

## NUMERICAL MODELING OF THE DYNAMIC BEHAVIOR OF A WIND TURBINE TOWER

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**Abstract.** *The high energy requirement due to population growth and economic development needs the exploration of alternative forms of energy generation, in order to reduce the effects of excessive emission of greenhouse gases. One solution is to generate clean electricity through wind power. In this work a tower with theoretical dimensions was studied using the finite element package ANSYS. First a simplified modeling of the tower was considered, reducing the system to a single degree of freedom. Next this was modeled using bar elements and a mass element concentrated at the top of the tower. A third modeling of the tower was considered using shell elements and mass elements distributed over the nodes of the top. Thus a response comparison was made in order to verify if the simplified model affected the accuracy of the final solution. The natural frequencies and the mode shapes of the models were obtained as well as transient and harmonic analysis results to determine the dynamic response of the tower in time domain. Finally the results obtained numerically were compared with those resulting from the analytical solution of a cantilever beam free with a tip mass attached at the end in order to verify the consistency of results obtained using different modeling.*

## 1 INTRODUCTION

Wind power generation is a source of renewable, clean and low environmental impact energy. It is available at many places around Brazilian territory. The use of this energy source to generate electricity commercially was driven by the global oil crisis in the 70s. Nowadays, Europe and the United States have developed this technology to reduce its dependence on fossil fuels. Currently maturity of wind as a source of electricity production is a reality in the world. This can be proven by the large number of wind farms connected to the power grid at all voltage levels. In late 2012, the installed capacity of wind turbines was about 282.5 GW, with a growth of over 10% market share compared to 2011 and representing an investment of 56 billion euros [1].

The wind turbine is supported by towers, which due to its geometry and high altitude are slender and flexible and may experience excessive vibration levels, caused by the operation of the turbine, as well as by wind loads. Detailed analysis of the structural behavior of the support tower reveals of great importance due to the cost factor, since this represents about 30% of the total system cost [2].

The height of a wind turbine support tower varies between 1 and 1.5 times the rotor diameter [3]. In 2012, the largest wind turbine support tower onshore available was the E-126 belonging to ENERCOM ^ ®, presented the Hub rotor positioned 135 meters from the ground and produced a power of 7.5 MW [2].

A flexible wind tower presents its natural frequency greater than the natural frequency of the rotor and lower than the rotating rotor blades frequency [3]. The flexible towers are typically less expensive than rigid towers, since they have less mass and require less material for its manufacture. However, considerations regarding dynamic effects must be considered, since these towers are normally exposed to complex distributions of aerodynamic loads [4,5].

In this work a tower with theoretical dimensions was studied using the finite element package ANSYS. First a simplified modeling of the tower was considered, reducing the system to a single degree of freedom. Next this was modeled using bar elements and a mass element concentrated at the top of the tower. A third modeling of the tower was considered using shell elements and mass elements distributed over the nodes of the top. Thus a response comparison was made in order to verify if the simplified model affected the accuracy of the final solution. The natural frequencies and the mode shapes of the models were obtained as well as transient and harmonic analysis results to determine the dynamic response of the tower in time domain. Finally the results obtained numerically were compared with those resulting from the analytical solution of a cantilever beam free with a tip mass attached at the end in order to verify the consistency of results obtained using different modeling.

## 2 TOWER MODELING

The structural system studied in this work is a high, flexible, slender tower that supports at the top the set nacelle+blades. This system can be modelled as a cantilever beam with a tip mass at the end, as shown in Figure 1. Although the real structure is a system with infinite degrees of freedom, it can be modelled as discrete system with multi degrees of freedom (MDOF) in a Finite Element approach. The dynamic equilibrium equations of this MDOF system are

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{F}(t) \quad (1)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are mass, damping and stiffness matrices, respectively.  $\ddot{\mathbf{y}}(t)$ ,  $\dot{\mathbf{y}}(t)$  and  $\mathbf{y}(t)$  are acceleration, velocities and displacements vectors, respectively.  $\mathbf{F}(t)$  is the dynamic load

vector. The solution of the free undamped vibration problem provides the natural frequencies and associate vibration modes of the system. Generally, structures like the one studied in this work usually vibrate predominantly on the fundamental mode [6].

