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RESEARCH OF DYNAMICS OF A VERTICAL SYMMETRIC OUT-OF-BALANCE ROTOR WITH THE CAVITY PARTLY FILLED WITH LIQUID IN VIEW OF NONLINEARITY OF ELASTICITY OF SUPPORT AND FOUNDATION

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Abstract. *Nowadays most rotary machines which are widely used in industry and engineering rotate on frictionless bearings. Slide bearings with liquid or gas lubricant, despite having some advantages over frictionless bearings, do not find widespread application in certain areas of industry and engineering. The reason for that is the action of the lubricant layer, which causes occurrence of self-oscillations in the rotor system, which are accompanied by significant amplitudes and result in fast failure of bearing units. Here dynamics of the rotor established vertically on the elastic foundation, revolving on frictionless bearings is investigated. Frictionless bearings can be considered as a model of absolutely rigid support – pivoted mount, if a single-row ball-bearing is applied; and built-in support, if in each support a pair of twin frictionless bearings is established. However, today such an approach, while simplifying the engineering task, does not give the exact enough solution. As a mathematical model of frictionless bearing it is important to choose models which much fuller reflect distinctive features of frictionless bearings, such as geometry errors, in particular, backlashes influence, and their change while in service as well; nonlinear rigidity properties; influence of centrifugal forces of rolling bodies, mutual displacement and skew of bearing rings; gyroscopic phenomena; influence of friction force in the lubricant layer and factors, which determine thickness of the lubricant layer in the junction of rings and rolling bodies.*

1 INTRODUCTION

In connection with the increased requirements for rotation accuracy and the increase in speeds of rotor revolution, there is a necessity of the account of elastic properties of frictionless bearings. Here, as the most essential factor which influences the rotor dynamics, we accept nonlinear rigidity properties of the frictionless bearing, when the radial pliability arises due to deformation of rolling bodies on tracks in places of junction, in particular [1]. The solution for the given problem also becomes more complicated because of the fact, that movement of the revolving rotor and movement of liquid in its cavity are interconnected. That causes change of frequency of the forced oscillations and occurrence of instability, and the solved equations system consists of the connected equations of the solid body movement, equations of the continuum and boundary conditions for liquid. At research of dynamics of high-speed rotors it is required to make a comparative rating of the listed factors, to take the action of the most essential ones into account and to neglect less important ones, to make the problem foreseeable and solvable.

2 ELASTIC PROPERTIES OF ROLLING BEARING

Elastic deformations arise in radial and axial directions. The radial pliability of bearings arises due to deformations of rolling bodies and tracks in places of contact. Equations of static balance of the bearing are made up according to Hertz theory. According to this theory relative approach of internal and external rings of the bearing is in the direction of the vector of radial loading:

$$\delta = \frac{1}{k} p^n . \qquad (1)$$

Exponent n is accepted for ball bearings equal 2/3 and for roller bearings 9/10. The factor of proportionality k depends on the material and form of the surfaces, which contact with each other in the bearing. For various types of rolling bearings k has its own values. For example, for bearings of type 200 and 206 it is accordingly equal $0,409\times10^6$ and $0,746\times10^6$ kgc/sm^3. Apparently from formula (1) restoring force of the radial ball-bearing is equal:

$$P(x_1) = kx_1^{\frac{3}{2}} , \qquad (2)$$

where x_1 – absolute size of approach of rings. For reduction to a more suitable kind and convenience of application of expression (2) it is approximated by its power series:

$$P^*(x_1) = a^* x_1 + b^* x_1^3 + ...$$

Function $P^*(x_1)$ should be symmetric concerning x_1, therefore there are no members with even degrees x_1. We shall be limited to a member containing x_1 in the third degree. Factors a^* also b^* get out of the condition of minimization:

$$\begin{aligned} &\frac{\partial}{\partial a^*}\int_{x_0}^{x_1^*}[P(x_1) - P^*(x_1)]dx_1 = 0 , \\ &\frac{\partial}{\partial b^*}\int_{x_0}^{x_1^*}[P(x_1) - P^*(x_1)]dx_1 = 0. \end{aligned} \qquad (3)$$

Calculating integrals (3) in limits from 0 up to A and further partial derivatives, we find:

$$a^{*}=0,584kA^{\frac{1}{2}},\ b^{*}=0,455kA^{-\frac{1}{2}}. \tag{4}$$

If to accept a limit of change x_1 equal $A=1$,

$$P^{*}(x_1)=0,584kx_1+0,455kx_1^3\ . \tag{5}$$

3 DYNAMICS OF THE RIGID SYMMETRIC UNBALANCED EMPTY ROTOR ROTATING ON ROLLING BEARINGS AND ESTABLISHED ON THE MASSIVE ELASTIC BASE

Let us consider dynamics of a symmetric unbalanced vertical rotor with weight m and rotating on rolling bearings with nonlinear rigid characteristic of kind (5). The external ring is rigidly connected to the base of weight M, which is established on an elastic support, with linear factor of rigidity c_2.

