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INNOVATIVE CONCEPT MODELLING OF SANDWICH BEAM-LIKE STRUCTURES

Giovanni De Gaetano*[1,2], Francesco I. Cosco[2,3], Carlos Garre[1], Carmine Maletta[1], Stijn Donders[4], Domenico Mundo[1]

[1]Università della Calabria, Dipartimento di Meccanica, Energetica e Gestionale (DIMEG), Italy
carlos.garre@gmail.com, {giovanni.degaetano, carmine.maletta, domenico.mundo}@unical.it

[2]G&G Design and Engineering, Via G. Barrio, 87100 Cosenza, Italy
francesco.cosco@gegde.com

[3]PMA division, KU Leuven, Belgium

[4]Simulation Division - LMS International, Interleuvenlaan 68, 3001 Leuven, Belgium
stijn.donders@lmsintl.com

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Abstract. *Sandwich structures are widely used in many technical applications, because their composition combines high rigidity and strength with a good energy absorption, keeping low weights. Their static and dynamic behaviour can be studied by performing series of experimental tests, which, however, are expensive and require much setting and execution times. For this reason, it is common to use Finite Element (FE) simulation models, achieving good static and dynamic accuracy. However, difficulties in defining and modifying a complex model led to the development of simplified models, such as 3D equivalent models. These homogeneous models are based on specific laws and have geometric and stiffness characteristics equivalent to those of complex models. Many efforts have been spent to obtain models resembling the characteristics of honeycomb structures. These models have reached accurate static prediction performance, but obtaining a good accuracy for dynamic loads is still a challenge. Concept modelling approaches proved very useful for defining equivalent reduced models, able to reduce computational resources as well as the time needed for model modifications. In this paper, a dynamic FE-based method is used to obtain a concept model of honeycomb sandwich beams, that can reproduce accurately their static and dynamic behaviours. The method consists of two steps. First, a detailed FE model of one honeycomb beam-structure is developed and validated against experimental data obtained from literature. Its natural frequencies are estimated by means of a modal analysis in free-free conditions. Then, the analytical modal model of the beam is used to derive cross-sectional stiffness properties of the equivalent 1D concept beam from the frequencies estimated by analysing the original 3D model. The analysis of a sandwich beam with a honeycomb aluminium core is presented as an application case to assess the accuracy of the proposed method.*

1 INTRODUCTION

In the last decades, the industrial use of composite materials appeared as a promising technology, able to increase the structural lightness, maintaining and often even improving static and dynamic performances. In particular, honeycomb sandwich structures are widely used in weight-sensitive structures, where high energy absorption and good vibration damping are required [1-3]. These structures consist of three elements: the face sheets, the core and the adhesive interface layers. By choosing different materials and geometric configurations for face sheets and cores, it is possible to adjust the structural properties to the desired performance [4]. Analytical methods are sometimes unable to predict the behaviour of these structures, especially when they have complex geometry or boundary conditions and with some types of materials. Recently, analytical methods have been gradually replaced by numerical methods, such as the Finite Element Method (FEM), which can handle highly complex problems, achieving a good accuracy. However, the typical disadvantage of these methods, i.e. the long time required to define and modify the models, urges the engineers to model the structures through alternative approaches, such as equivalent methods and concept modelling methods.

The equivalent methods are based on the concept of homogenization. With this assumption, the complex component is considered as a continuous model, thus avoiding all the problems caused by the structure’s heterogeneity [5-7]. Such methods implement an equivalence of mass and stiffness between the honeycomb sandwich structure and the equivalent one. They are classified according to the mathematical assumptions on displacement or deformation fields (order of shear deformation, Bernoulli or Timoshenko hypotheses, etc) [8], or on the basis of the equality nature implemented by the method. The latter may concern the entire model (equivalent isotropic material) or just the honeycomb core (equivalent orthotropic material) [9].

