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INFLUENCE OF TEMPERATURE ON THE MODAL PARAMETERS OF LAMINATED GLASS BEAMS

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Abstract. *Laminated glass is a sandwich element consisting of two or more glass sheets, with one or more interlayers such as polyvinyl butyral (PVB). Laminated glass beam and plates show a viscoelastic behaviour being the response of the glass linear-elastic whereas that of the polymeric interlayer is viscoelastic and temperature dependent. In this work, it is studied the effect of temperature on the modal parameters of laminated glass elements. Several operational modal tests have been carried out on a laminated glass beam at different temperatures in the range 10ºC-45ºC. The modal parameters have been identified with Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic SSI method using ARTeMIS Extractor software. The results show that the natural frequencies decrease and the damping ratios increase with increasing temperature. Finally, the experimental modal parameters are compared with those determined using the analytical model of Ross, Kerwin and Ungar.*

1 INTRODUCTION

Laminated glass beams are sandwich elements which consist of two or more sheets of monolithic glass with one or more interlayers of a polymer such as polyvinyl butyral (PVB). The thickness of the PVB layer is usually 0.38 mm or a multiple of this value.

The main advantage of laminated glass compared with monolithic glass is the safety provided in case of breakage, since the PVB interlayer holds the glass fragments together, thus attenuating the accident risk. Moreover, the PVB interlayer increases considerably damping, reducing the magnitude of the vibrations due to dynamic loadings. PVB is an amorphous thermoplastic which shows linear-viscoelastic behaviour. A fundamental characteristic of viscoelastic materials is that the mechanical properties are frequency (or time) and temperature dependent [**1**, **2**]. This fact makes more difficult the structural analysis of laminated glass elements. On the other hand, glass is usually modelled as a linear elastic material.

The dynamic response of laminated glass elements is temperature and frequency dependent [**3**, **4**, **5**]. The response of laminated glass elements present two borderline cases: (1) the monolithic limit, where the flexural inertia is that corresponding to the total thickness of the element; 2) the layered limit for which the flexural inertia is that corresponding to the sum of the inertia of the two glass layers [**6**, **7**]. The monolithic limit appears at low temperatures and high frequencies whereas the behaviour is close to the layered limit a high temperatures and low frequencies.

In this paper, Operational Modal Analysis (OMA) has been used to estimate the bending modal parameters of a non-symmetric laminated glass beam in free-free configuration. The modal parameters have been identified with Enhanced Frequency Domain Decomposition (EFDD) and the Stochastic SSI method using ARTeMIS Extractor software. As the PVB shows a viscoelastic behaviour, the modal parameters are temperature dependent and several operational modal tests were performed at different temperatures in the range 10ºC-45ºC.The experimental modal parameters are compared with those determined using the analytical model of Ross, Kerwin and Ungar [**6**] and the effect of temperature is investigated.

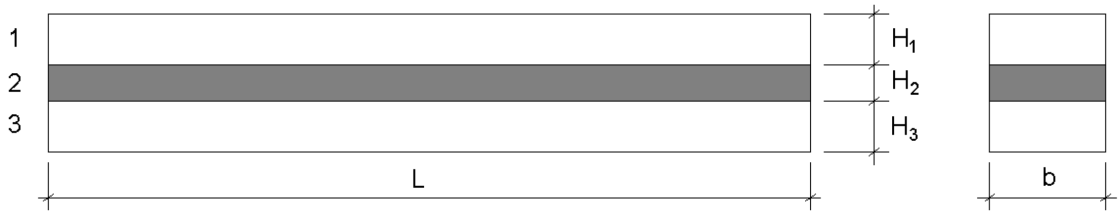

Figure 1: Laminated glass beam.

2 VISCOELASTIC BEHAVIOR

The polyvinyl butyral (PVB) can be considered a linear-viscoelastic material in with mechanical properties are frequency (or time) and temperature dependent [**8**, **9**]. In the frequency domain, the complex tensile modulus is given by:

$$E_2^*(\omega) = E_2^{'}(\omega) + i\,E_2^{''}(\omega) = E_2^{'}(\omega)\left(1 + i\,\eta_2(\omega)\right) \tag{1}$$

where $E^{'}(\omega)$ y $E^{''}(\omega)$ are the storage and the loss tensile moduli, respectively, and

$$\eta_2(\omega) = \frac{E_2''(\omega)}{E_2'(\omega)} \tag{2}$$

is the loss factor that relates both moduli. The subscript “2” is hereafter used to reference the viscoelastic material (PVB). On the other hand, superscript “*” indicates complex.