3.1 The Equations of Movement of the System and their Solution

For drawing up of the equation of movement of the system we shall enter motionless system of coordinates *OxyZ*. We consider that in an equilibrium state the geometrical center of the shaft (rotor) and the centre of gravity of the base coincides with axis *Oz*. We shall designate coordinates in the displaced position of the center of a shaft (rotor) O_1 as x and y, and the centre of gravity of a rotor as x_s and y_s. We shall designate coordinates of the centre of gravity of base O_2 as x_2 and y_2. We believe that the rotor makes plane-parallel movement, and rotation of the base around of coordinate axes is absent. We consider angular speed of the rotor (shaft) $\Omega_0=const$ large enough, that it lies beyond the critical speed of the rotor, c_0 and c_1 – factors of rigidity of the support of the rotor (rigidity of the rolling bearing), χ and χ_0 – factors of external frictions.

After calculation of the kinetic energy of the system, potential energy of the isotropic elastic field of the support and isotropic nonlinear-elastic field of the rolling bearing, in view of force of external resistance and dissipative function the equations of movement of the system look like:

$$\begin{aligned}
&m\ddot{x}+2c_0(x-x_2)+2c_1(x-x_2)^3+\chi\dot{x}=me\Omega_0^2\cos\Omega_0 t,\\
&m\ddot{y}+2c_0(\mathrm{y}-y_2)+2c_1(\mathrm{y}-y_2)^3+\chi\dot{y}=me\Omega_0^2\sin\Omega_0 t,\\
&M\ddot{x}_2+2c_2x_2-2c_0(x-x_2)-2c_1(x-x_2)^3+\chi_0\dot{x}_2=0,\\
&M\ddot{y}_2+2c_2y_2-2c_0(\mathrm{y}-y_2)-2c_1(\mathrm{y}-y_2)^3+\chi_0\dot{y}_2=0.
\end{aligned} \tag{6}$$

The first and third equations of system (6) are not connected with the second and fourth equations and it is possible to search for their solutions independently from each other. Further on we shall use solutions of the first and third equations. We shall write down the first and third equations as:

$$\begin{aligned}
&\ddot{x}+k_0^2(x-x_2)+k_1(x-x_2)^3+2n\dot{x}=e\Omega_0^2\cos\Omega_0 t,\\
&\ddot{x}_2+k_2^2x_2-k_{01}^2(x-x_2)-k_{10}(x-x_2)^3+2n_0\dot{x}_2=0,
\end{aligned} \tag{7}$$

here $k_0^2=\dfrac{2c_0}{m},\ k_1=\dfrac{2c_1}{m},\ 2n=\dfrac{\chi}{m},\ k_{01}^2=\dfrac{2c_0}{M},\ k_{10}=\dfrac{2c_1}{M},\ 2n_0=\dfrac{\chi_0}{m},\ k_2^2=\dfrac{2c_2}{M}.$

For definition of periodic solutions of nonlinear system (7) it is possible to take advantage of decomposition in Fourier series

$$x_i = a_{0i} + a_{1i}\cos\omega t + b_{1i}\sin\omega t + a_{2i}\cos 2\omega t + b_{2i}\sin 2\omega t + \ldots \,, \qquad (8)$$

when equating factors at identical harmonics of functions of cosine and sine it is possible to receive an infinite system of nonlinear-algebraic equations concerning unknown factors $a_{0i},\, a_{ji},\, b_{ji}$.

Considering that the system makes harmonious fluctuations, at the approached calculations in (8) it is enough to take into account the first harmonics.

3.2 Research of proper fluctuations of the system

At research of proper fluctuations of the system we shall present the solution as

$$x = a\cos\omega t,\; x_2 = b\cos\omega t \,, \qquad (9)$$

where a, b – amplitudes, ω – frequency of proper fluctuations of the system.

