

## A REVIEW OF LOAD MODELS FOR THE PREDICTION OF FOOTFALL-INDUCED VIBRATION IN STRUCTURES

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**Abstract.** *Crowd induced dynamic loading in large structures, such as gymnasiums or stadium, is usually modelled as a series of harmonic loads which are defined in terms of their Fourier coefficients. Different values of these coefficients that were obtained from full scale measurements can be found in codes. Recently, an alternative has been proposed, based on random generation of load time histories that take into account phase lag among individuals inside the crowd. This paper presents the testing done on a structure designed to be a gymnasium. Two series of dynamic test were performed on the gym slab. For the first test an electrodynamic shaker was placed at several locations and during the second one people located inside a marked area bounced and jumped guided by different metronome rates. A finite element model (FEM) is presented and a comparison of numerically predicted and experimentally observed vibration modes and frequencies has been used to assess its validity. The second group of measurements will be compared with predictions made using the FEM model and three alternatives for crowd induced load modelling.*

## 1 INTRODUCTION

The interest in modelling human induced loads has been recurrent since the first accidents on suspension bridges in the nineteenth century like Broughton (1831) in the U.K. or Angers (1850) in France. Accidents and the use of new materials allowing the design of slender structures, the simultaneous interest in the structural serviceability performance such as the one that occurred during the opening ceremony of the London Millenium Footbridge (10 June 2000) made it mandatory to carry out an in-depth analysis of the equivalent actions to be used in the design of structures.

In the last years, interesting contributions are due to Ellis and Ji [1] and Sim [2], but also European research projects [3, 4]. The publication of SCI Guide P354 [5], which incorporates new results such as the Fourier coefficients representing the crowd activities (point 3.1.3) is one of the starting points of the research contained in this paper.

An alternative has been proposed by Sim [6] who has worked on the statistical characterization of phase lag among individuals inside a crowd, based on test results. Thus, the load depends on random factors and is no longer the addition of pure harmonic loads.

In this paper we present a series of tests done on a structure designed to be a gymnasium. Test results will be compared with predictions based on the two abovementioned load modelling alternatives and a new methodology for modelling jumping loads proposed due to the coherence lack between experimental data at high frequency range.

## 2 TEST STRUCTURE

The test we shall present was performed on a stand-alone building, designed to be a gymnasium, which belongs to the School of Industrial Engineering of the Technical University of Madrid. The structure was finished in the 1950's and, unfortunately, the original as-built drawings are not available.

The photograph displayed in Figure 1 a general view of the structure, which is neither regular nor symmetric. Many structural details are responsible for that. For example, a large window or an access stair that seems to constrain horizontal displacements at the second bay.

Interior walls are set to split the building in two parts. The big windows on the left belong to the gym, while the small windows on the right, just above the access stairs, belong to the dressing rooms, toilets and other small rooms distributed in two floors. Tests and measurements were only performed on the left hand side.



Figure 1: External photograph of the structure

## 3 TEST LAYOUT AND MEASUREMENT EQUIPMENT

Two series of dynamic tests were performed on the gym slab. In the first one, an electrodynamic shaker was placed at different locations while accelerations and displacements were recorded at selected points of the structure (Figure a). In the second one, groups of people

placed at inside a reduced area bounced and jumped guided by a metronome. In the latter tests, different combinations of crowd number and metronome frequency were considered (Figure b).

The electrodynamic shaker APS 113HF is used together with an amplifier APS 145 and a signal generator Hameg HM 8150 that provides the suitable commands. The latter are harmonic signals displaying variable frequencies (chirp), in the range of interest (between 1 and 40 Hz), 100 s long. During the second test, the jumpers were distributed in several groups of three individuals. Combining these groups, tests with different numbers of people were carried out: 6, 12, 18, 24 and 30 people, i.e.: densities between 0.5 and 2.5 people/m<sup>2</sup>. All jumpers were weighted before starting the tests in order to assess the total load in each set of jumps. The participants had to jump guided by a metronome under three main frequencies (1.5, 2 y 2.5 Hz).



Figure 2: Two photographs taken while performing the tests.

#### 4 STRUCTURAL FE MODEL

A finite element (FE) model of the whole building has been set using ANSYS. Figure displays different views of it. The plots on the left display the structure without walls in order to allow the visualizing all the structural components: the structural columns, and the beams supporting the slab and bound the bays. The roof slab is structurally similar to the floor one.