As regards the shear behavior, the complex shear modulus is given by:

$$G_2^*(\omega) = G_2'(\omega) + iG_2''(\omega) = G_2'(\omega)(1 + i\eta_2(\omega)) \tag{3}$$

where $G'(\omega)$ and $G''(\omega)$ are the storage and the loss shear moduli, respectively.

Both shear and tensile moduli can be related by means of the correspondence principle [**10**, **11**]. This principle allows us to use the expressions of the linear elasticity theory with linear-viscoelastic materials but introducing the corresponding complex viscoelastic properties, i.e.:

$$G_2^*(\omega) = \frac{3E_2^*(\omega) \cdot K_2^*(\omega)}{9K_2^*(\omega) - E_2^*(\omega)} \tag{4}$$

Where K_2^* (ω) is the complex bulk modulus.

PVB mechanical behavior can be established by relaxation or creep tests in the time domain or its corresponding dynamic tests in the frequency domain [**1**, **12**]. For obtaining the dynamic moduli, a time-frequency conversion can be used from relaxation test data [**13**, **14**]. The relaxation master curve is usually fitted with a generalized Maxwell model [**2**] which can be represented with a Prony series given by [**15**]:

$$E_2(t) = E_{2\infty} + \sum_{i=1}^{n} e_i \cdot \exp\left(-\frac{t}{\tau_i}\right) \tag{5}$$

where e_i and τ_i are the Prony series coefficients to be estimated. The store and loss components of the complex modulus can be determined directly from the relaxation Prony series coefficients by:

$$E_2'(\omega) = E_{2\infty} + \sum_{i=1}^{n} \frac{\tau_i^2 \cdot \omega^2 \cdot e_i}{\tau_i^2 \cdot \omega^2 + 1} \tag{6}$$

and

$$E_2''(\omega) = E_{2\infty} + \sum_{i=1}^{n} \frac{\tau_i \cdot \omega \cdot e_i}{\tau_i^2 \cdot \omega^2 + 1} \tag{7}$$

Similar expressions to Eqs. (5, 6 and 7) can be used to determine the complex shear moduli from shear relaxation data.

3 THE MODEL OF ROSS, KERWIN AND UNGAR (RKU)

Ross, Kerwin, and Ungar [**7**, **6**] developed one of the earliest damping models for three layered sandwich beams based on damping of flexural waves by a constrained viscoelastic layer. They considered that the beam is simply supported and vibrating at a natural frequency or the beam is infinitely long so that the end effects may be neglected. This means that the flexural deformation of the beam during vibration is spatially sinusoidal in shape. Furthermore, the modulus of elasticity of the glass is high compared to that of the PVB so that it is commonly assumed that $E_2^* \approx 0$. With these considerations, the complex natural frequencies are estimated with the expression:

$$\omega^{*2} = \omega^2(1 + i\eta) = \mathrm{k}^4 \cdot \frac{\mathrm{EI}^*}{\overline{\mathrm{m}}} \quad (8)$$

Where ω is the natural frequency, η is the loss factor, k is the wavenumber, $\overline{\mathrm{m}}$ is the mass per unit length and EI* is an effective stiffness given by:

$$\mathrm{EI}^* = \left[\frac{\mathrm{E}_1 \cdot \left(\mathrm{H}_1^3 + \mathrm{H}_3^3\right)}{12} + \frac{\mathrm{E}_1 \cdot \mathrm{H}_1 \cdot \mathrm{H}_3 \cdot H_{31}^2 \cdot g_R^*}{\mathrm{H}_1 + g_R^* \cdot (\mathrm{H}_1 + \mathrm{H}_3)} \right] \quad (9)$$

Which depends on the thickness of each layer (H_1, H_2 and H_3), the glass Young modulus E_1 and the parameters:

$$H_{31} = H_2 + \frac{H_1 + H_3}{2} \quad (10)$$

$$\mathrm{g}_\mathrm{R}^* = \frac{\mathrm{G}_2^*}{\mathrm{E}_3 \cdot \mathrm{H}_3 \cdot \mathrm{H}_2 \cdot \mathrm{k}^2} \quad (11)$$

On one hand, the effective stiffness EI* is function of the wavenumber k and the properties of the cross section. On the other hand, the wavenumber depends, in turn, on the flexural stiffness EI*. This means that the wavenumber must be known in Eqs. (9) and (11). For a simply supported laminated sandwich beam, the damped normal mode has the simple sinusoidal form, with k having discrete values:

$$\mathrm{k} = \frac{\pi\, \mathrm{n}}{\mathrm{L}} \;(\mathrm{n} = 1, 2, 3, ...,) \quad (12)$$

For other boundary conditions, correction factors have been proposed to use with wavenumbers of the Euler Bernoulli beam [**16**, **17**].