As time obviously does not enter the system of the equations (7) (the system is self-contained), choosing in an appropriate way the reference mark of time, it is possible to secure factors at $\sin\omega t$ turn into zero. As the system is symmetric we can also accept free factors in (9) as equal to zero.

Substituting Eqs.(9) in Eqs.(7) and using Galerkin’s method, we shall receive the system of nonlinear (algebraic) equations concerning a and b:

$$\begin{aligned} &-\omega^2 a + k_0^2(a-b) + \frac{3}{4}k_1(a-b)^3 = 0, \\ &-\omega^2 b + k_2^2 b - k_{01}^2(a-b) - \frac{3}{4}k_{10}(a-b)^3 = 0, \end{aligned} \qquad (10)$$

resolving which we have

$$a^2 = \frac{4[\omega^2(k_2^2-\omega^2) - k_0^2(k_2^2-(1+\mu)\omega^2](k_2^2-\omega^2)^2}{3k_1(\mathrm{k}_2^2-(1+\mu)\omega^2)^3} \,, \qquad (11)$$

$$b^2 = \frac{4\omega^4\mu^2[\omega^2(k_2^2-\omega^2) - k_0^2(k_2^2-(1+\mu)\omega^2]}{3k_1(\mathrm{k}_2^2-(1+\mu)\omega^2)^3} \,. \qquad (12)$$

Here $\mu = \dfrac{m}{M}$ – relation of weight of the rotor to weight of the base.

From expressions (11) and (12) we define amplitudes of proper fluctuations of the system depending on frequency of proper fluctuations ω (skeletal curve systems).

At $k_1 = 0$ and $k_{10} = 0$ from system Eqs.(10) we shall receive the equation for definition of critical speeds of linear system

$$\omega^4 - (k_0^2 + k_2^2 + k_{01}^2)\omega^2 + k_0^2 k_2^2 = 0 \,, \qquad (13)$$

whence

$$\omega_{1,2}^2 = \frac{1}{2}(k_0^2 + k_2^2 + k_{01}^2) \pm \frac{1}{2}\sqrt{(k_0^2 - k_2^2)^2 + 2k_2^2 + k_{01}^2 + k_{01}^4 + 2k_0^2 k_{01}^2} \,. \qquad (14)$$

If the factor of rigidity c_1 (factor at x^3) for the rolling bearing is positive, the characteristic of the system will be rigid.

Apparently from formulas (11), (12), at $\omega \to \sqrt{\dfrac{2c_1}{m+M}}$ amplitudes of proper fluctuations *a* and *b* infinitely grow, i.e. the phenomenon of resonance will take place. Straight line $\omega = \sqrt{\dfrac{2c_1}{m+M}}$ is the vertical asymptote of skeletal curves, which are determined by formulas (11) and (12). The amplitude of proper fluctuations of rotor *a* will be equal to zero, when proper frequency of system ω will accept value

$$\omega = \sqrt{\frac{2c_2}{M}} \quad . \tag{15}$$

Expression for amplitude of proper fluctuations of the base will become

$$b^2 = \frac{4\mu^2\omega^4 k_0^2 \mu\omega^2}{3k_1\mu^3\omega^6} = \frac{4k_0^2}{3k_1},$$

thus the base goes in the direction opposite to the direction of movement of the rotor.

3.3 The Compelled Nonlinear Fluctuations of the Rotor on the Elastic Base Rotating on Rolling Bearings

At research of the compelled fluctuations of the rotor and base, caused by unbalance of the rotor, for simplification of research of the compelled fluctuations of the system and for convenience of calculations, we include phase corner ε in the expression of revolting force $me\Omega_0^2\cos(\Omega_0 t+\varepsilon)$, i.e. equations of the compelled fluctuations are represented as

$$\begin{aligned} &\ddot{x} + k_0^2(x-x_2) + k_1(x-x_2)^3 + 2n\dot{x} = e\Omega_0^2\cos(\Omega_0 t+\varepsilon), \\ &\ddot{x}_2 + k_2^2 x_2 - k_{01}^2(x-x_2) - k_{10}(x-x_2)^3 = 0. \end{aligned} \tag{16}$$

