11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.* (eds.)
Lisbon, Portugal, 9–12 September 2013

ROBUST CONTROL FOR TRAJECTORY TRACKING OF 2D FLEXIBLE LINK MANIPULATORS USING STABLE INVERSION METHOD

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Keywords: Planar Flexible Link Manipulators, Robust Control, Stable Inversion Method.

Abstract. *End effector trajectory control of flexible manipulators system (FMS) is traditionally complex in nature because of the coupled nonlinear dynamics and non-minimum phase nature of the system. In case of trajectory tracking, model inversion method is the common approach to find the desired control signal to the system. This method is simple and stable in case of rigid manipulators but for the flexible manipulators model inversion is unstable due to internal dynamics. Stable inversion technique can be applied to convert non minimum phase system into stable minimum phase system by solving internal dynamics for a bounded elastic deformation to a given end effector reference trajectory. In this paper, a robust controller is designed for an accurate trajectory tracking of 2D flexible manipulator with model uncertainty. Inverse kinematic for a given end effector trajectory is solved using Damped Least Square Method (DLS). A PD type Iterative learning method is used to solve internal dynamics for a given end effector trajectory. The proposed controller consists of feed forward compensator derived from stable inversion model and a robust feedback control to stabilize the unmodeled dynamics. The performance of the controller is demonstrated with 3 link planar flexible manipulator with unmodeled end-effector payload mass. Dynamics of the flexible link manipulator is derived using Lagrangian formulation and flexibility of links is defined using Euler-Bernoulli beam equation. Flexible links are discretized using finite element method to get finite dimensional dynamic model for a model based controller design.*

1 INTRODUCTION

Rigid manipulator systems have high stiffness and inertial properties, which make it easier to design control for different tasks. However, the bulky nature of manipulators need more power to drive the system, and restricts the operation speed. To improve the performance of rigid manipulators, the focus on flexible manipulators is increased in recent years. The flexible manipulators have higher payload to arm weight ratio and consume less energy to operate, but low stiffness and inertial properties leads to additional elastic vibrations in manipulator links. The control objective of flexible manipulators is to follow the desired trajectory and minimized vibrations along trajectory. The most challenging problem in control design for flexible manipulator is under actuation and non-minimal phase nature. Under actuation is due to finite number of actuators to control infinite degrees of freedom that arise due to link flexibility [1]. Non-minimum phase nature occurs because of non-collocation of actuators and sensors. A Singular perturbation method [2,3] is proposed to solve trajectory tracking of flexible manipulators by considering that the elastic modes are well separated from rigid body motion. This method reduces the system into slow and fast dynamics using a perturbation parameter. Since the coupling between rigid and elastic modes are nonlinear, the procedure to find the perturbation parameter is not straight forward [2]. Stable Inversion Method [4-7] is used for trajectory tracking of flexible manipulator by converting the non-minimum phase system into minimum phase by solving unstable internal dynamics. Nonlinear regulation, Iterative learning in time domain and frequency domain is proposed in [4], to solve internal dynamics for the bounded elastic deformation. Exact model Inversion [6] is proposed for trajectory tracking of flexible manipulator by converting the system into input-output form by coordinate transformation and the internal dynamics is solved as a two sided boundary problem using MATLAB solver bvp5c. The advantage of stable inversion technique is that it ensures the endeffector trajectory tracking without measuring the vibrations of endeffector. Thus, a simple closed loop feedback control with arbitrary gains at manipulator joints is sufficient to ensure an accurate endeffector trajectory tracking. However, robustness of feedback control loop in the presence of model uncertainties is not addressed.

In this paper, a two stage control for an endeffector trajectory tracking in the presence of model uncertainties is presented. First stage involves the computation of modified joint trajectories using damped least square method. From these modified joint trajectories, the unstable internal dynamics is solved for bounded elastic deformations using PD iterative learning method. In second stage, the feed forward compensator and robust feedback control acts to ensure the stability of the system for unmodeled dynamics. The feed forward compensator is derived using stable inversion technique, and the robust feedback control is designed using Lyapunov function. The performance of proposed controller is demonstrated on a three link planar flexible manipulator with unmodeled payload mass. Dynamics of the flexible link manipulator is derived using Lagrangian formulation and flexibility of links is defined using Euler-Bernoulli beam equation. Flexible links are discretized using finite element method to get finite dimensional dynamic model for a model based controller design.

