 8% corresponding to a signal to noise ratio of $22dB$ if the noise arises only from the measurements at the microphone array). Figure 5 shows graphs obtained after model adaptation of $\tilde{\psi}_{\min,j}(itest) = ArcCos \frac{\left\|\tilde{\boldsymbol{E}}_{jm,itest}\tilde{\boldsymbol{E}}_{jm,itest}{}^{\dagger}\tilde{\boldsymbol{p}}_{itest}\right\|}{\left\|\tilde{\boldsymbol{p}}_{itest}\right\|}$ (in black) and $\psi_{res,j}(itest) = ArcCos \frac{\left\|\tilde{\boldsymbol{E}}_{jm,itest}\tilde{\boldsymbol{E}}_{jm,itest}{}^{\dagger}\boldsymbol{p}\right\|}{\left\|\boldsymbol{p}\right\|}$ (in red). There are as many tests as perturbed pressure fields.

In Figure 5 it is first noted that, for a model of a given form, the relation $\psi_{res} < \tilde{\psi}_{\min}$ does not always hold true, except roughly for the nominal model. Therefore it could occur than the adaptation may lead to a model less distant from the erroneous objective than from the exact objective. However, for the nominal model $\tilde{E}_0$, the results confirm in general what had been observed (i.e. $\psi_{res} < \tilde{\psi}_{\min}$) with simple sources [19] where the reason of being is intuitively understood. On the contrary, the fact that $\tilde{\psi}_{\min}$ is around ψ_{res} and the fact that ψ_{res} seems to be almost constant whatever the test (except for $\tilde{E}_0$), are not understood yet. Let us present a first intuitive idea: when considering the error in the model consisting of both an epistemic and a structured uncertainty, it is probable that to dispose of the first to reach the exact objective is more difficult than to dispose of the second. Once the first problem is solved, the effect of the second lever is of the second order.

Graphs associated to $\tilde{E}_1, \tilde{E}_3, \tilde{E}_4$ are quasi-parallel. Graphs $\tilde{E}_1, \tilde{E}_5$ are almost merged. Each of these remarks receives an explanation after having noticed that the dipoles seen by the microphones were almost monopoles, and that the dipole image contributes little to the pressure field.

For models $\tilde{E}_0, \tilde{E}_2$ the hierarchy would be clearer with ψ_{res} than with $\tilde{\psi}_{\min}$ but $\tilde{\psi}_{\min}$ is accessible while ψ_{res} is not. Nevertheless with ψ_{res} or with $\tilde{\psi}_{\min}$, the hierarchy $E_0 > E_2 > E_1 > E_5 > E_3 > E_4$ is still kept.

To respect the maximum of 10 pages for this paper, more results in the same vein obtained with other configurations will be shown during the oral presentation.

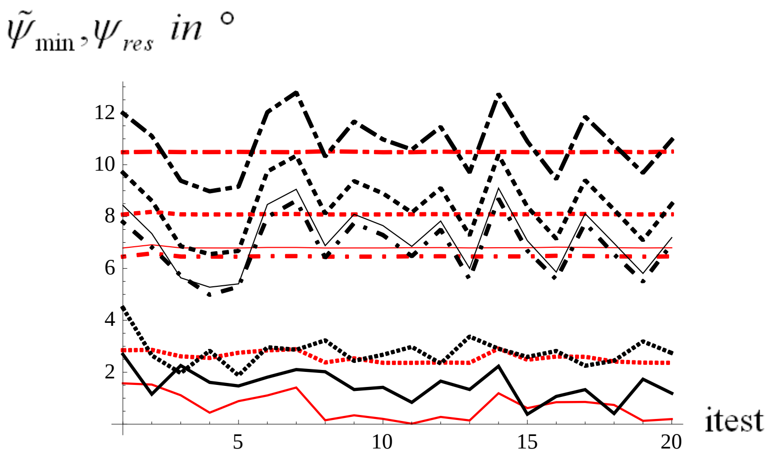

Figure 5: ranking of five adapted models; case of perturbed objectives; in black the angles $\tilde{\psi}_{\min}$; in red the angles ψ_{res}. $\tilde{E}_0$ continuous line; $\tilde{E}_1$ dot/dash/dot; $\tilde{E}_2$ dotted line; $\tilde{E}_3$ dashed line; $\tilde{E}_4$ short line/long line/short line; $\tilde{E}_5$ thin continuous line.

6 CONCLUSION

The present work concludes with the possibility (or more rigorously with the non-impossibility) of ranking various models even when they have structured uncertainty and even in presence of an erroneous objective. This information is new. The hierarchy rests on an angle given by the geometrical interpretation of the resolution of inverse problems in the L_2 sense. The main idea is that the perturbed objective is essentially made up of the exact pressure due to the nominal model, presently a dipole, a monopole and a rigid boundary. The less representative of the true configuration is the model, the greater is the angle between model and objective. Furthermore, the structured uncertainty on models, thus not acting on their form, seems to be of the second order relative to the epistemic error.

The extension to more complicated configurations with a greater number of sources and microphones seems to be limited by the calculation time required to minimize angles ψ or $\tilde{\psi}$. For the 20 tests of Figure 5, the calculation time is quite short, but with 3 sources and 7 microphones, still with 5 models of various forms, almost 3 hours were necessary to achieve the hierarchy through 5 tests (it is true that one test may be sufficient).

