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# DYNAMIC GROUND STRESS-PATH EVOLUTION DUE TO THE RAILWAY TRAFFIC: A PARAMETRIC STUDY.

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**Abstract.** *Historically, the design of railway track has been performed with bases on empirical rules. However, with increase of the train speed, namely due to the development of modern high speed railway tracks, more scientific procedures are required in order to guarantee a satisfactory behaviour of the track and a minimization of maintenance operations.*

*One the aspect that demands a deeper analysis is related to the geotechnical behaviour of the track-ground layers when submitted to dynamic moving loads. Although, preliminary studies have been presented by Yang et al.* [1], *this subject is here re-analysed with special emphasis for the study of the stress paths induced by the railway traffic. In the present paper, a 2.5D model based on FEM-PML (Finite Element Method-Perfect Matched Layers)* [2], *is used for the 3-D simulation of the stress changes in the track-ground layers due to the train passage. Moreover, it should be referred that the model has capability to simulate all the elements of the track-ground system as well as the rolling stock and its dynamic interaction with the railway infrastructure.*

*At the first stage, a parametric study is developed where only the influence of the quasi-static mechanism is analysed. The stress paths are evaluated for different geometric points in the foundation and for distinct train speeds. For an easy interpretation of the results, the stress-paths are represented into the p–q space. Subsequently, the analysis is complemented with the consideration of the dynamic excitation mechanism, being its influence compared with the quasi-static mechanism.*

# 1 INTRODUCTION

Structural behavior of the railway track is directly related to the level of stresses that are transmitted to the ground, through the upper layers of wear and they must be reduced to an acceptable level to prevent the rapid deterioration of the railway. The evaluation of these stresses is a complex process that it is still very dependent of empirical rules [3, 4]. So, the numerical evaluation of the geomechanical stress state in structure subject to rail traffic is extremely important for a systematization of the structural design of the railway infrastructure, comprising both the short and long term behavior.

In the last years several authors have focused their attention on this topic in order to deepen the knowledge about the same [1, 5-7]. The passage of successive axels of the train causes rotations of the principal stresses direction in the elements of the soil. These rotations have a three-dimensional character, occurring both in the longitudinal in transversal planes (unless the point of analysis is in the plane of symmetry of the track), and they induce an aggravation of permanent deformations in the soil, in comparison with an analysis that doesn't take into account with this specificity. This effect has been highlighted by several studies [8, 9].

Regarding the numerical modeling of the effects expressed above, it should be referred that when it is plausible to consider the structure as invariant and infinite along the longitudinal direction, the adoption of numerical models based on the concept 2.5D can present a clear advantage to carry out parametric studies. This concept combines the processing speed of calculation and consideration of the three-dimensionality in the problem, being clearly an advantage for this type of analysis [10-13]. The application of the concept 2.5D, in the conditions listed above, only requires the discretization of the transversal section of the railway track and it takes advantage of the Fourier transform in relation to time and space variables (in longitudinal direction). This combination of procedures permits to obtain a three-dimensional response of the problem, repeated for different wavenumbers / frequencies.

The vibrations induced in the railway track and adjacent ground by the passage of railway traffic may be caused by different mechanisms of excitation. Usually, a distinction is made between the quasi-static and dynamic excitation mechanisms [14, 15]. The first mechanism is related with the magnitude of the moving loads corresponding to the distribution of the train weight by its axles. The dynamic character is only due to the temporal variation in the field of stress and strain in relation to a fixed reference in the domain. The second mechanism is due to the dynamic interaction between the train and the track that induces vertical acceleration on the rolling-stock and consequent dynamic loads on the track.

In order to evaluate and interpret the changes of the stress path in the ground due to the railway traffic this paper presents initially the numerical model adopted, which is based on the concept 2.5D adapted to the finite element method. Then a case study is presented, where a set of parametric studies are developed taking into account the dynamic interaction between the railway and the foundation, as well as, the problem of interaction between the train and the track. These studies allow evaluating the evolution of the stress paths on the ground due to the railway traffic, and to better discern the influence of velocity of circulation of the train, the geometrical position of the points of soil considered in the analysis and unevenness of the track.