2 MATHEMATICAL MODELING OF FLEXIBLE LINK MANIPULATOR

2.1 Kinematics of flexible link manipulator

A three link planar flexible manipulator is shown in Figure 1. Kinematics of a flexible manipulator is described using floating reference frame formulation [8]. XY is the global reference frame, and X_iY_i $[i = 1, 2, 3]$ are the rigid coordinate frame associated to link i. $\hat{X}_i\hat{Y}_i$ is the

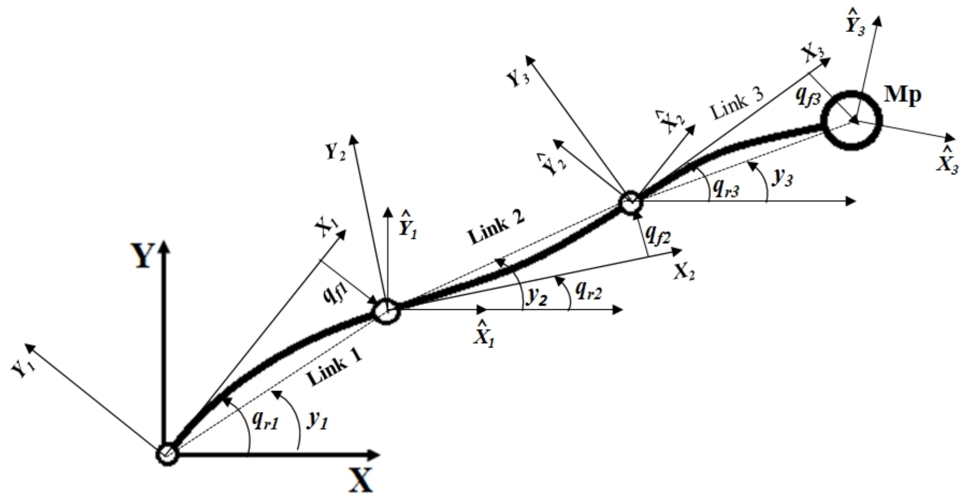

Figure 1: 3-Link Manipulator (Floating Reference Frame)

elastic coordinate frame defined w.r.t rigid coordinate frame X_iY_i. The configuration of each link is defined as a set of rigid and elastic coordinates i.e. $[q_r\ q_f]$. The rigid coordinates q_r represent the joint rotations of each link, and the elastic coordinates q_f represent the deformations of each link. The actual position of an endeffector is a nonlinear function in terms of joint rotations q_r and elastic coordinates q_f. To simplify the coupling between joint rotations and elastic deformations, the actual tip position is approximated using joint angles and link tip elastic deformations by a weighted parameter. The advantage of this approximation is it does not require all the internal elastic coordinates to find the endeffector position. Using weighted parameter, the modified joint angles are defined as a linear combination of joint angles and link tip elastic coordinates [3-6]

$$y_i = q_{r_i} + \Gamma q_{f_i} \tag{1}$$

where y_i, q_{r_i}, and q_{f_i} are respectively the modified joint angle, actual joint angle, and link tip deformation for each i=1,2,3. Γ is the weighted parameter matrix, derived using geometry of link [4]. With this approximation, the endeffector position of a flexible link manipulator $r_{ef}(q_r, q_f) \approx r(\hat{y})$ is expressed as

$$r_{ef}(q_r, q_f) \approx r(\hat{y}) \approx \begin{bmatrix} l_1\cos(y_1) + l_2\cos(y_2) + l_3\cos(y_3) \\ l_1\sin(y_1) + l_2\sin(y_2) + l_3\sin(y_3) \end{bmatrix} \tag{2}$$

where $\hat{y} = [y_1\ y_2\ y_3]^T$ is the modified joint coordinates vector. l_1, l_2, and l_3 are the lengths of link 1, link 2, and link 3 respectively. For the given endeffector trajectory r_{ef}, rigid inverse kinematics is solved for modified joint angles using damped least square method [9]. This method finds the incremental value Δy for each time step, that minimizes the quantity

$$\parallel J(\Delta y) - e \parallel^2 + \lambda^2 \parallel (\Delta y) \parallel^2 \tag{3}$$

where $\lambda > 0$ is non-zero damping constant, J is jacobian matrix, and e is the endeffector position error.