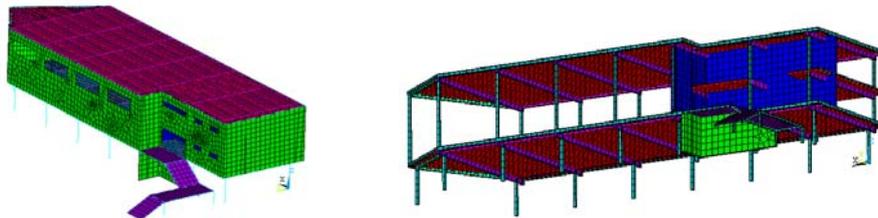


Figure 3: Views of the finite element model.

The model has been tuned based on test results: vibration frequencies and frequency response functions among the shaker sensor and those on the slab. Flexural vibration modes associated with vertical motions of both slabs are presented in Table 1 for both the model and the identification through SSI. Relative errors are below 4% in all cases.

Mode	Experimental Freq. (Hz)	Numerical Freq. (Hz)	Error (%)
1st Floor bending	5.74	5.69	0.9
2nd Floor bending	6.75	6.60	2.2
3rd Floor bending	8.52	8.20	3.8
4th Floor bending	10.85	10.77	0.7

Table 1: Flexural eigen frequencies comparison

## 5 DYNAMIC LOADING AND RESULT

The loading induced by the crowd has been modeled considering two existing alternative methodology

The first one may be found in SCI P354 guide [5], where the acting load follows the procedure explained by Ellis [1]:

$$F(t) = W \left\{ 1 + \sum_{j=1}^3 \alpha_j \sin(\omega_j t + \phi_j) \right\} \quad (1)$$

where  $W$  is the weight of the jumpers,  $\omega_j$  is  $j$  times the jumping frequency,  $\phi_j$  is the phase lag of the  $j$ th term and  $\alpha_j$  is the Fourier coefficient (or dynamic load factor) of the  $j$ th term.  $\alpha_j$  and  $\phi_j$  values of the  $j$ th term are shown in Table 2 ( $p$  is the number of participants)

$j$	$\alpha_j$	$\phi_j$
1	$1.61 \cdot p^{-0.082}$	$\pi/6$
2	$0.94 \cdot p^{-0.24}$	$-\pi/6$
3	$0.44 \cdot p^{-0.31}$	$-\pi/2$

Table 2:  $\alpha_j$  and  $\phi_j$  values of the  $j$ th term

The second methodology is based on the works of Sim [6], which considers randomness in the phase lag among individuals in the crowd. The load contribution associated with the  $i$ th jump of each individual has the following form:

$$F(t) = W k_{p,i} \cos^2 \left( \frac{\pi (t - t_{\text{eff},i})}{t_{p,i}} \right); \quad \text{for } \frac{-t_{p,i}}{2} \leq (t - t_{\text{eff},i}) \leq \frac{t_{p,i}}{2} \quad (2)$$

where  $W$  is the weight of the jumper,  $k_{p,i}$  is the impact factor,  $t_{p,i}$  is the contact period and  $t_{\text{eff},i}$  is the effective time. Those three parameters [ $k_{p,i}$ ;  $t_{p,i}$ ;  $t_{\text{eff},i}$ ] are set for each individual and each jump with a statistical model proposed by Sim [6], which is dependent on the main jumping frequency.

Figure 4 displays sample loadings considering both alternative methodologies.