# 2 NUMERICAL MODEL

As mentioned before, when the structure is invariant the three-dimensional solution of the problem can be found by a 2.5D approach. This is the modeling strategy followed for the simulation of the track ground system. However, it should be highlighted that the numerical approach followed also allows the simulation of the rolling stock and of its dynamic interaction with the remaining system. This aspect is reached by a modular algorithm, where the track-ground system is modeled by a 2.5D finite elements approach, being the structural behavior of the train simulated by a multi-body approach. The main steps of the computation are schematically depicted in Figure 1.

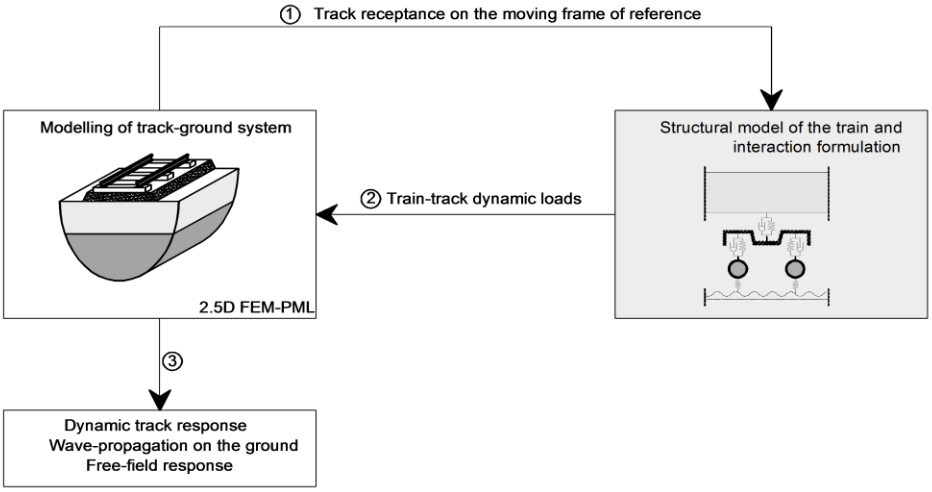


Figure 1 – Computational scheme adopted for the train-track interaction.

The dynamic interaction between both systems is performed by a compliance approach, where the compatibility of displacements and equilibrium of loads is guaranteed

Regarding the numerical modeling of the train, a detailed description of the numerical procedure followed can be found in the following references: [16, 17].

Concerning the modeling of the track-ground system an additional explanation should be performed: it is assumed that the dynamic response of the system is elastic and linear. Taking into account that assumption, the three-dimensional solution is found through a 2.5D procedure based on the finite elements approach.

The main concept behind the proposed solution is the use of a method which is between the two and the three dimensional domain, where the analysis can be carried out in the wavenumber/frequency domain. All the variables, i.e., loads (action) and displacements (response), must be transformed to the wavenumber/frequency domain by means of a double Fourier transform, related with the direction along the track (x direction) and with time. Transformed quantities are functions of the Fourier images of x and t, defined as wavenumber and frequency and are represented by $k_1$ and $\omega$, respectively.

Following the usual steps of the finite element procedure, namely the strong and weak formulations, the following equilibrium equation can be derived for any point of a three dimensional domain. Additional information about the procedure can be found in [15, 16, 18]. However, in spite of the advantages provided by a finite element approach, a topic of particular relevance is the formulation of special procedures to treat the boundary effects that are inherent to the truncation of the domain associated with the finite element discretization. In order to avoid this spurious reflexion of waves, a 2.5D PML approach is adopted. The whole domain, i.e., the cross section, is discretized into finite elements, being the external layers that bound the "box domain" formed by PML’s. This numerical device has the function of absorbing, without spurious reflection, the arbitrary direction waves that impinge the boundary between the domains described by the 2.5D FEM and by the 2.5D PML. Additional information about the modelling strategy can be found in Lopes et al. [18].