2.2 Dynamics of flexible link manipulator

Each flexible link is modeled as a finite element beam using Euler-Bernoulli beam formulation with clamped-free boundary conditions and the dynamics of the system is derived using

Lagrangian approach. The following assumptions are considered to derive equations of motion of flexible manipulator

- Motion of each link is planar
- Each link is assumed long and slender, so shear effects are neglected
- Deformation of each link is assumed to be small and in horizontal plane
- Each link has constant cross-section area and uniform material properties

Equations of motion of a flexible manipulator are written as

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q,\dot{q}) = B\tau \tag{4}$$

Where $q = [q_r \; q_f]^T$ are rigid and elastic coordinates of the manipulator. q_r is the $n \times 1$ vector that represents rigid body rotations of the n manipulator joints, and q_f is the $m \times 1$ vector that represent elastic coordinates of the link. The number of elastic coordinates depends on number of finite elements used to discretize the link. M(q) is the Inertia matrix, $V(q,\dot{q})$ is the Coriolis and centrifugal vector, $G(q,\dot{q})$ represents the internal forces due to body elasticity and structural damping. Input matrix B distributes the torque at manipulator joints. The equations of motion explicitly written in rigid and elastic coordinates are

$$\begin{bmatrix} M_{rr}(q) & M_{rf}(q) \\ M_{rf}^T(q) & M_{ff}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_f \end{bmatrix} + \begin{bmatrix} V_r(q,\dot{q}) \\ V_f(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ D_{ff}\dot{q}_f + K_{ff}q_f \end{bmatrix} = \begin{bmatrix} B_r \\ B_f \end{bmatrix} \tau \tag{5}$$

Since the actuators are placed at manipulator joints, the input matrix is expressed as $B_r = I_{nxn}$ and $B_f = 0_{mxn}$. The model inversion of Eq. 5, that maps input torque and desired output trajectory, depends on rigid and elastic coordinates of the system. If the desired output is the endeffector trajectory, the system is unstable due to non-minimum phase nature. The unstable internal dynamics is then solved in a passive way by using modified joint trajectories that approximately define endeffector position. The substitution of actual joint coordinates q_{r_i} in terms of modified joint coordinates y_i and link tip elastic coordinates q_{f_i} using Eq. 1 gives

$$M_{rf}^T(\hat{y} - \Gamma\hat{q}_f)(\ddot{\hat{y}} - \Gamma\ddot{\hat{q}}_f) + M_{ff}(\hat{y} - \Gamma\hat{q}_f)\ddot{q}_f + V_f(\hat{y} - \Gamma\hat{q}_f, \dot{\hat{y}} - \Gamma\dot{\hat{q}}_f) + D_{ff}\dot{\hat{q}}_f + K_{ff}q_f = 0 \tag{6}$$

where $\hat{y}$ are the modified joint coordinates, and $\hat{q}_f = [q_{f_1} \; q_{f_2} \; q_{f_3}]^T$ are the link tip deformations. The bounded internal elastic coordinates q_f is calculated using a simple PD type iterative learning method in time domain [4] for the given modified joint trajectories $\hat{y}$.

3 CONTROLLER DESIGN

A two stage controller, Figure 2, is used for the endeffector trajectory tracking. First stage involves the computation of modified joint reference trajectory $\hat{y}(t)$, for a given endeffector trajectory r_{ef}, using the damped least square method and solving the internal dynamics for a bounded elastic deformation q_f using PD type iterative learning method. Second stage consists of feed forward compensator and a robust feedback controller.

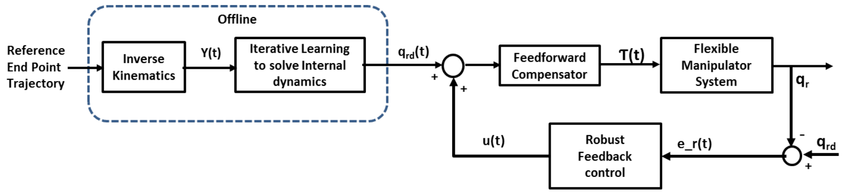

Figure 2: A two stage controller architecture.