The configuration dealt with here leads to a well-conditioned problem. In the case of ill-conditioned problems which need regularization, it has been shown that angles ψ or $\tilde{\psi}$ were still accessible [11] and we do not expect new problems on this aspect (of course this has to be confirmed).

As mentioned in the text, the behaviour of $\tilde{\psi}_{\min}$ and ψ_{res} is not quite clear.

Finally, the very interesting and, from our point of view, important observations brought here are still at the narrative level. They remain to be demonstrated, if this is possible.

REFERENCES

[1] S.F Wu, Methods for reconstructing acoustic quantities based on acoustic pressure measurements, *Journal of the Acoustical Society of America* **124**(**5**), 2680-2697, 2008.

[2] P.A. Nelson, A review of some inverse problems for acoustics, *International Journal of Acoustics and Vibration* **6**(**3**), 118-134, 2001.

[3] J.D. Maynard, E.G. Williams, Y. Lee, Nearfield acoustic holography; I. Theory of generalized and the development of NAH, *Journal of the Acoustical Society of America* **78**(**4**), 1395-1412, 1985

[4] X. Zhao, S.F. Wu, Reconstruction of vibro-acoustic fields using hybrid nearfield acoustic holography, *Journal of Sound and Vibration* **282**, 1183-1199, 2005.

[5] T.F. Brooks, W.M. Humphreys, A deconvolution approach for the mapping of acoustic sources (DAMAS) determined from phase microphone arrays, *Journal of Sound and Vibration* **294** , 856-879, 2006.

[6] V. Martin, C. Cariou, Improvement of active attenuation by modifying the primary field, in : Proceedings of the *Congress Active 97*, Budapest, Hungary, 1997.

[7] V. Martin, T. Le Bourdon, J.R.Arruda, Geometrical interpretation of acoustic holography : adaptation of the propagator and minimum quality guaranteed in the presence of errors, *Journal of Sound and Vibration* **331**, 3493-3508, 2012.

[8] A. Kirsh, *An introduction to the mathematical theory of inverse problems*, Springer, 1996.

[9] P.C.Hansen, *Rank-deficient and discrete ill-posed problems; numerical aspects of linear inversion*, Siam, 1998

[10] E.G. Williams, *Fourier Acoustics, Sound radiation and nearfield acoustical holography*, Academic Press, 1999.

[11] V. Martin, The fundamental elements in certain inverse acoustic problems : their roles and interactions, Keynote Lecture in : Proceedings of the *1st International Symposium on Uncertainty Quantification and Stochastic Modelling*, 26th Feb-2nd March 2012, Maresias, Sao Sabastiao, SP, Brazil, 2012.(To be published in the *Journal of the Brazilian Society of Mechanical and Science Engineering*, 2013)

[12] T. Le Bourdon, C. Picard, V. Martin, A parameter to classify different propagators for identifying source strengths, in : Proceedings of *Novem09*, Oxford, United Kingdom, 2009.

[13] M.D. Collins, W.A.Kuperman, Environmental focusing and source localization, *Journal of the Acoustical Society of America* **90**(**3**),1410-1422, 1991.

[14] P.Castellini, M. Martarelli, Acoustic beamforming : analysis of uncertainty and metrological performances, *Mechanical Systems and Signal Processing*, **22**, 672-692, 2008.

[15] V. Martin, T. Le Bourdon, A.M. Pasqual, Numerical simulation of acoustic holography with propagator adaptation: application to a 3D disc, *Journal of Sound and Vibration* **330**, 4233-4249, 2011.

[16] L.P.R. de Oliveira, M.M. da Silva, J.A.P. Sanchez, L.A.M. Gonçalves, Sound quality scattering due to vibro-acoustic model variability in : Proceedings of the *1st International Symposium on Uncertainty Quantification and Stochastic Modelling*, 26th Feb-2nd March 2012, Maresias, Sao Sabastiao, SP, Brazil, 2012.

[17] M.D.Redel-Macias, F.Fernandez-Navarro, P.A. Gutiérrez, A.J. Cubero-Atienza, C. Hervas-Martinez, Improvement of accuracy in a sound synthesis method using evolutionary product unit networks, *Expert Systems with Applications* **40**, 1477-1483, 2013.

[18] V. Martin, F. Cohen-Tenoudji, Identification of acoustic sources with uncertain data, in : Proceedings of the *International Congress of Acoustics*, June 2013, Montreal, Canada, 2013.

[19] V. Martin, Qualification of source velocities/controls in inverse acoustic problems with model and objective uncertainties via the geometrical interpretation, article of 27 pages *to be submitted*.

[20] P.A.G. Zavala, K. Janssens, J.R.F.Arruda, Dipole source identification on subsonic airflow through structures, in : Proceedings *ISMA* 2012, Leuven, Belgium, 2012.

[21] P.A.G. Zavala, F.Bueno, J.R.F.Arruda, V. Martin, 3D Source localization on a truck pass-by noise test using generalized inverse beamforming, in : Proceedings of the *XVth International Symposium on Dynamic Problems of Mechanics*, M.A. Savi (editor), February 17-22, 2013, Buzios, RJ, Brazil, 2013.

[22] P.M.Morse, K.U. Ingard, *Theoretical acoustics, Section 7.1*, MacGraw-Hill, 1968.

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