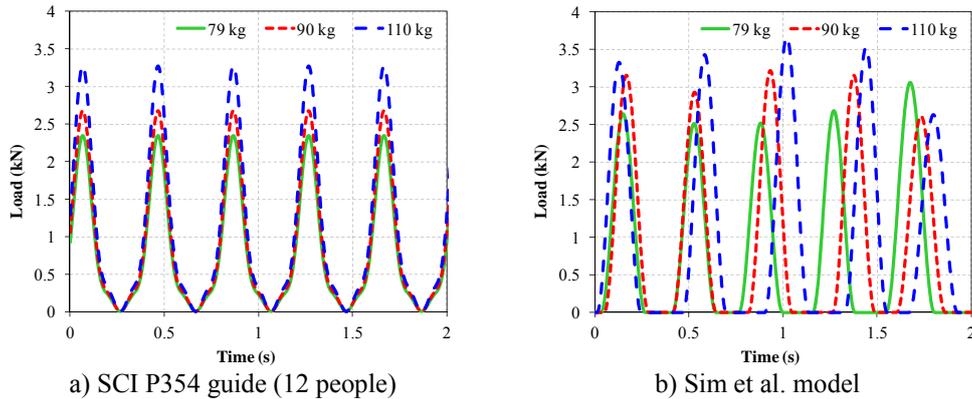


Figure 4 Dynamic loads of three jumpers at 2.5 Hz beat frequency

Figure 4a) is consistent with a main jumping frequency of 2.5 Hz, and Fourier coefficients associated with a group of 12 people.

Figure 4b) has been built under the same assumptions, but with the second methodology. The time lags and contact duration differences result in a much different pattern.

The loadings are applied to the FE model at selected nodes of the jumping area, one per individual. In each case attention has been paid to identifying the jumper weight at each position during the test. With these loads, a transient analysis is performed by applying modal superposition. The sub-steps are 5 ms long.

Figure 5 displays a comparison of acceleration time histories obtained applying both loadings at the same sensor location (number 8), together with a portion of the one recorded during the test. In this test there were 30 jumpers and the main jumping frequency was set to 2 Hz. As expected, predictions based on SCI P354 become periodic after a few seconds, while Sim's load model leads to a variable pattern, which is closer to the experimental record. It is also interesting to point out that neither load model is able to reproduce the highest amplitude spikes in the experimental record, since the latter are associated to high frequency pulses acting during very short time intervals.

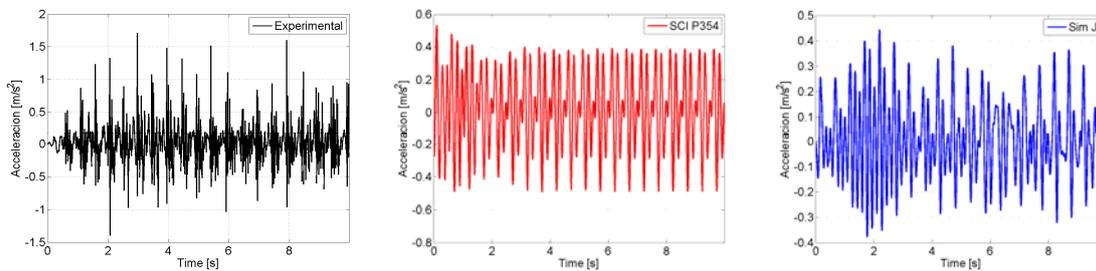


Figure 5 Comparison of acceleration time histories (30 people - 2 Hz)

Figure 6 presents alternative comparisons. Figure 6a) displays the root mean square (RMS) acceleration of the records displayed in Figure . They have been obtained considering a square window 1s long. All records display an almost uniform pattern around 110 dB, with average differences below 5 dB.

Figure 6b) displays spectral densities. The agreement between predictions and experimental values is remarkable at the jumping frequency and its harmonics (2.5, 5 and 7.5 Hz).

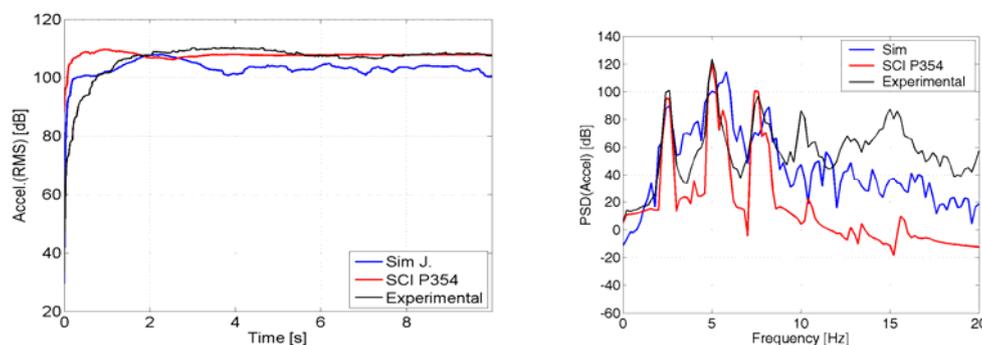


Figure 6 Comparison of running RMS and PSD results (30 people – 2.5 Hz)