Instead, concept modelling techniques, which in recent years have increased their impact in different fields of industry, especially in the automotive field, allow to obtain simplified models from a 3D model of complex structures, reducing the required computational resources as well as the time needed for their modification. For elements showing a beam-like global behaviour, it is possible to reduce a 3D model into a 1D model with equivalent stiffness properties, by following a static FE-based approach or a dynamic FE-based approach.

In the first case, a set of static load cases is applied to the original structure and the linear elastic load-deformation relationships are resolved, in which the stiffness characteristics are the only unknown quantities [10-11]. The second method, discussed in this work, is based on a dynamic approach [12], in which the stiffness characteristics of equivalent 1D beam elements are estimated using the natural frequencies computed by a modal analysis of the original 3D FE model of the structure.

This paper aims at extending the dynamic FE-based approach for the case of complex structures, such as honeycomb sandwich beam. To assess the accuracy of the method, a comparative dynamic analysis based on natural frequencies is carried out between a detailed 3D model of a honeycomb beam, a 3D equivalent model and the proposed dynamic-based 1D concept beam.

The paper is organized as follows: Section 2 describes the reference models, i.e. the full 3D model and the equivalent 3D model, which will be used for comparison with our method; both models are validated against experimental data taken from the literature. Section 3 presents the proposed method for 1D concept modelling of beams, describing both the analytical model and the numerical implementation. An application case is reported in Section 4, comparing the dynamic performance of our method with respect to the reference models. Section 5 concludes the paper, summarizing the presented work and the obtained results.

2 3D FINITE ELEMENT MODELLING OF HONEYCOMB SANDWICH BEAMS

In this section an application model of a honeycomb sandwich structure, derived from the literature [13] is described. It will be used as reference case in the following sections, with the aim of assessing the predictive capabilities of the proposed concept modelling method. The sample structure is a honeycomb sandwich beam, with rectangular cross-sectional area and hexagonal honeycomb core. The top and bottom panels have equal facing skin thickness. The frequencies of the first vertical and lateral bending modes of the structure, measured in clamped-free conditions, are reported in [13]. The geometric characteristics of the beam are indicated in Table 1, while the three-dimensional geometrical model of the unit honeycomb cell is shown in Figure 1.

beam length (a)	beam width (b)	thickness of cell (t_c)	cell size (d)	core height (h_c)	thickness of face skin (t_f)	angle of cell (ϑ)
290 mm	40 mm	0.2 mm	2 mm	9 mm	1 mm	60°

Table 1: Geometric characteristics of honeycomb sandwich beam.

The same material has been assumed for both face skins and core parts, i.e. an aluminium having the following properties:

- Young's modulus: E = 72 GPa;
- Poisson's ratio: ν = 0.33;
- mass density: $\rho = 2.8\cdot10^{-9}$ ton/mm^3 .

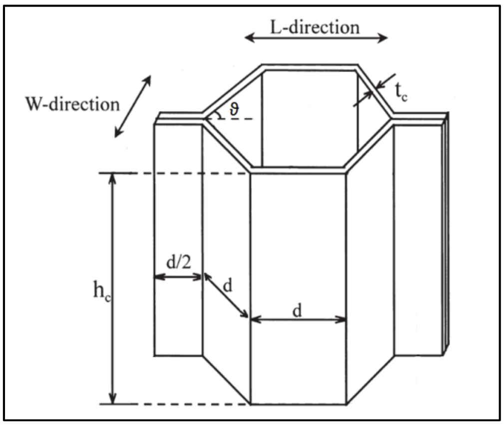
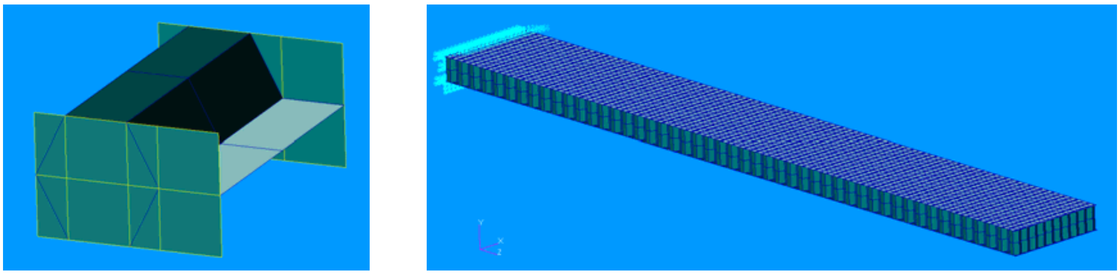


Figure 1: Unit cell geometry of honeycomb core.