4 EXPERIMENTAL TESTS AND ANALYTICAL PREDICTIONS

Operational modal analysis (OMA) was applied to a laminated glass beam with the following geometrical data: H_1=3.75mm; H_2=0.38mm; H_3=7.90mm; L=1000mm; b=100mm. The beam was tested on free-free configuration at different temperatures from 12ºC to 45ºC.

The beams were excited applying many hits along the beam, random in time and space and the responses were recorded for approximately 5 minutes using a sampling frequency of 2000 Hz. Seven light accelerometers (0.8 grams each) with a sensitivity of 100mV/g and uniformly distributed along the beam, were used to record the acceleration responses of the beam.

Modal parameters were estimated using both Frequency Domain Decomposition (EFDD) [**18**, **19**] and Stochastic Subspace Identification method (SSI) [**20**, **21**]. The natural frequencies and loss factors estimated by the EFDD technique, together with those predicted by the RKU model, are presented in Tables 1 and 2 and in figures 2 to 10. Similar results were obtained with SSI. It has been assumed that loss factor η and the modal damping ζ, are related by [**17**]:

$$\eta = 2\zeta \tag{13}$$

In Fig.10 the singular value decomposition of the responses at 20ºC and 40ºC are presented. As damping increases with increasing temperature, peaks become less clear for the higher modes as the temperature increase.

With respect to the natural frequencies, they decrease with increasing temperature for all the modes. The error between the experimental and the predicted natural frequencies is always less than a 10%, being the predicted values higher than the experimental ones. Thus, the RKU model predicts with a good accuracy the natural frequencies of laminated glass beams.

As far the loss factor, it is small for low temperatures and increases with increasing temperature. Large discrepancies have been encountered between the results provided by the analytical models and the experimental results, being the maximum errors around 47%.

According to the RKU model, the mode shapes of a laminated glass beam coincide with those corresponding to an Euler Bernoulli Beam. At low temperatures the beam behaves like a monolithic beam so that the mode shapes at these temperatures should be quite similar to those of monolithic beam. In figures 11-14 it is shown the MAC (Modal Assurance Criteria) of each mode at 12ºC with the same mode at different temperatures. It can be observed that the MAC is very close to 1 for all the modes and all temperatures so that we can conclude the effect of temperature in the mode shapes of a laminated glass beam is very small.

	Tª [ºC]	f_n [Hz] RKU MODEL	f_n [Hz] OMA	% Difference
MODE 1	12	66.16	66.65	-0.73
	20	66.02	66.47	-0.68
	25	65.78	66.29	-0.76
	30	65.38	65.97	-0.89
	35	64.86	65.37	-0.78
	40	63.15	63.70	-0.86
	45	60.40	60.25	0.24
MODE 2	12	181.74	182.90	-0.64
	20	180.80	182.00	-0.66
	25	179.70	181.10	-0.77
	30	177.90	179.30	-0.78
	35	174.81	175.90	-0.62
	40	169.67	168.10	0.93
	45	157.97	153.70	2.78
MODE 3	12	354.38	357.10	-0.76
	20	351.37	354.50	-0.88
	25	348.73	351.30	-0.73
	30	342.92	346.40	-1.00
	35	334.00	335.50	-0.45
	40	323.03	311.80	3.60
	45	295.71	276.80	6.83
MODE 4	12	582.48	587.60	-0.87
	20	575.53	581.50	-1.03
	25	569.63	574.50	-0.85
	30	556.58	563.60	-1.24
	35	539.59	543.50	-0.72
	40	516.20	498.00	3.65
	45	473.32	432.70	9.39