Based on the above, simplifications can be made in the analysis. One of these techniques is to reduce the MDOF system to a single degree of freedom SDOF through modal analysis [7]. [8] determine an equivalent SDOF to a dynamic system with distributed mass and stiffness. The generalized mass and the generalized stiffness of the tower are computed, respectively as:

$$K^* = \int_0^L EI [\Phi''(z)]^2 dz = \int_0^L EI \frac{\pi^4}{16L^4} \cos^2\left(\frac{\pi z}{2L}\right) dz \quad \therefore \quad K^* = \frac{\pi^4}{32L^3} EI \quad (2)$$

and

$$M^* = M + \int_0^L \left[1 - \cos\left(\frac{\pi z}{2L}\right)\right]^2 dz = M + \frac{mL}{2\pi} (3\pi - 8) \quad \therefore \quad M^* = \frac{mL}{2\pi} \left[ \pi \left(3 + 2\frac{L_e}{L}\right) - 8 \right] \quad (3)$$

where the tip mass  $M = mL_e$  is defined proportional as an equivalent length  $L_e$ .

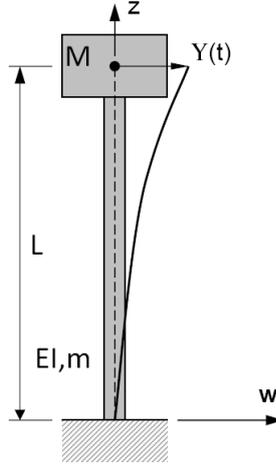


Figure 1: Cantilever beam with a tip mass at the end

### 3 ANALYTICAL FORMULATION

The equation of motion associated for a continuous beam with a tip mass, subjected to a distributed force  $F(x,t)$ , like the one shown in Figure 1 is

$$[m + M\delta(x - L)]\ddot{w}(x, t) + 2\xi m\omega\dot{w}(x, t) + EIw^{iv}(x, t) = F_0 x \sin(\omega_f t) \quad (4)$$

where the primes and overdots denote partial derivatives with respect to  $x$  and time  $t$  respectively.  $EI$  is the bending stiffness and  $m$  is the mass per unit length of the beam.  $\delta(x)$  denotes the Dirac function. It is advantageous to analyze continuous systems by transforming them into discrete ones by the Galerkin method [9]. After the discretization we obtain

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \quad (5)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are, respectively, the mass, damping and stiffness matrices,  $q$  is the vector of generalized coordinates, and  $Q$  is the vector of imposed forces; the overdot denotes differentiation with time.

#### 4 FINITE ELEMENT MODELING

The numerical modeling via Finite Element Modeling was performed using ANSYS 12.0 software. ANSYS simulation software is able to solve a wide range of mechanical problems. These problems include static and dynamic analysis (linear and nonlinear), heat transfer, and fluid problems as well as acoustic and electromagnetic ones.

In this work various types of ANSYS library finite elements were tested in order to achieve a better numerical result of the wind turbine tower behavior: SHELL93, BEAM4 and MASS21. It was also used on the modeling the ANSYS CERIG command, which creates a massless web of rigid bars. This web restrains some nodes called “slaves” attached to a single “master” node.

#### 5 NUMERICAL RESULTS

##### 5.1 MODAL ANALYSIS

A simple model is considered for tower representation with realistic dimensions to illustrate this work. The proposed model consists of a wind turbine tower connected to a nacelle, as presented in Figure 1. The system is modelled as a uniform cantilever beam of circular hollow cross-section with a tip mass (the nacelle) at its free end. The tower is constructed from steel with a hub height of 60m, a width of 3m and a shell thickness of 0.015m. The elastic modulus and density of the steel are assumed to be  $2.1 \times 10^{11} \text{N/m}^2$  and  $7850 \text{kg/m}^3$ , respectively. The tower carries a nacelle and a rotor system mass of 19.876kg. The drag coefficient used for the tower was 1.2, with the density of air of  $1.25 \text{kg/m}^3$ . We assume none modal damping ratios. Eq. (2) and (3), using the precedent data, produces the following generalized stiffness and mass parameters:

$$M^* \ddot{Y} + K^* Y = F^*(t), \text{ where } M^* \cong 34,899 \text{kg} \text{ and } K^* \cong 46,3671 \text{N/m} \quad (6)$$

The estimated frequency is  $\omega_{reduced} \cong 0.58 \text{Hz}$ , close to the literature result  $\omega_{reduced} \cong 0.567 \text{Hz}$  [10]. The relative error, inferior to 2.5%, justifies the simplified Rayleigh model.