## 3 CASE STUDY

### 3.1 General description

The case study presented here, for which is elaborated a theoretical study of the evolution of the geomechanical stress state, adopts the geometric and geomechanical properties of an existing trench of the Linha do Norte incorporated in the Campo Experimental do Carregado [19]. The train characteristics of Alfa-Pendular are considered in the modeling.

In Figure 2 the cross-section of the problem is presented, as well as, its discretization into 2.5D finite elements. The surrounding layer of the foundation represents the boundary conditions and it is modeled using the PML (*Perfect Matched Layers*) method. In order to reduce the computational effort required, the modeling of the problem taken advantage of its symmetrical properties, which allow to simulate only half of the cross-section of the track-ground system.

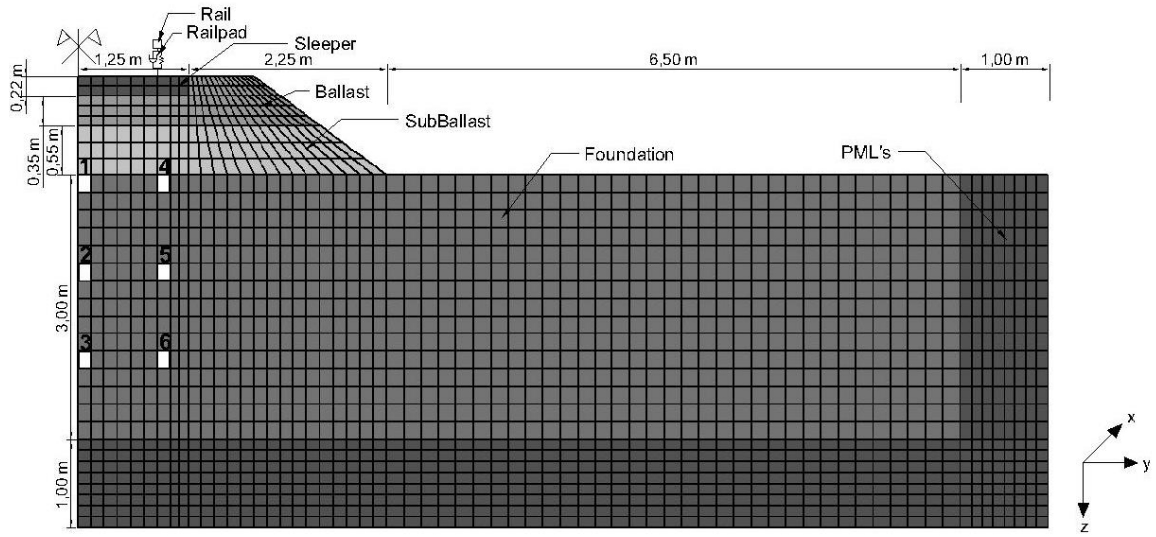


Figure 2 – Transversal section of the railway track.

The geomechanical properties of the track-ground layers are presented in Table 1. Properties adopted for the foundation are compatible with a velocity of propagation of S waves equal to 150 m/s.

Table 1 – Geomechanical properties of the track-ground layers (Adapted from [13]).

| | Stiffness | Poisson's ratio | Damping | Mass |
|---|---|---|---|---|
| Rail | $EI_r$=6.45 MPa | (-) | (-) | $m_r$=64 kg/m |
| Railpad | $K_p$=600 KN/mm | (-) | $C_p$=22.5 KNs/mm | (-) |
| Sleeper | E=30000 MPa | v=0,20 | ξ=0,01 | ρ=1833,3 kg/m$^3$ (m=300 kg/un) |
| Ballast | E=97 MPa | v=0,12 | ξ=0,061 | ρ=1591 kg/m$^3$ |
| Sub Ballast | E=212 MPa | v=0,30 | ξ=0,054 | ρ=1913 kg/m$^3$ |
| Foundation | E=127,4 MPa | v=0,49 | ξ=0,03 | ρ=1900 kg/m$^3$ |

As stated above, and in order to take advantage of the potential of the numerical model adopted, it is presented below a study focused on the evolution of stress state for different elements of the foundation. This study is developed for different train speeds, taking into account the quasi-static and also the dynamic mechanism of excitation induced by the track unevenness.