We shall present the approached compelled fluctuations of the system as

$$x = a_1\cos\Omega_0 t, \quad x_2 = b_1\cos\Omega_0 t. \tag{17}$$

After substitution Eqs.(17) in system Eqs.(16) and equating of factors at identical harmonics of functions $\cos\Omega_0 t$ and $\sin\Omega_0 t$, we shall receive:

$$\begin{aligned} &-a_1\Omega_0^2 + k_0^2(a_1-b_1) + \frac{3}{4}k_1(a_1-b_1)^3 = e\Omega_0^2\cos\varepsilon, \\ &2n\Omega_0 a_1 = e\Omega_0^2\sin\varepsilon, \end{aligned} \tag{18}$$

$$-b_1\Omega_0^2 + k_2^2 b_1 - k_0^2(a_1-b_1) + \frac{3}{4}k_{10}(a_1-b_1)^3 = 0. \tag{19}$$

Here, as well as before, factor at $\cos 3\Omega_0$ is unbalanced. This assumption is justified by the fact, that the amplitude of compelled fluctuations of the third harmonic is much smaller than the amplitude of the basic harmonic.

From system Eqs.(18), excluding phase corner ε, we shall receive:

$$[(k_0^2-\Omega_0^2)a_1-k_0^2b_1+\frac{3}{4}k_1(a_1-b_1)^3]^2+4n^2\Omega_0^2a_1^2=e\Omega_0^4\ . \quad (20)$$

Having divided the second equation of system Eqs.(18) into the first one, we shall receive expression for ε:

$$tg\varepsilon=\frac{2na_1\Omega_0}{(k_0^2+\Omega_0^2)a_1-k_0^2b_1+\frac{3}{4}k_1(a_1-b_1)^3}\ . \quad (21)$$

Adding up two equations of system (6) we shall receive one equation, from which with the account of (6) and (17) we find:

$$\left[(2c_2-M\Omega_0^2)b_1-m\Omega_0^2a_1\right]^2+\chi^2\Omega_0^2a_1^2=m^2e^2\Omega_0^4. \quad (22)$$

From here we have:

$$(b_1)_{1,2}=\frac{\pm\Omega_0\sqrt{\mu^2e^2\Omega_0^2-4n_1{}^2a_1^2}+\mu\Omega_0^2a_1}{(k_2{}^2-\Omega_0^2)}\ . \quad (23)$$

Substituting Eq.(23) in Eq.(19) we shall receive the equation concerning amplitude of the rotor as:

$$\frac{(\ k_2^2+k_{01}^2-\Omega_0^2)(\pm\Omega_0\sqrt{\mu^2e^2\Omega_0^2-4n_1^2a_1^2}+\mu\Omega_0^2a_1)}{k_2^2-\Omega_0^2}-k_{01}^2a_1-$$
$$-\frac{3}{4}k_{10}(a_1+\frac{\pm\Omega_0\sqrt{\mu^2e^2\Omega_0^2-4n_1^2a_1^2}+\mu\Omega_0^2a_1}{k_2^2-\Omega_0^2})^3=0\ , \quad (24)$$

here $2n_1=\dfrac{\chi}{M}$ – the factor describing damping of external friction.

With the use of equation (24) we construct the resonant curve, i.e. dependence of amplitude of compelled nonlinear fluctuations of the rotor on angular speed of the rotor Ω_0. Considering that factor n_1 is much smaller than unit, which allows to neglect value $4n_1^2a_1^2$ in comparison with value $\mu^2e^2\Omega_0^2$, we find from Eq.(24):

$$\frac{(k_2^2+k_{01}^2-\Omega_0^2)(\Omega_0\mu(e+a_1)-k_{01}^2(k_2^2-\Omega_0^2)a_1}{k_2^2-\Omega_0^2}-\frac{3k_{10}\left\{\left[(1+\mu)\Omega_0^2-k_2^2\right]\Omega_0^2\mu e\right\}^3}{4(k_2^2-\Omega_0^2)^3}=0\,. \quad (25)$$

This expression defines the amplitude-frequency characteristic of the rotor without taking damping into account.