Table 1: Experimental and predicted natural frequencies

	Tª [ºC]	η% RKU MODEL	η% OMA	% Difference
MODE 1	12	0.15%	0.28%	-47.22
	20	0.43%	0.57%	-24.48
	25	0.90%	0.96%	-6.11
	30	1.87%	2.11%	-11.54
	35	4.15%	4.56%	-9.03
	40	8.55%	8.98%	-4.81
	45	13.44%	18.92%	-28.96
MODE 2	12	0.32%	0.45%	-28.19
	20	0.73%	0.98%	-25.42
	25	1.75%	1.81%	-3.13
	30	3.16%	3.76%	-15.87
	35	5.53%	9.40%	-41.20
	40	12.14%	22.98%	-47.17
	45	18.35%	20.34%	-9.78
MODE 3	12	0.51%	0.50%	1.47
	20	1.18%	1.20%	-1.78
	25	2.42%	2.63%	-7.98
	30	4.20%	5.31%	-20.84
	35	7.61%	11.87%	-35.90
	40	13.13%	19.54%	-32.80
	45	21.95%	25.06%	-12.41
MODE 4	12	0.77%	0.68%	12.87
	20	1.62%	1.70%	-4.94
	25	2.86%	3.63%	-21.21
	30	5.47%	6.49%	-15.72
	35	9.37%	12.96%	-27.72
	40	13.70%	24.96%	-45.11
	45	23.50%	29.82%	-21.19

Table 2: Experimental and predicted natural frequencies

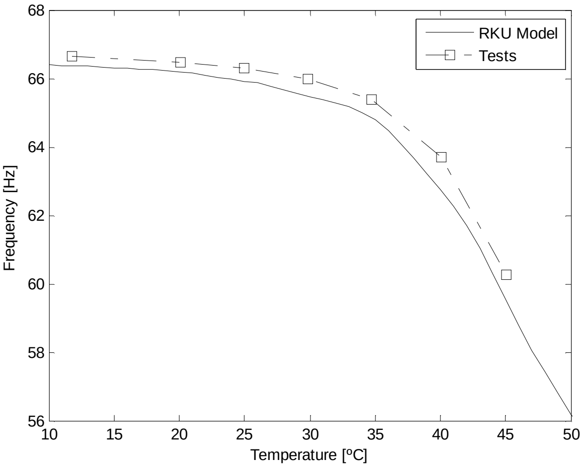

Figure 2: Experimental and predicted natural frequencies for mode 1.

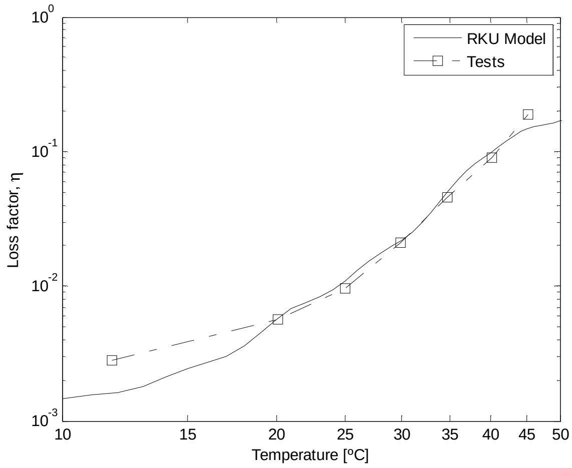

Figure 3: Experimental and predicted loss factor for mode 1.

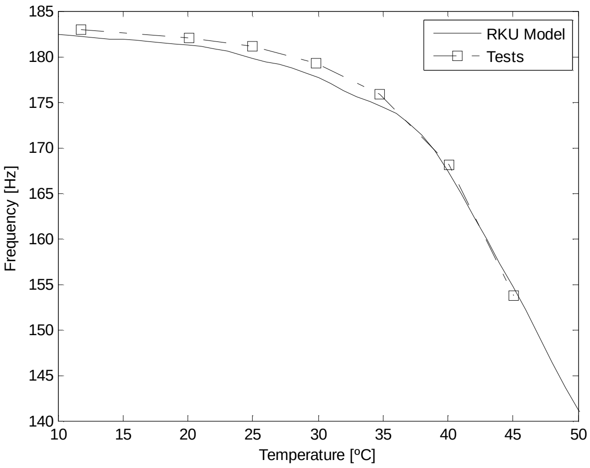

Figure 4: Experimental and predicted natural frequencies for mode 2.