2.1 Detailed 3D model

A detailed 3D FE model of the beam, with the same characteristics of the experimental sample analysed in [13], has been created. Both the skins and the core parts were meshed using 2D shell elements, with four corner nodes. The two adhesive layers, linking the face sheets of the skin and the core, were not taken into account, because their effect on the dynamic behaviour of the structure can be considered negligible in normal conditions. Therefore, an ideal contact between the parts, without the presence of any delamination, has been assumed. This condition can be achieved with a perfect coincidence in the contact surfaces between the extreme nodes of the core and those of the skin faces. For this purpose, the mesh of a single hexagonal unit has been created, as shown in Figure 2a), and subsequently it has been replicated along two directions, in order to obtain the whole beam model.



a) b)

Figure 2: Mesh of a single cell (a) and detailed FE model of clamped honeycomb sandwich beam (b).

The mesh of the core has been created as consisting of two elements along the height of each cell unit. The final cantilever model, resulting in approximately 110000 Degrees of Freedom (DoFs), is shown in Figure 2b).

By executing a numerical modal analysis on this model, the natural frequencies of the structure have been estimated and the first three values have been compared with the experimental data reported in [13].

Bending Modes	**Natural Frequencies (Hz)**		
	Experimental Data [13]	3D Finite Element Model	Frequency Difference (%)
1^{st} vertical	134.5	126.88	-5.67%
1^{st} lateral	311	323.62	4.06%
2^{nd} vertical	711	713.02	0.28%

Table 2: Dynamic results comparison between experimental data and finite element model.

Table 2 shows a good agreement between the experimental data and the frequencies predicted by the FE model, with a relative error which remains below the value of 6%. The third frequency is particularly accurate and the difference is less than 0.5%.

The higher errors in the first two natural frequencies are probably due to the different boundary conditions between the modelled and the experimental case, determined by the clamping system. However, the model with this mesh morphology can be considered reliable and can be used as a reference model for the comparison between the 3D equivalent model and our 1D concept model.

2.2 Equivalent 3D model

The so-called 3D equivalent model consists of a homogeneous single-skin model with characteristics that are equivalent to those of the detailed 3D model, determined by imposing static equalities. The equivalent rigidity method [5,14] has been used in the work presented in this paper. Figure 3 shows a scheme of the equivalent rigidity method.

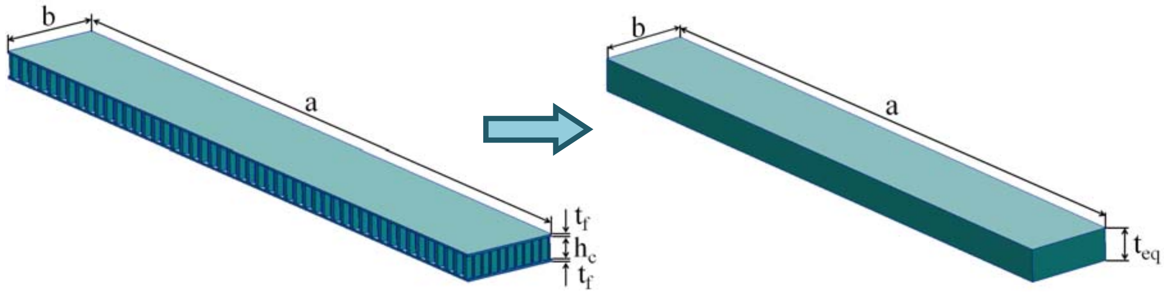


Figure 3: A schematic of the equivalent rigidity method.