3.1 Feedforward Compensator

Feed forward compensator is derived using a stable inversion model. Since the flexible states q_f, $\dot{q}_f$, $\ddot{q}_f$ are computed from iterative learning method, the model inversion is stable for the given output reference trajectory r_{ef}.

$$\tau = M_{rr}\ddot{q}_r + M_{rf}\ddot{q}_f + V_r(q,\dot{q}) \tag{7}$$

However, the computed torque in Eq. 7 works fine for exact model. A simple PD feedback control with arbitrary gains can give accurate trajectory tracking at manipulator joints.

$$\tau = M_{rr}(\ddot{q}_r + K_p e_r + K_v \dot{e}_r) + M_{rf}\ddot{q}_f + V_r(q,\dot{q}) \tag{8}$$

Where, K_p and K_v are the position and velocity error gain, respectively. e_r is the joint trajectory error i.e.

$$e_r = q_{rd} - q_r \tag{9}$$

Where q_{rd} is the desired joint trajectory, that are computed using bounded elastic coordinates and modified joint trajectory from the first control stage. To account for model uncertainties and unknown payload mass M_p, a robust feedback loop is needed to ensure stability along the trajectory.

3.2 Robust feedback control

Many methods are available in literature to design robust controller. Here Lyapunov function [10] is used to design feedback gains to guarantee the stability along the trajectory with uncertainty in the model. Consider a nominal model

$$\overline{M}\ddot{q} + \overline{V}(q,\dot{q}) + \overline{G}(q) = B\tau \tag{10}$$

A feedback linearization to a nominal model gives the tracking error dynamics for the joint variable q_r as

$$\begin{bmatrix} \dot{e}_r \\ \ddot{e}_r \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_r \\ \dot{e}_r \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{11}$$

Due to the uncertainties present in M(q) and $V(q,\dot{q})$, the tracking error dynamics have an additional nonlinear term η, which is nonlinear function of both e_r and u.

$$\dot{e}_r = Ae_r + B(u + \eta) \tag{12}$$

$$\eta = \Delta(u - \ddot{q}_r) + M^{-1}\delta \tag{13}$$

Where

$$\Delta \;=\; M^{-1}\overline{M} - I_n \tag{14}$$

And

$$\delta = V - \overline{V} \tag{15}$$

To derive stability conditions the following assumptions are made with finite constants defining the size of uncertainty

$$\frac{1}{\mu_2} \leq \parallel M^{-1} \parallel \leq \frac{1}{\mu_1} \tag{16}$$

$$\parallel \Delta \parallel \leq a \leq 1 \tag{17}$$

$$\parallel \delta \parallel \leq \beta_0 + \beta_1 \parallel e_r \parallel + \beta_2 \parallel e_r \parallel^2 \tag{18}$$

$$\parallel \ddot{q_{rd}} \parallel \leq c \tag{19}$$

Consider a feedback controller

$$u = -Ke_r \tag{20}$$

Such that

$$\dot{e_r} = Ae_r + B(u + \eta) = (A - BK)e_r + B\eta = A_c + B\eta \tag{21}$$

By placing the poles far from the left-half of the plane, the stability of closed loop system in the presence of η is guaranteed. Solving Lyapunov equation

$$A_c{}^T P + PA_c = -Q \tag{22}$$

with the choice of

$$Q = \begin{bmatrix} 2K_p^2 & 0 \\ 0 & 2K_v^2 - 2K_p \end{bmatrix} \tag{23}$$

and

$$K_v^2 > K_p \tag{24}$$

The positive definite solution of Eq. 22is written as

$$P = \begin{bmatrix} 2K_pK_v & K_p \\ K_p & K_v \end{bmatrix} \tag{25}$$

and feedback gains is defined as

$$K = B^T P = \begin{bmatrix} \frac{K_v{}^2}{a} & K_v \end{bmatrix} \tag{26}$$

The closed loop system Eq. 21 is uniformly bounded if $e_r(0) = 0, \dot{e_r}(0) = 0$ and

$$a > 1 + \frac{1}{\mu_1}[\beta_0 + 2(\beta_2\beta_0 + \beta_2(\mu_1 + \mu_2)c)^{\frac{1}{2}}] \tag{27}$$

where

$$K_v = 2aI \tag{28}$$

and

$$K_p = 4aI \tag{29}$$

Where K_p and K_v are position and velocity error gains, respectively. I is a $n{\times}n$ identity matrix.