### 3.2 Influence of the Quasi-static excitation mechanism

Initially, the parametric studies consider only the influence of the quasi-static excitation mechanism in the evolution of the stress state on the ground. These studies are performed for various points of the foundation, as indicated in Figure 2. The analysis mentioned are presented in the form of stress paths and represented in the *p-q space*, in which *p* refers to the mean stress and *q* to the shear stress, both defined by Equation (1).

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$q = \sqrt{\frac{1}{2}} * \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (1)$$

where the variables $\sigma_1$, $\sigma_2$ and $\sigma_3$ represent the principal stresses [20].

The evolution of the stress paths in depth during the train passage for two vertical alignments, containing, respectively, the elements 1, 2 and 3 (Alignment 1) and the elements 4, 5 and 6 (alignment 2) are represented in Figure 3. The analyses consider the velocity of the train passage equal to 30 km/h.

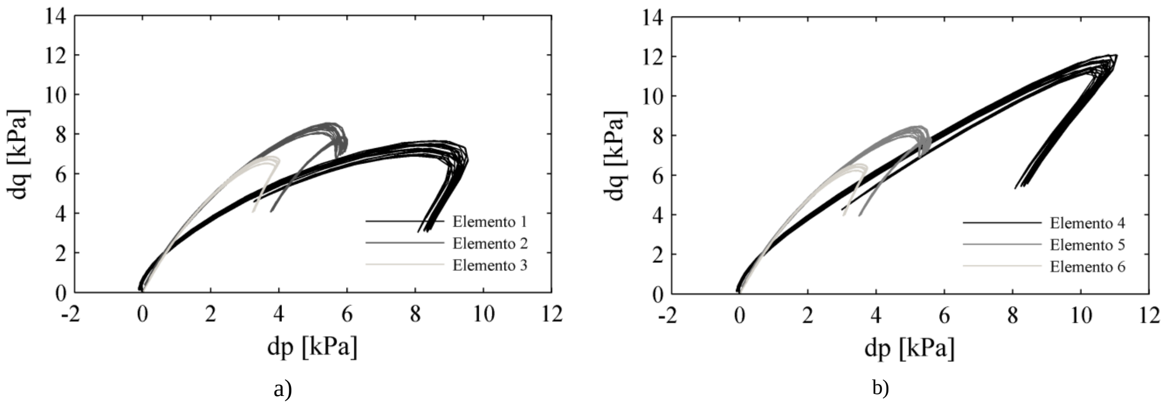


Figure 3 – Evolution of the stress paths in depth during the train passage (V=30km/h): a) Alignment 1; b) Alignment 2.

The shape of the stress paths presented is directly related to the train geometry. The space between the axles of the train causes load/unload cycles with different configurations. The depth of the point in analysis is also an important factor, due to degradation of stresses into depth (see Figure 4). While recognizing the importance of all these aspects, this paper focuses mainly on quantitative analysis, being the reader advised to consult other documents published by the authors [21] .

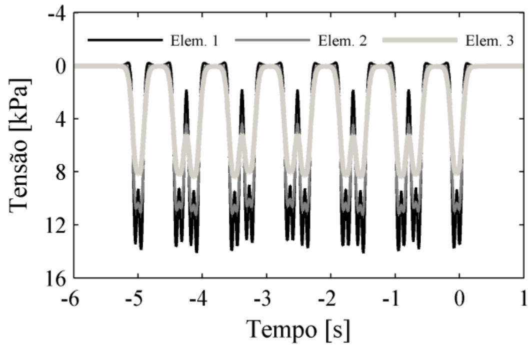


Figure 4 – Degradation of normal stress $\sigma_z$ in depth (Alignment 1, V=30m/s).

A quantitative evaluation of the results achieved is summarized in Table 2, where the maximum values of shear and mean stresses increments are indicated ($dq_{máx}$ and $dp_{máx}$, respectively), as well as the corresponding stress ratio ($dq_{máx}$/ $dp_{máx}$).