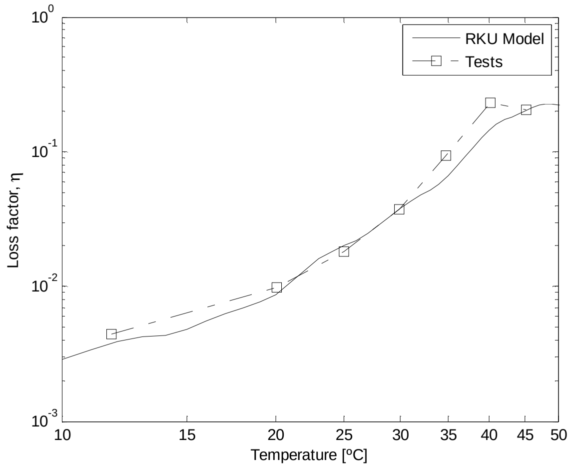

Figure 5: Experimental and predicted loss factor for mode 2.

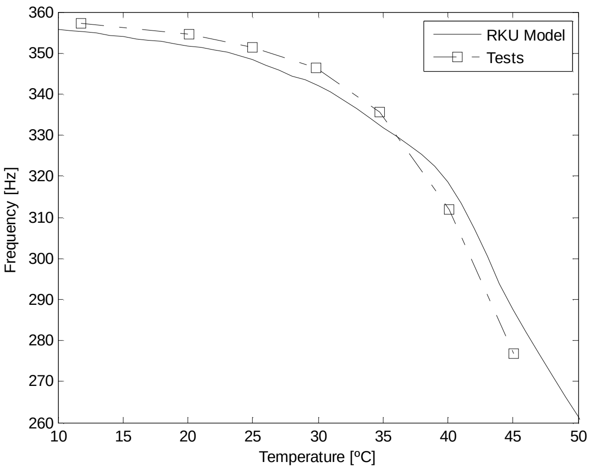

Figure 6: Experimental and predicted natural frequencies for mode 3.

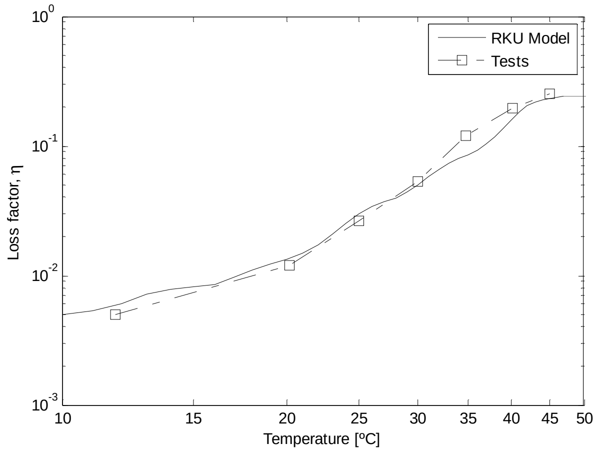

Figure 7: Experimental and predicted loss factor for mode 3.

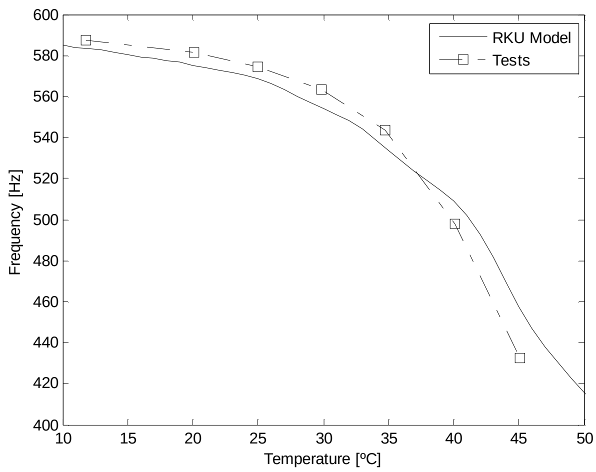

Figure 8: Experimental and predicted natural frequencies for mode 4.

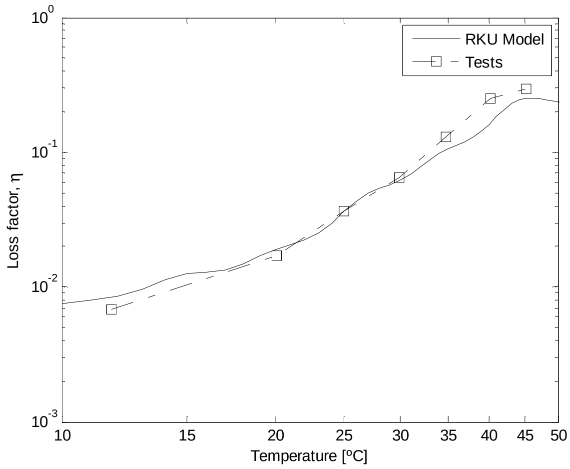

Figure 9: Experimental and predicted loss factor for mode 4.