Randomness in Sim's model translates into a spread of energy across a wider frequency range, which leads to a better fit below 10 Hz. Above this limit, both models underestimate the response. Figure Figure 6b) shows an interesting improvement of predictions using Sim's model in the ranges of frequencies between harmonics of the jumping frequency. However, when natural vibration frequencies are close to the harmonic frequencies associated to jump-

ers, sometimes, not always, the spread leads to a poorer fit. This feature is very dependent on the degree of synchronization of jumpers during the tests.

## 6 PROPOSED LOAD MODEL

From Figure 6, it is evident that the SCI and Sim Jump Force Models do not adequately predict the energy contained at high frequencies within the jump force profile. Therefore, the profile would need to be altered in order to more accurately predict the response of the structure.

In order to analyze the jump force profile with more clarity, jump force trials were conducted. Subjects performed sets of jumps at frequencies that included 1.5, 2, 2.5, 2.8, and 3 Hz. The force plate on which the subjects performed the jumps recorded the temporal force profile of the impact. An image of these trials and an example of the temporal responses are depicted in Figure 7.

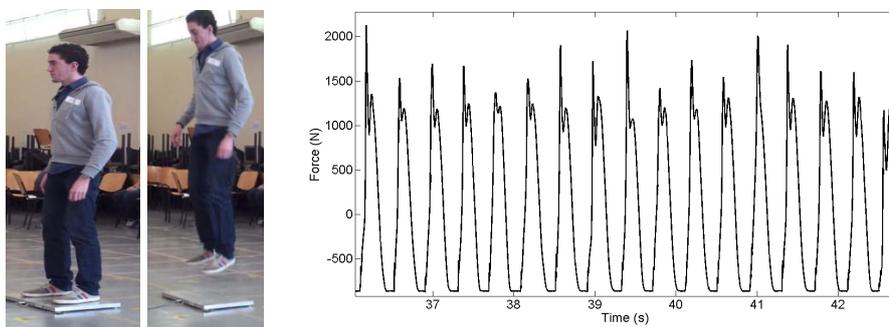


Figure 7: Force plate jump trials and an example of the temporal force history produced

From the force history as shown above in Figure 7, the collection of isolated force profiles are shown below.

The results of the force plate trials were analyzed in an attempt to better characterize the profile of the jump force and determine how best to approximate it. As shown in figure 7, the jump force can be decomposed into the summation of 1 peak with a large period and a number of peaks with smaller periods. Utilizing a similar model to that of the Sim Model, the approximation will still be comprised of the summation of two square cosines functions. Hence, the same equation will be used as a model.

$$F(t) = A_1 \cos^2(\omega_1(t-t_1)) + A_2 \cos^2(\omega_2(t-t_2)) \quad (3)$$

The idea moving forward was to find average values for the parameters of  $A_1$ ,  $A_2$ ,  $\omega_1$ ,  $\omega_2$ ,  $t_1$ , and  $t_2$ .

In order to do so, an attempt was made to minimize the error between the experimental results of the force plate trials and the approximation function with respect to each individual parameter by an iterative least-squares procedure.

Initially, each jump force profile is isolated from the history of the force plate trial as depicted in Figure 8 and Figure 9 shows a side by side comparison of the experimental force profiles and the numerical approximations.