Three finite element models were studied using elements BEAM4, SHELL93 and CERIG command. Figure 2 illustrate them. The third model uses SHELL93 with concentrated mass at the free end. Massless rigid bars are inserted at the structure top, like shown on Figure 3. Displacement restraints are set at the tower nodes linked to these bars. The master central node has concentrated mass of 19876 kg and the master node displacement is transferred to slave nodes.

The set of natural frequencies obtained are shown on Table 1 comparing the three FEM models with the SDOF approximation.



Figure 2: Finite element models.

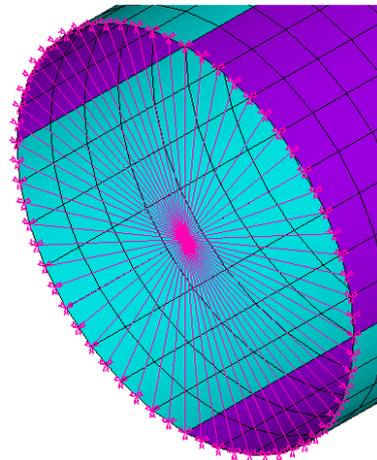


Figure 3: Massless rigid bars at the top of the structure connecting “master” node to “slave” ones.

Figure 4 shows a comparison with the two models using BEAM4 and SHELL93 with respect to mode vibration shapes. A comparison was also made with the two SHELL93 models using CERIG or not, Figure 5 show mode deformation at the tower top in both cases.

Mode	SDOF	BEAM4	SHELL93	SHELL93 with CERIG command		
				401 elem.	1601 elem.	6401 elem.
1	<b>0,58</b>	<b>0,56926</b>	<b>0,5714</b>	<b>0,57104</b>	<b>0,57138</b>	<b>0,57140</b>
2		<b>0,84789</b>	<b>0,8492</b>	<b>0,84881</b>	<b>0,84916</b>	<b>0,84917</b>
3		-	2,8451	<b>4,1630</b>	<b>4,1652</b>	<b>4,1654</b>
4		-	2,8560	4,8397	4,7806	4,7768
5		<b>4,231</b>	<b>4,1622</b>	4,8397	4,7806	4,7768
6		-	4,6242	<b>5,1892</b>	<b>5,1916</b>	<b>5,1918</b>
7		-	4,6245	6,5366	6,4571	6,4517
8		<b>5,2910</b>	<b>5,1918</b>	6,5366	6,4571	6,4517
9		-	5,6234	10,171	10,049	10,040
10		-	5,6235	10,171	10,049	10,040