This method assumes that the top and bottom surface layers offer resistance to axial and bending deformations, while the core offers resistance only to shear deformation and satisfies the Bernoulli hypotheses.

So, equivalent beam thickness (t_{eq}) and Young's modulus (E_{eq}) are defined in such a way that the axial and bending stiffness of the skin beam are equivalent to those of the sandwich structure. Therefore, the equivalent characteristics of the beam are estimated from the following equations:

$$\begin{aligned} t_{eq}E_{eq} &= 2t_f E_f \\ \frac{1}{12}t_{eq}^3 E_{eq} &= \frac{1}{12}\left[\left(h_c + 2t_f\right)^3 - h_c^3\right]E_f \end{aligned} \tag{1}$$

where E_f represents the Young's modulus of facing material. The length (a) and the width (b) of the beam remain the same as those of the original model.

By solving these equations, t_{eq} and E_{eq} are obtained as follows:

$$\begin{aligned} E_{eq} &= \frac{2t_f}{t_{eq}}E_f \\ t_{eq} &= \sqrt{3h_c^2 + 6h_c t_f + 4t_f^2} \end{aligned} \tag{2}$$

Regarding the material characteristics, the equivalent mass density (ρ_{eq}) is obtained by identifying a unit cell and deriving the volume fraction occupied by metal:

$$\rho_{eq} = \frac{2\rho_f t_f + \rho_{ca}h_c}{t_{eq}} \tag{3}$$

where ρ_f is the density of facing material and ρ_{ca} is the average density of honeycomb core.

If we neglect the contribution of the adhesive used for joining honeycomb core cells, such average density can be written as:

$$\rho_{ca} \cong \frac{8}{3\sqrt{3}}\frac{t_c}{d}\rho_c \tag{4}$$

where ρ_c represents the density of honeycomb core material. These three equivalent parameters (E_{eq}, t_{eq} and ρ_{eq}) characterize the single skin model of equivalent honeycomb beam.

3 DYNAMIC FE-BASED 1D METHOD FOR BEAM-LIKE STRUCTURES

The dynamic FE-based method [12] aims at defining a 1D FE concept model of beam-like structures, starting from the targeted natural frequencies of 3D detailed beam model and the differential equations of beam vibrations, based on the analytical Timoshenko model [15]. Considering a beam with double-symmetric section, the cross-section centre of gravity is coincident with the shear centre. This assumption leads to consider a set of uncoupled differential equations for flexural and torsional vibrations.

3.1 Analytical Model

First, a Cartesian reference frame (x, y, z) is defined, with the origin at the geometric centre of beam section, where x is the neutral axis, y and z are the principal and the secondary bending directions respectively. By setting the external excitations to zero in the flexural and torsional equations of motion, the uncoupled ordinary differential equations, governing the modal behaviour of the beam, are as follows [15]:

$$\begin{gathered} EI_b \frac{d^4 w_b}{dx^4} + \omega_f^2 m \frac{EI_b}{GA_s} \frac{d^2 w_b}{dx^2} - \omega_f^2 m w_b = 0 \\ EI_w \frac{d^4 \psi}{dx^4} - GI_t \frac{d^2 \psi}{dx^2} - \omega_t^2 J_t^o \psi = 0 \end{gathered} \tag{5}$$

where the upper equation refers to flexural vibrations, whereas the second to torsional vibrations. The physical meaning of each parameter is given in Table 3.

A_s	shear area	J_t^o	polar moment of inertia about centre of gravity
l	half length of the beam	I_t	torsional modulus
L	full length of the beam	I_w	warping modulus
m	distributed mass	E	young's modulus
w	beam deflection	G	shear modulus
ψ	twist angle of cross-section	ω_f	generic flexural natural frequency
I_b	cross-section moment of inertia	ω_t	generic torsional natural frequency

Table 3: Nomenclature of characteristics of general beam.