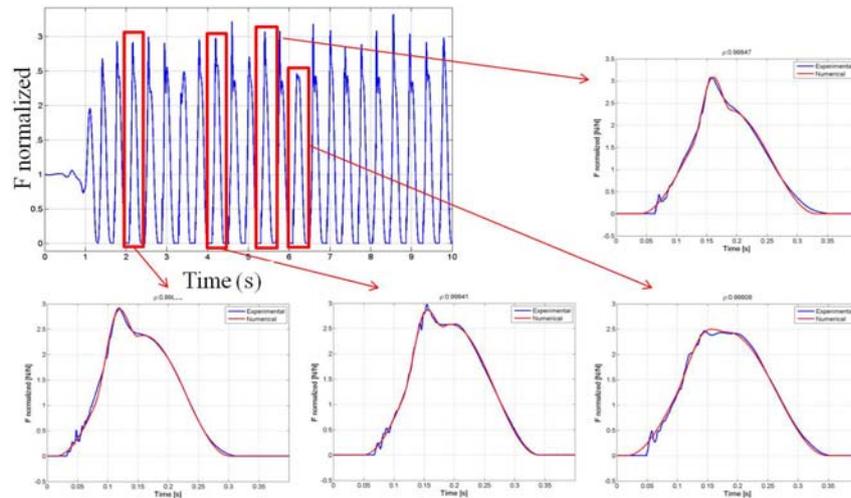


Figure 8: Examples of the process of isolating each jump force profile

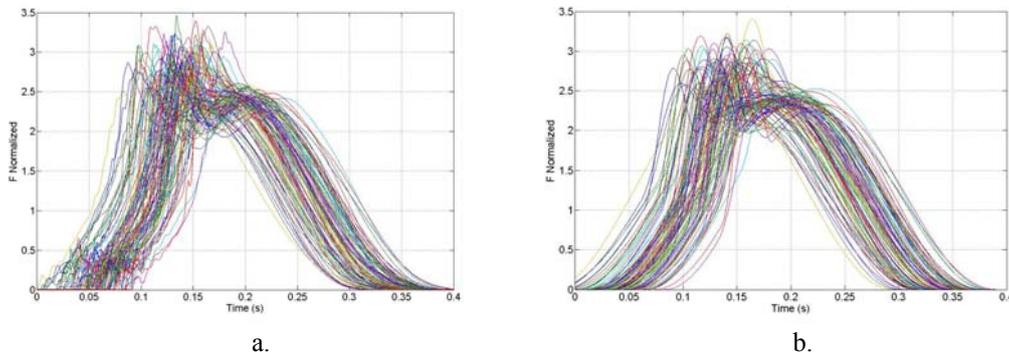


Figure 9: A comparison of a) the array of experimental jump force profiles and b) the array of approximations for the jump

For each jump profile in the collection, the total impulse for the experimental and approximate force profiles was calculated. The graph of these impulses is shown in Figure 9a).

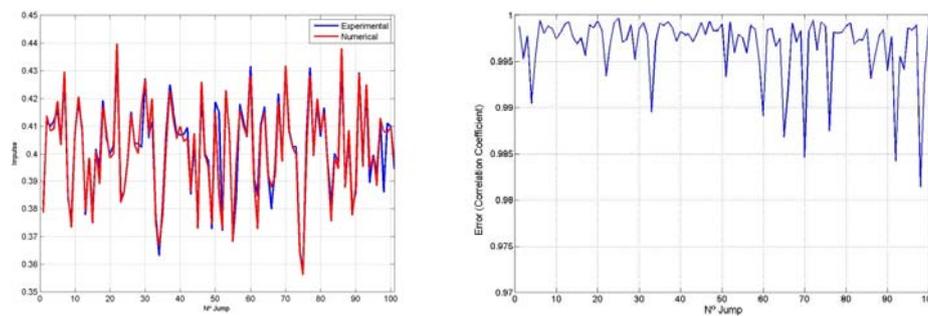


Figure 10: The correlation coefficients between the experimental jump force profile and the numerical approximation of the jump force profile.

As can be seen in Figure 10a), the error between the approximations and the experimental results rarely approaches 1%. In addition, the correlation coefficients between the experimental and numerical force profiles were calculated for each jump, (Figure 10b) and those jumps where the correlation coefficient was less than 0.993 were removed from any subsequent analyses. The correlation coefficients were calculated as follows.

The next step in the process was to obtain a representative statistical distribution for each variable utilized in the approximation. In order to do so, the histograms of the final values for each variable were plotted. Figure 11 displays these histograms as well as the parameters for the normal curves that were fitted to each of the variables.