Table 1: Tower natural frequencies – a comparison

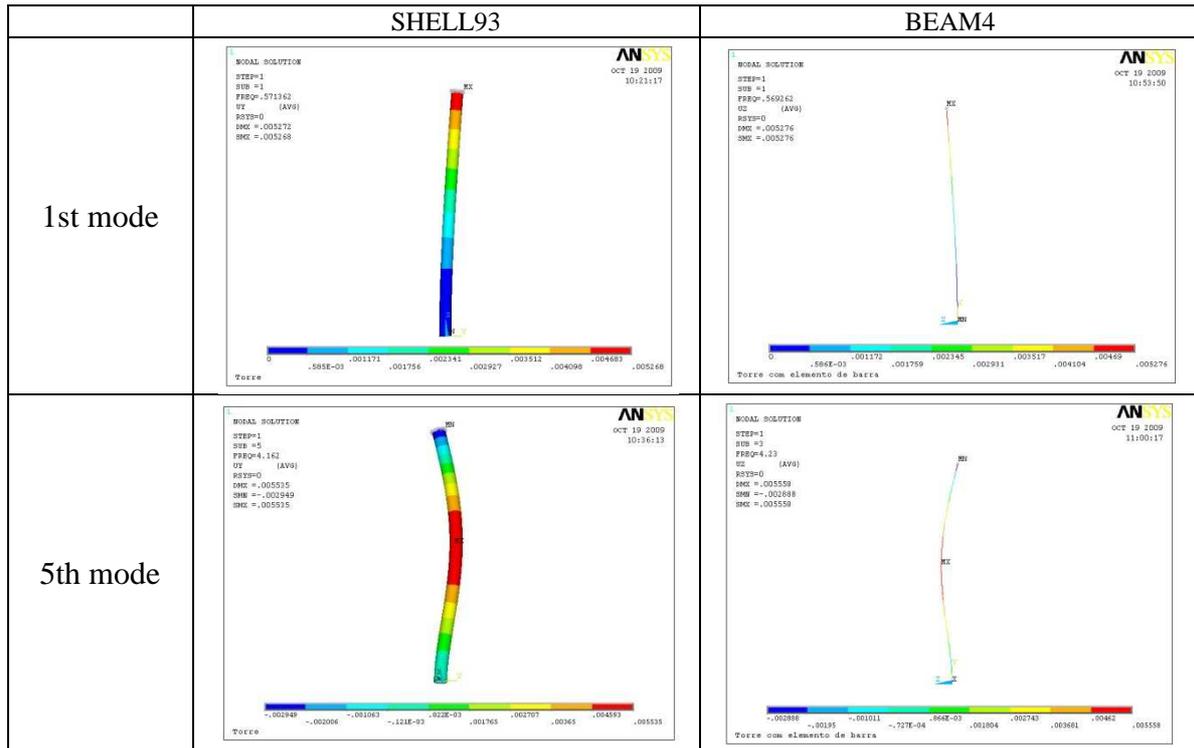
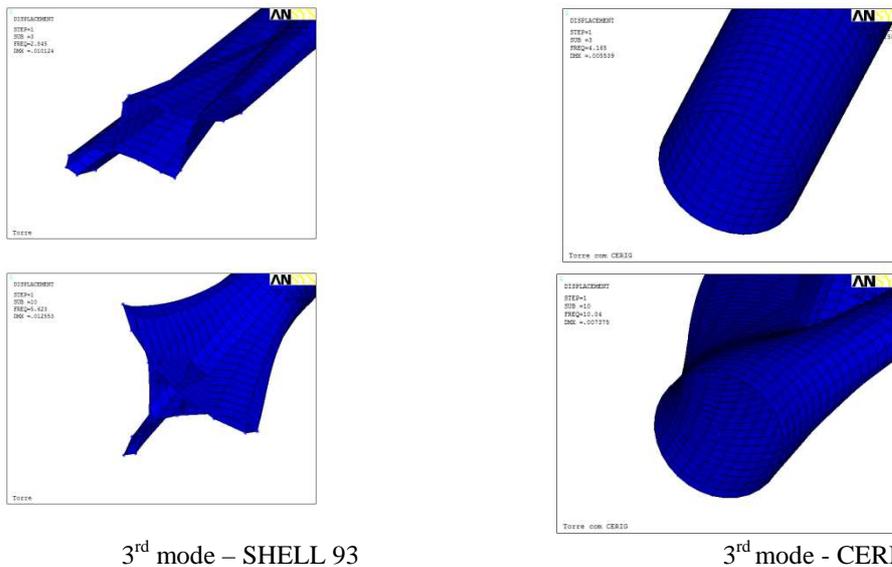


Figure 4: Comparison of mode vibration using beam4 shell93 elements



3<sup>rd</sup> mode – SHELL 93

3<sup>rd</sup> mode - CERIG

Figure 5: Comparison of mode vibration using shell93 elements with CERIG or not

CERIG command was used to model the fixation of the set nacelle+blades, creating a rigid region at the tower top. The mass at the top was considered and the results were satisfactory. From the results obtained it was concluded that the proposed numerical model is quite adequate, since ensures a good precision in the expected frequency values.

## 5.2 HARMONIC ANALYSIS

An harmonic analysis was performed on ANSYS with the SHELL93 with CERIG command model. Figure 6 shows the frequency response curve obtained. In order to validate this result, the frequency response for a SDOF system, using the generalized stiffness and mass  $K^*$  and  $M^*$ , above was obtained. The frequency response of a SDOF is given by

$$h(\omega) = \frac{1}{-\omega^2 m + i\omega c + k} \quad (7)$$

Figure 7 presents the corresponding frequency response plot. Comparing the two curves it can be observed a very good agreement between the SDOF and FEM models.