For flexural vibrations, by solving the characteristic equation and imposing the free-free boundary conditions, the following non-linear frequency equations are obtained:

$$\begin{gathered} \alpha\, tanh(\alpha \cdot l) + \beta tan(\beta \cdot l) = 0 \\ \alpha\, tan(\beta \cdot l) - \beta tanh(\alpha \cdot l) = 0 \end{gathered} \tag{6}$$

where the first equation refers to symmetric modes and the second to the anti-symmetric modes. In both equations, the functions α and β are defined as follows:

$$\begin{gathered} \alpha = \sqrt[4]{p} \cdot \sqrt{\sqrt{1+q^2} - q} \\ \beta = \sqrt[4]{p} \cdot \sqrt{\sqrt{1+q^2} + q} \end{gathered} \tag{7}$$

Here, the coefficients p and q are defined as:

$$p = \frac{\omega_f^2 m}{EI_b}$$
$$q = \frac{\omega_f \sqrt{EI_b m}}{2GA_s} \tag{8}$$

For the torsional vibrations, the characteristic equation leads to the following frequencies:

$$\omega_{t,n} = \frac{n\pi}{L}\sqrt{\frac{GI_t}{J_t^o}}\sqrt{1+\left(\frac{n\pi}{L}\right)^2\frac{EI_w}{GI_t}} \qquad n = 0,1,2,\ldots \tag{9}$$

In the last equation, frequencies for both symmetric and anti-symmetric modes are considered, depending on whether the value of n is even or odd, respectively.

3.2 Numerical Method

Equations (6) and (9) are used as a basis to develop two different numerical methods for the flexural and torsional modes, respectively. In both methods, an unconstrained nonlinear minimization algorithm is used and the first three flexural or torsional natural frequencies are used, in order to estimate the beam section properties.

For flexural vibrations, I_b and A_s are the design parameters that must be optimized; instead, for torsional vibration, such parameters are I_t and I_w. The optimal values are computed after setting initial guess values and adopting the Nelder-Mead simplex algorithm [16].

The goal is to minimize the squared sum of the differences between the reference frequencies vector, obtained from the initial dynamic simulation on the detailed 3D FE model, and the frequencies vector iteratively computed by applying equations (6) and (9). The material properties of the beam are assumed to be known, while the value of the polar moment of inertia is assumed to be given by the sum of the two moments of inertia, estimated in the flexural analysis cases, multiplied by the density of the material.

Once the optimal values of the shear areas A_{sy} and A_{sz} are computed, the shear factors of the conceptual model in the two bending directions are calculated as:

$$K_y = \frac{A_{sy}}{A_{eq}}$$
$$K_z = \frac{A_{sz}}{A_{eq}} \tag{10}$$

where A_{eq} is the equivalent cross-sectional area, calculated with the following analytical formula:

$$A_{eq} = \frac{V_{real}}{L} \tag{11}$$

In this formula, the real volume, V_{real}, is the volume fraction concerning the walls of the core and the skins. It can be calculated as:

$$V_{real} = \frac{M_c}{\rho_c} + \frac{M_f}{\rho_f} \tag{12}$$

where M_c and M_f represent respectively the total mass of core and skins.

4 APPLICATION CASE

This section presents an application case, aimed at comparing the dynamic accuracy of the equivalent 3D model and the accuracy of the proposed dynamic FE-based 1D model. For this purpose, an FE honeycomb beam model has been analysed, having the same geometric characteristics described in Table 1, with the exception of the beam length, which has been assumed equal to 1 meter in order to provide the structure with a beam-like behaviour. The detailed 3D model has about 3300 honeycomb cells and over 370000 DoFs.

For the equivalent 3D model, the formulas given in Section 2.2 have been used and the following values of the equivalent parameters have been calculated:

- equivalent Young's modulus: E_{eq}= 8.3 GPa;
- equivalent thickness: t_{eq}= 17.35;
- equivalent mass density: ρ_{eq}= $5.46 \cdot 10^{-10}$ ton/mm^3 .