Table 2 – Stress ratio qmáx/pmáx for the elements in alignments 1 and 2.

| | | | | |
|---|---|---|---|---|
| Alignment 1 | Element | 1 | 2 | 3 |
| | $dq_{máx}$ [KPa] | 7,68 | 8,56 | 6,88 |
| | $dp_{máx}$ [KPa] | 9,55 | 6 | 4,02 |
| | Stress ratio | 0,8 | 1,43 | 1.71 |
| Alignment 2 | Element | 4 | 5 | 6 |
| | $dq_{máx}$ [KPa] | 12,09 | 8,48 | 6,57 |
| | $dp_{máx}$ [KPa] | 11,06 | 5,62 | 3,74 |
| | Stress ratio | 1,09 | 1,51 | 1.76 |

The values obtained in the stress ratio allows to conclude that with the increase of the depth, the decrease of the mean stress increments are more significant than the shear stress increments, which indicates that the ground still having a considerable distortion.

Comparing the values of the stresses for the two alignments analyzed, it can be observed that it is followed the same trend of increasing of the stress ratio, being the highest difference verified in the elements located closer to the surface. Thus, the increased of the shear stress is most pronounced for the element 4, due to its location near to the surface and out of the plane of symmetry, implying the presence of tangential stresses in the three orthogonal planes and, consequently, a higher degree of distortion.

Nevertheless the previous, it is recognized that the velocity of the train is a crucial aspect for the stress state evolution in the foundation. Therefore, in order to assess the behavior of the foundation for different velocities of the train passage, it is simulated the passage of the train for speeds close to the critical velocity of the track-ground system, as depicted in Figure 5.

In the results presented below it is assumed is also considered the initial stress state of the ground, assuming an at rest earth pressure coefficient, $K_{0,}$ equal to 1.0 ($K_0$ = 1.0). In Figure 5 are presented two indicative failure lines, corresponding to trajectories of extension and compression, considering a soil with friction angle with a value equal to 31º. Mention should be made that the failure criteria lines depicted in the figures are only indicative, since the analysis is performed in the field of the elasticity.

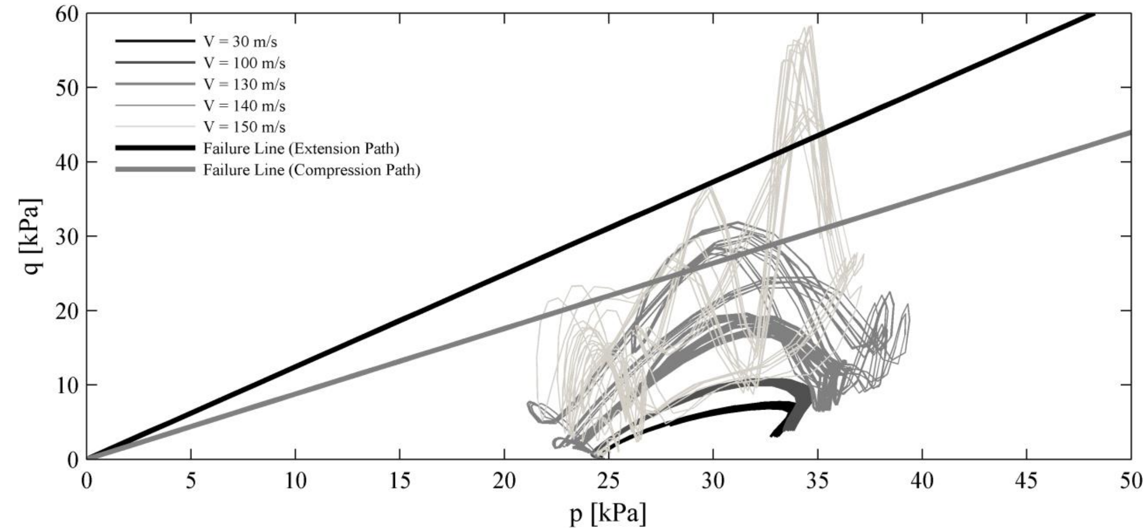


Figure 5 – Evolution of the stress paths during the train passage with different speeds (Element 1).