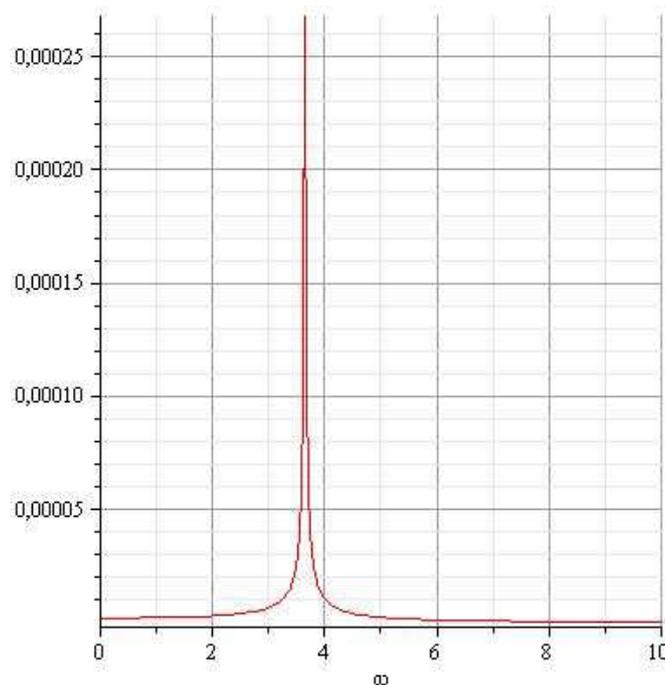


Figure 6: Frequency response S1DOF (frequency in rad/s)

## 5.3 TRANSIENT ANALYSIS

Finally a transient analysis was performed, considering an harmonic triangular distributed load;  $F(t) = F_0(x)\sin(\omega_f t)$ . Figure 8 shows the top displacement time evolution obtained through the analytical solution of Eq. 4. The numerical solution via FEM using ANSYS element BEAM4 is shown in Figure 9. It can be observed that although the two results are similar, a better FEM model would get a better approximation between numerical and analytical solutions.

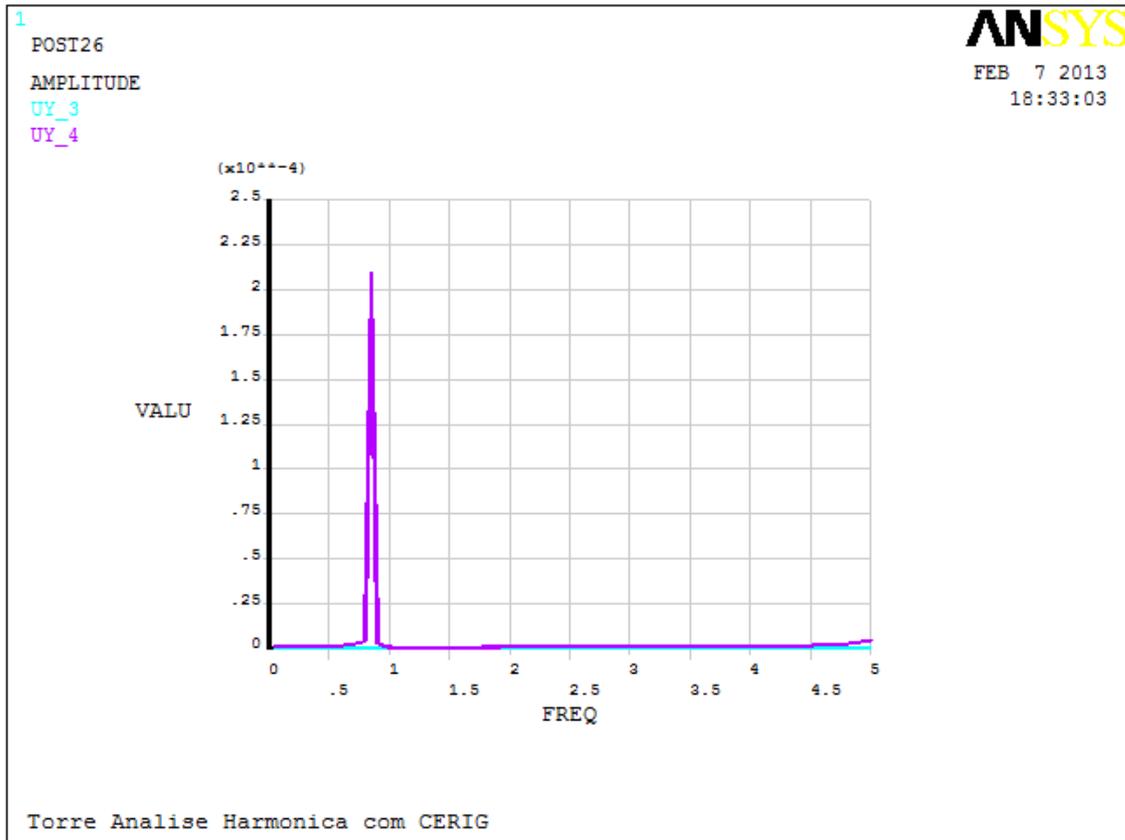


Figure 7: Frequency response – SHELL93 with CERIG (frequency in Hz)