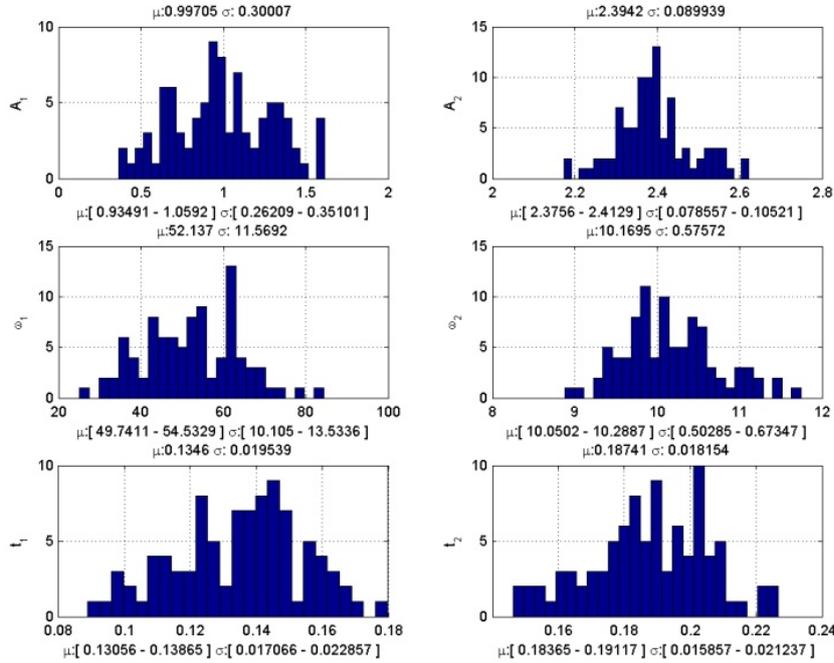


Figure 11: Histograms of variable magnitudes for the jump force approximations

As before, the model for the jump force profiles were programmed so as to simulate the jump forces from the trials conducted within the gymnasium. The resulting simulations of the force histories were used with the finite element model to obtain the structural response to the simulated profiles generated by this new technique. The resulting simulated PSDs were compared with both the experimental results and the PSDs from the jump simulations using the Sim Model. The results are displayed below in Figure 12.

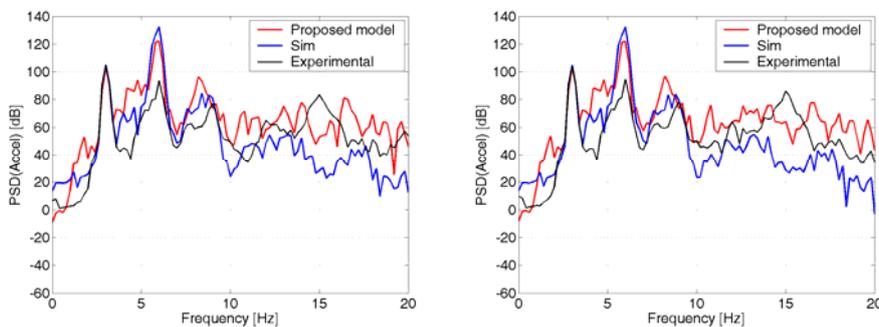


Figure 12 Comparisons of the PSDs produced from the new model, Sim model, and the experimental results for 30 subjects jumping at 3 Hz for two sensor positions.

## 7 CONCLUSIONS

The main goal of this paper is to assess three different human induced vibration load models (jumps). To this end a series of tests have been conducted on a real structure designed to be a gymnasium, which has natural frequencies within the range of the excitation. Groups of people jumped synchronously guided by a metronome. Also a Finite Element model has been built and fitted considering the vibration modes and frequencies identified using SSI method.

Load models have been found able to reproduce with good accuracy average acceleration levels in the time domain (RMS). However, in the frequency domain, Sim's model is less prone to underestimating response close to structural vibration modes. When compared with SCI's load model, randomness in time lags and contact time durations result on a spread of energy across a wider frequency range, which is closer to actual recorded values.

In regards to the Sim Model, the proposed model provides a clear improvement in the energy contained within higher frequencies of the PSD diagrams. This can be seen clearly in PSDs presented in Figure 12. In addition, the proposed model is able to portray the energy contained within the first 4 harmonics of the jump frequencies with a high degree of accuracy.

Another clear advantage of this new modeling technique in regards to the Sim and Modified Sim Models is in the ease of programming and obtaining temporal jump force simulations. The previous models utilized a program where each jump was dependent upon the previous jump and in doing so, a more complex program is required. However, the programming of the new model is accomplished through inputting the statistics for the normal distributions of the variables as well as the correlation coefficients.

## ACKNOWLEDGEMENTS

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