Transforming Eq.(25), we shall receive

$$a_1^2(\omega^2)+Q=0\ , \quad (26)$$

here
$$Q_\pm=\frac{4\left(\pm\Omega_0\sqrt{(\frac{\mu e\Omega_0}{a_1})^2-4n_1^2}\right)(k_2^2+k_{01}^2-\Omega_0^2)(k_2^2-\Omega_0^2)}{3\left[(k_2^2-\Omega_0^2(1+\mu)\right]^3k_{10}}+$$

$$+a_1^2\left\{\frac{\pm\Omega_0\sqrt{(\frac{\mu e\Omega_0}{a_1})^2-4n_1^2}}{\left[k_2^2-(1+\mu)\Omega_0^2\right]}-1\right\}^3, \tag{27}$$

$$a_1(\omega^2)=\frac{4\left[\mu\Omega_0^2(k_2^2-\Omega_0^2)-k_{01}^2\left(k_2^2-(1+\mu)\Omega_0^2\right)\right](k_2^2-\Omega_0^2)}{3k_{01}\left(k_2^2-(1+\mu)\Omega_0^2\right)^3}. \tag{28}$$

The curve constructed with the help of the first item of equation (26), as it is known, defines the amplitude-frequency characteristic of proper fluctuations of the rotor (skeletal curve).

For nonlinear system an opportunity of occurrence of some periodic modes at change of angular speed of the rotor Ω_0 in certain limits is an essential feature. Vertical lines $\Omega_0 = k_2$ are asymptotes of amplitude-frequency characteristic of the system.

Substituting the obtained values of amplitude of the rotor a_1 in Eq.(23), we shall receive expression for b_1, on which we construct dependence $b_1(\Omega_0)$, i.e. the amplitude-frequency characteristic of the base.

3.4 Effect of Self-Centering

For studying of the effect of self-centering we define limit values of amplitudes a and b with the help of equalities (23) and (26) at unlimited increase of angular velocity of the rotor Ω_0. At $\Omega_0 \to \infty$ from (23) and (26) we shall receive

$$\lim_{\Omega_0\to\infty} a_1 = -e,\ \lim_{\Omega_0\to\infty} b_1 = -(a_1 \pm e)\mu \tag{29}$$

At direct precession we have $\lim\limits_{\Omega_0\to\infty} a_1 = -e$, $\lim\limits_{\Omega_0\to\infty} b_1 = 0$. Hence, at unlimited increase of angular velocity of rotation of the rotor, the vector of static unbalance goes to point O and aspires to combine the axis of rotation of the rotor with the vertical coordinate axis Oz. Thus static steadiness of the rotor is eliminated.

At substitution of (29) with the account of (17) in the expression of coordinate of the centre of gravity (inertia) of system $x_c = \dfrac{(mx_s + Mx_2)}{(m+M)}$, we shall receive that $x_c = 0$; similarly, we have $y_c = 0$. That also confirms the existence of effect of self-centering in the given rotor system.

The effect of self-centering of the rigid rotor, established on rolling bearings with elastic base, is a prominent feature of the specified system.

3.5 Pressure between the Rotor and the Base. Pressure upon Support.

For definition of pressure between the rotor and the base (bearings), "having rejected" mentally the rotor, it is possible to replace its action upon the base with required reactions. As the rotor is symmetric, it is enough to find reactions for one bearing of the rotor.

As each support of bearings has identical reactions, the rotor makes plane-parallel movement. The equation of movement of the base looks like:

$$M\ddot{x}_2 + 2c_2x_2 = 2R_{1x},$$

$$R_1 = \frac{(2c_2 - M\Omega_0^2)b_1}{2}.$$

From here it follows, that if between weight of the base, factors of rigidity of elastic field of support and angular velocity of the rotor there exist dependences of kind

$$M = \frac{2c_2}{\Omega_0^2},$$

then radial pressure between the rotor and the base equals zero. Hence, if operating speed is constant, it is possible to obtain factor of rigidity of elastic field of weight of the base, so that on the set frequency of rotation the pressure between the rotor and bearings equals zero.

For definition of pressure upon support, "having rejected" mentally the base, it is possible to replace its effect on support with its reactions. Then reaction of the base to support or the pressure between the base and its support becomes:

$$R_2 = c_2 b_1 ,$$

where b_1 – amplitude of fluctuations of the base. Pressure upon support will equal zero at unlimited increase of angular velocity of rotor Ω_0, i.e. at occurrence of the phenomenon of self-centering of the rotor system.

4 CONCLUSIONS

- The equations of movement of the unbalanced rotor are made up and solved in view of nonlinearity of the elastic support.
- The expressions allowing simply enough to carry out research of proper and compelled fluctuations of the system are received.
- Conditions of existence of effect of self-centering of the system are defined.
- Conditions of the minimal pressure upon support of the rotor system are determined.

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