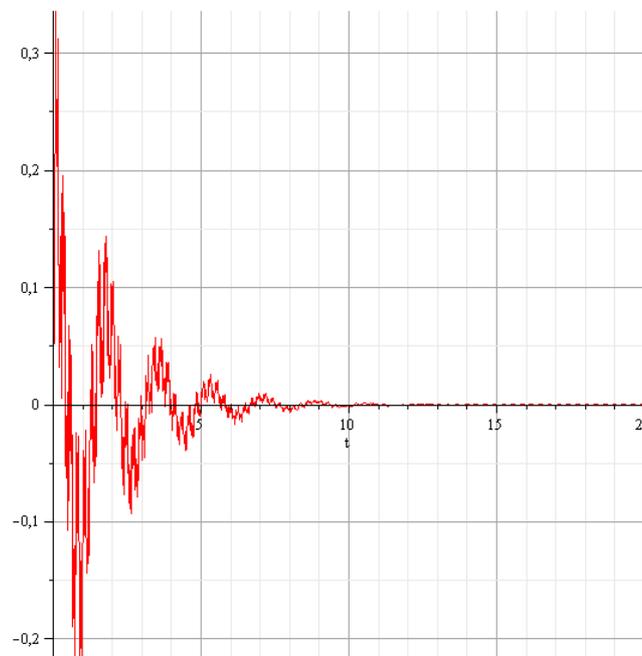


Figure 8: Tower top displacement – analytical solution

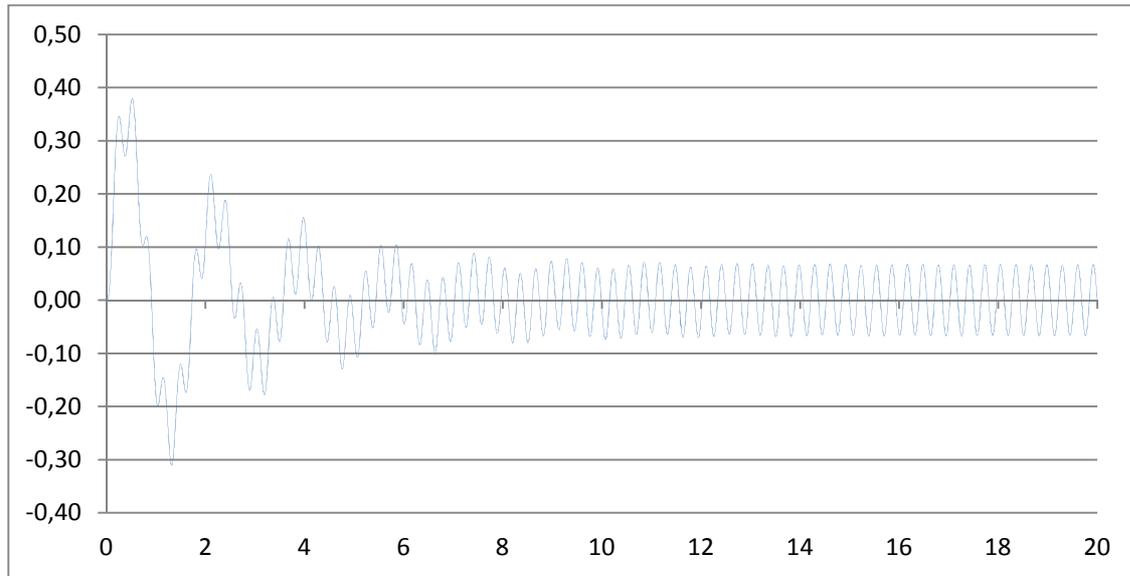


Figure 9: Tower top displacement: BEAM4 ANSYS element

## 6 CONCLUSIONS

Wind turbine towers use to be high, flexible, slender structures. Such systems turn to be more vulnerable to excessive vibration caused by dynamic loading like wind forces. In this work the dynamic behavior of a theoretical wind turbine tower was studied considering different modeling approaches: single degree of freedom reduction, analytical solution of a continuous cantilever beam with a tip mass and using finite element method comparing also different types of elements. Three types of analysis were performed: modal, harmonic and transient analysis. Good agreement between the models was found out.

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