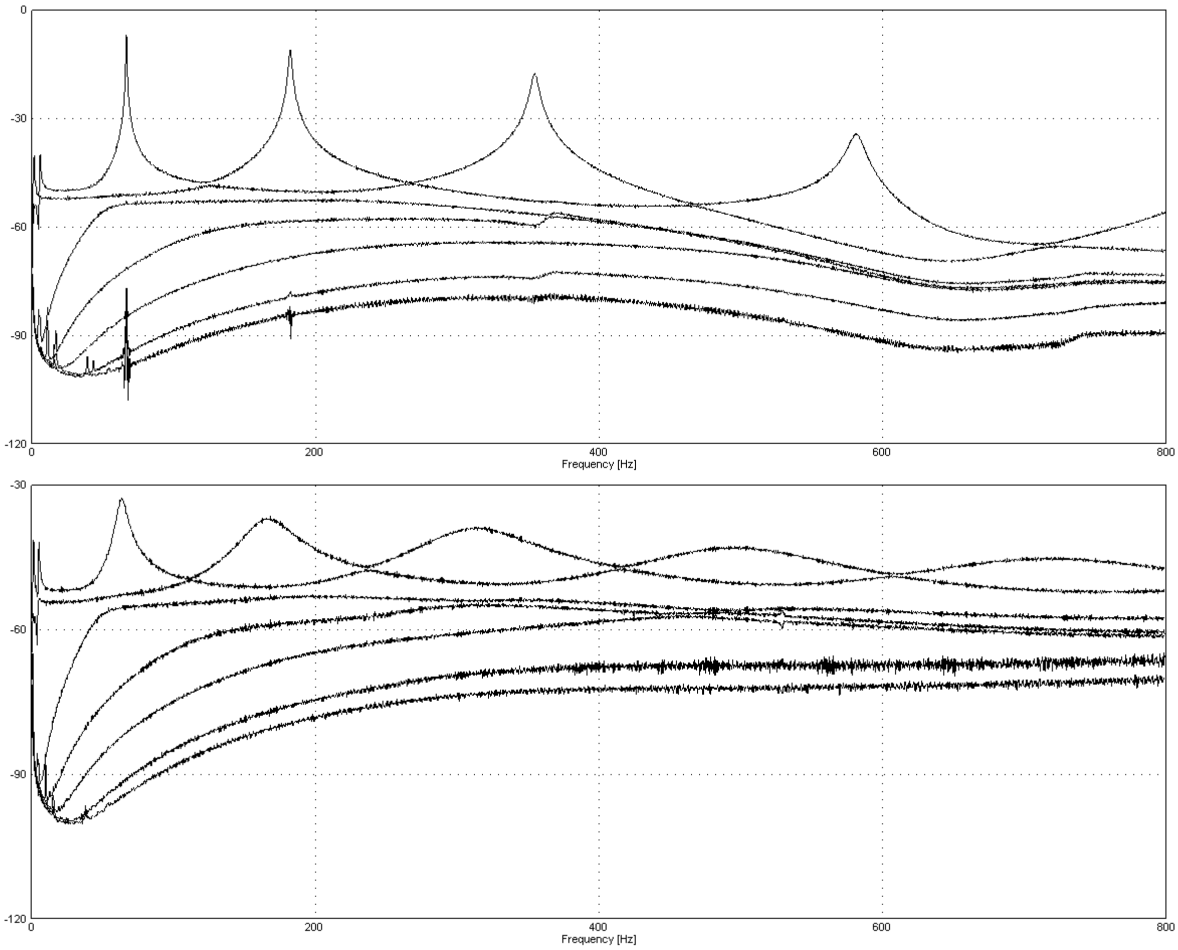

Figure 10: Singular value decomposition of the responses at 20ºC (top) and 40 ºC (bottom).