As can be seen in Figure 5, when the train speed approaches the critical velocity of the track-ground system, it began to appear behaviors that contrast with the pattern seen of several load/unload cycles typical of the stress paths induced by the passage of the train at lower speed. Indeed, when the speed of the train becomes closer to the critical speed of the system it is not possible anymore to identify the cycles of load and unload induced by the passage of each individual axle.

This increasing of the complexity is largely due to the effects of dynamic amplification. For lower speeds, the dynamic nature of the problem is due only to the temporal variation of the position of the geometric loading and the dynamic amplification effect becomes reduced. On the other hand, when the train speed becomes closer to the critical speed of the system, a dynamic amplification phenomenon occurs with high amplification of the dynamic response of the system. In addition to the increased complexity of the stress paths, it is also notorious that the high amplification deviatory stress is followed by a slight change of the mean stress. This effect is responsible for the approach of the stress paths to the failure surface.

Summarizing, it is possible to draw some lessons from the parametric studies presented: i) the evolution of the stress paths in depth shows an increase of the stress ratio and a higher degree of distortion for the elements away from the plane of symmetry; ii) the changes of the stress state are more pronounced for higher train speeds; iii) the increase of the train speed leads to the generation of dynamic effects that are translated into a significant increase in shear stress, iv) with the increase of the train speed, the configuration of the stress paths undergoes profound changes, increasing the level of complexity of the analysis.

### 3.3 Influence of the Dynamic Mechanism

Similarly to previous parametric studies, where only the influence of the quasi-static mechanism was analyzed, in the present section it will be evaluated the influence of the dynamic excitation mechanism in the response of the foundation due to the passage of the railway traffic.

In this study, it is assumed that the source of generation of vibrations and dynamic loads is related to the presence of irregularities along the track (Figure 6). The presence of these defects involve the generation of dynamic forces on the vehicle, causing changes in the stress state of the ground. In order to simplify the modeling of these defects shall be considered different profiles, generated by a sinusoidal function, which can be written in complex notation as follows:

$$y_i(x) = A_i e^{ik_i x} \quad (2)$$

where Ai represents the magnitude of the irregularity and $k_i$ ($k_i=2\pi/L_i$) is the wave number, being the $L_i$ the wavelength.

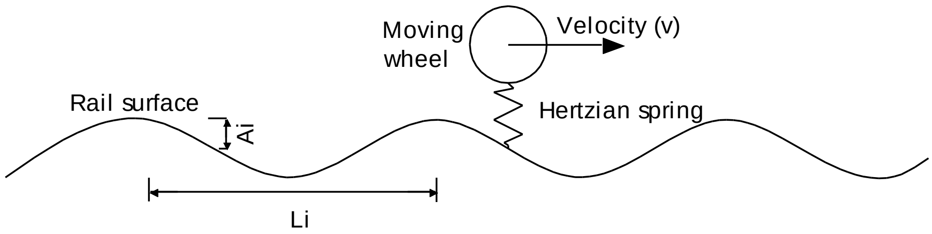


Figure 6 – Idealized track defects and wheel/rail interaction (Adapted by [1]).

In this study, it is investigated the influence of the five different sinusoidal profiles exposed in Table 3, covering a range of typical track geometry variations [1, 4].

Table 3 – Characteristics of the irregularities profiles.

| | $L_i$ [m] | $A_i$ [mm] |
|---|---|---|
| Profile 1 | 30.0 | 5.0 |
| Profile 2 | 7.5 | 2.5 |
| Profile 3 | 1.5 | 0.3 |
| Profile 4 | 1.1 | 0.2 |
| Profile 5 | 0.5 | 0.1 |

So, in order to systematize the influence of the presence of track defects, the previous analyses, perform with the quasi-static mechanism, are complemented with the consideration of the dynamic excitation mechanism for a velocity equal to 70 m/s. For this velocity are evaluated a "scaling factor of tensions" in function of the irregularities profiles.