The equivalent model consists of 3D solid elements, each of size 5 x 5 x 4.34 mm, and a number of DoFs approximately equal to 54000. The size of the mesh in the equivalent model has been chosen as a compromise between predictive capability and model dimension after running a convergence test.

Instead, for the equivalent 1D concept model, geometric and stiffness properties have been estimated by using the natural frequencies computed by a modal analysis on the 3D model and the numerical procedures described in Section 3.2.

In Table 5 the values estimated for the relevant stiffness parameters of the equivalent beam are listed. It is worth of notice that the warping modulus (I_w) converged to zero, which is consistent with the closed-section nature of the structure under study.

Parameters	**Values for 1D beam model**
A_{eq}	122.05 mm^2
K_y	0.190
K_z	0.332
I_y	9280.05 mm^4
I_z	1527.09 mm^4
I_t	2310.09 mm^4
I_w	0 mm^6

Table 5: Equivalent beam properties estimated by the dynamic FE-based method.

The parameter values listed in Table 5 have been used to define a concept model of the beam, formed by 10 one-dimensional, 100 mm long, beam elements. Each beam element has 2 nodes, for a total of only 66 DoFs. This number is much lower than that of the detailed 3D model, which results in a very significant reduction of the computational time needed for FE simulations.

After the three FE models have been created, a modal analysis in free-free conditions has been executed for each of them, in order to assess the accuracy of the two reduced models (equivalent 3D and concept 1D) in predicting the modal behaviour of the detailed 3D beam model.

The predicted flexural and torsional natural frequencies are listed in Table 6, together with the relative percentage error, calculated as the percentage difference between the frequencies of each reduced model and the values predicted by the detailed model. It can be observed that the 3D equivalent model approximates very precisely the lateral bending modes, with an error that remains below 2%. However, the differences for vertical bending and especially torsional vibrations, with values close to 10% and 15% respectively, are not negligible.

Modes		*Detailed 3D Model*	*Equivalent 3D Model*	*Frequency Difference (%)*	*Dynamic 1D Model*	*Frequency Difference (%)*
Vertical Bending	*1st*	63.804	69.438	8.83%	63.805	0.00%
	2nd	175.37	191.08	8.96%	175.43	0.03%
	3rd	342.3	373.67	9.16%	342.7	0.12%
Lateral Bending	*1st*	156.88	159.34	1.57%	156.88	0.00%
	2nd	428.13	434.71	1.54%	428.38	0.06%
	3rd	827.37	839.78	1.50%	828.57	0.15%
Torsion	*1st*	719.01	819.82	14.02%	721.71	0.38%
	2nd	1437.8	1640.5	14.10%	1461.2	1.63%
	3rd	2156.0	2462.9	14.23%	2236.7	3.74%

Table 6: Dynamic comparison between the detailed, the 3D equivalent and the 1D concept model, in terms of natural frequencies and percentage differences

Instead, the proposed dynamic concept model shows a very good accuracy for all modes, with a maximum difference with respect to the detailed model of 0.15% and 3.74% for bending and torsional modes respectively.

5 CONCLUSIONS

In this paper, a dynamic FE-based concept modelling methodology, enabling to define accurately concept beam models of complex structures, like honeycomb sandwich beams, has been proposed and validated.

A detailed 3D model of honeycomb sandwich beam has been used as reference model, since it has been validated by comparing its flexural frequencies with experimental data taken from the literature. In order to assess the accuracy of the dynamic concept method, a concept beam model has been defined and its predictive performances compared with those of a an equivalent 3D model. The comparison showed a significant improvement of accuracy, for both the flexural and the torsional frequencies, with respect to the equivalent 3D model.

The presented method also showed a significant reduction of the number of DoFs and, hence, of computational time demanded to execute FE simulations.

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