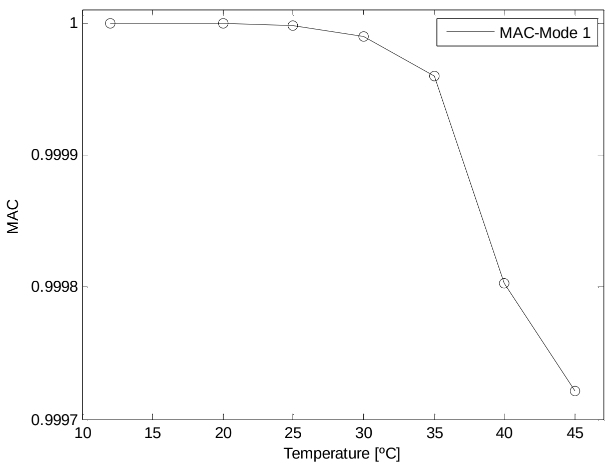

Figure 11: MAC for mode 1.

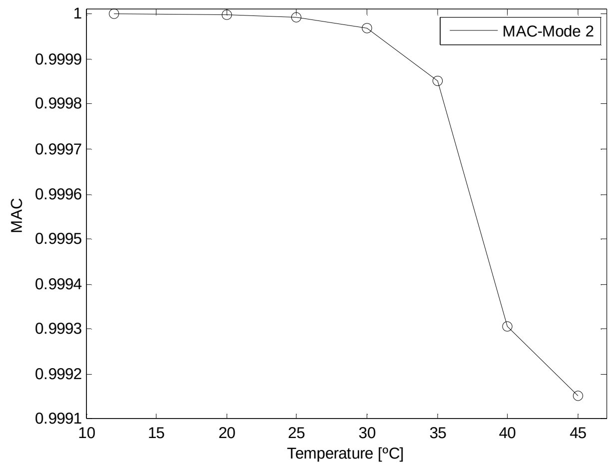

Figure 12: MAC for mode 2.

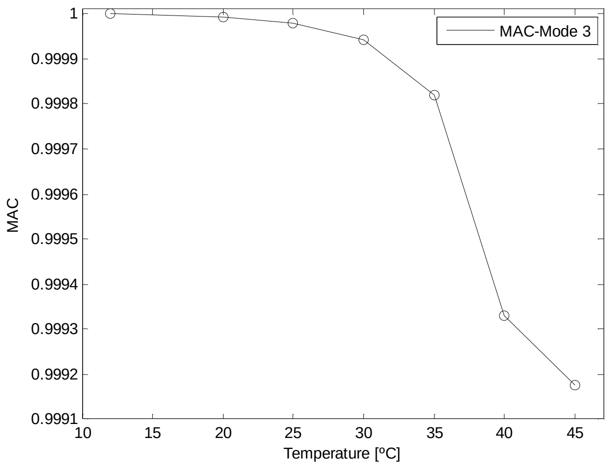

Figure 13: MAC for mode 3.

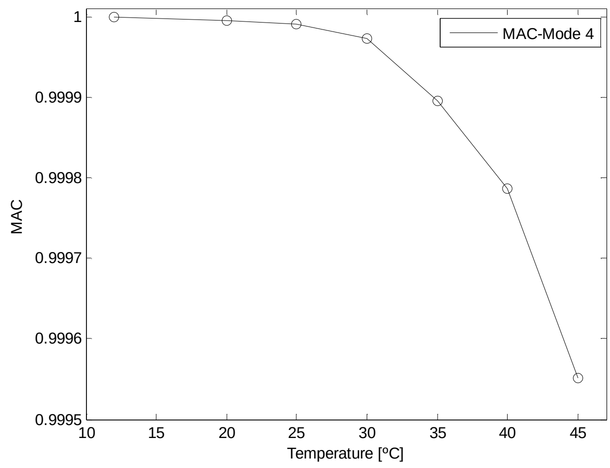

Figure 14: MAC for mode 4.

5 CONCLUSIONS

- Operational modal analysis has been used to estimate the modal parameters of a free-free laminated glass beam at different temperatures. The results have been used to validate the predictions given by the analytical model of Ross, Kerwin and Ungar.
- The analytical predictions for the natural frequencies are in good agreement with the experimental values obtained by OMA. However, significant discrepancies have been encountered between the experimental loss factors and those predicted with the RKU model.
- The natural frequencies decrease with increasing temperature whereas the loss factors increase with increasing temperature. On the other hand, there are not significant discrepancies in the mode shapes.

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