These coefficients are evaluated based an initial situation, which represents the situation in which the train running at a very low speed (10 m/s) and there are no irregularities in the rails (taking into account only the mechanism quasi-static). In this situation the dynamic effects are minimized. For different excitation frequencies induced by track defects (different irregularities profiles), the "scaling factor of tension" is obtained by dividing the tensions increments calculated for a velocity of 70 m/s by the increments obtained in the initial situation.

In this study it is considered the assessment of these coefficients for the maximum value of the incremental normal stress $\sigma_z$, the shear stress increment (dq) and mean stress increment (dp) for the Element 1 (see Figure 2). Table 4 presents the information necessary for their evaluation. A first analysis of the results present in the Table 4 reveals that considering only the effect of the increase of the circulation speed from 10 m/s to 70 m/s, the increase of the scaling factor isn't very significant, in other words, the effects of the dynamic amplification for this velocities range are practically negligible. Recall that, for these two cases, it is only considered the influence of the quasi-static mechanism in the stress state change.

Table 4 – Evaluation of the scaling factor of tensions (Element 1).

| | V=10 m/s | V=70 m/s | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Frequency ($f=V/L_i$) [Hz] | | | | | | | | | | | | |
| | 0 Hz | 0 Hz | | 2.3 Hz | | 9.3 Hz | | 46.7 Hz | | 63.6 Hz | | 140 Hz | |
| | [kPa] | [kPa] | Coef. de Maj. | [kPa] | Coef. de Maj. | [kPa] | Coef. de Maj. | [kPa] | Coef. de Maj. | [kPa] | Coef. de Maj. | [kPa] | Coef. de Maj. |
| $dp_{máx}$ | 9,39 | 9,61 | 1,02 | 9,94 | 1,06 | 11,21 | 1,19 | 17,73 | 1,89 | 26,95 | 2,87 | 26,00 | 2,77 |
| $dq_{máx}$ | 7,51 | 8,43 | 1,12 | 8,80 | 1,17 | 12,08 | 1,61 | 18,95 | 2,52 | 32,79 | 4,37 | 19,51 | 2,60 |
| $d\sigma_{z,máx}$ | 14,01 | 14,66 | 1,05 | 15,31 | 1,09 | 17,73 | 1,27 | 24,25 | 1,73 | 31,14 | 2,22 | 23,86 | 1,70 |

When considering the effect of dynamic excitation mechanism for the various excitation frequencies of the load, in agreement with the profiles of irregularities considered, the stress increments in the soil suffer significant increases. In fact, the consideration of the geometrical defects of the track originates incremental stresses that increase with the "passing frequency" to a maximum which lies in the range of 60-70 Hz. This peak is related to the resonance of the unsprung masses of the train, that occurs for values within this range of frequencies, as observed by Alves Costa et al [22]. However, in the lower frequency range, the influence of the dynamic mechanism is minor and it can even be considered negligible for large wavelengths of the track unevenness.

In short, in the context of the studies performed, the consideration of the dynamic excitation mechanism in the analyses it's relevant, especially for frequencies of oscillation of the load in the order of 60-70 Hz.

## 4 CONCLUSIONS

In this paper the changes of stress state of the railway track foundation caused by the traffic are evaluated. The knowledge of how these changes are processed, are a relevant aspect for a correct interpretation of this problem. The option by a numerical modeling technique based on 2.5D FEM-PML proved to be crucial for the simulation of the three-dimensional behavior of the problem.

The studies carried out considering the distinction between quasi-static and dynamic the excitation mechanisms allowed to obtain a perception of their individual influence. Regarding the to studies developed based on the first mechanism expressed above, it is important to refer that to levels of train speed distant from the critical velocity of the soil, the short term behavior of the railway poses no serious problems. However, the degradation of resistant capabilities of the constituent elements of the railway by repetitive loading/unloading cycles may cause consequences to its behavior. Note that, when considering speeds close to the critical velocity of the track-ground system, it is necessary to take into account the effects of the dynamic amplification, because these effects involve serious consequences for the behavior of the railway infrastructure.

The preservation of the railway should also be a parameter to take into account by the railway administrations. As shown, the presence of track defects, especially for shorter wavelengths, is responsible for an increase of the stress increments in the foundation, which can induce the degradation of the track more quickly.

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