Table 1: The physical parameters of three link planar manipulator.

Parameter	Link 1	Link 2	Link 3
Link Length (L)	4.5 m	4.0 m	3.5 m
C/s Area(A)	0.0028 m^2	0.0020 m^2	0.0008 m^2
Tensile Modulus	206000 MPa		
Density (ρ)	8253 Kg/m^3		

4 SIMULATION RESULTS

A three Link planar manipulator is considered to demonstrate the end point trajectory control using robust feedback controller for unmodeled payload mass. Each link is modeled as a two finite element beams with two DOF on each node and one DOF for rigid body rotations. Uniform cross-section and material properties are assumed on each link. The physical parameters of flexible manipulator is presented in Table 1.

4.1 Reference trajectory

Figure 3 shows an endeffector trajectory in XY plane. The defined endeffector reference trajectory is a circle of 2 m diameter starts at point (X,Y) = (9,3) and ends at same point after completing one rotation anti-clock wise direction. Figure 4, Figure 5 and Figure 6 are the joint position, velocity and acceleration trajectories, respectively. These joint trajectories are computed in stage one, from the given reference endeffector trajectory.

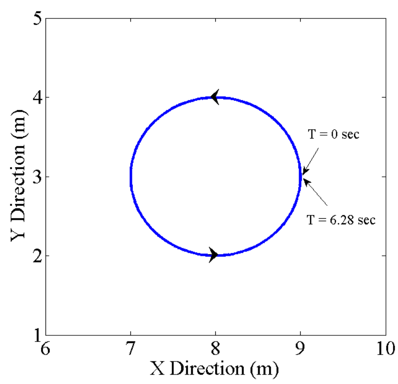

Figure 3: Endeffector reference trajectory in XY plane

4.2 Robust feedback controller design

In this simulation, the uncertainty in Inertia matrix M(q) is considered due to payload mass where as uncertainty in coriolis and centrifugal matrix is neglected. The finite constants considered to define the size uncertainty of inertia matrix M(q) are $\mu_1 = 0.02$ and $\mu_2 = 0.1$. Since the uncertainties in $V(q, \dot{(q)})$ are neglected, small values of β_0, β_1 and β_2 are considered, which is $\beta_0 = 0.1$, $\beta_1 = 0.1$, $\beta_2 = 0.1$. The constant c = 7 for the given reference trajectory. With these consideration a=38 satisfies condition Eq. 27. Hence, the gains K_p and K_v are selected as

$$K_p = 4aI = 152I \; and \; K_v = 2aI = 76I \tag{30}$$

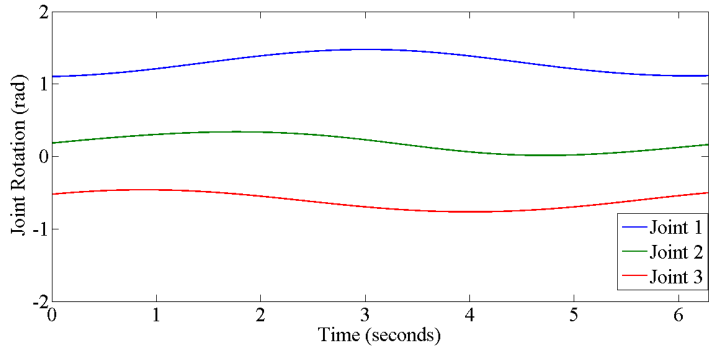

Figure 4: Joint reference position q_{rd}

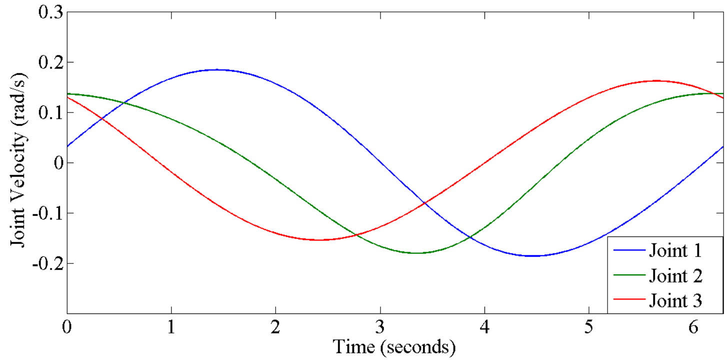

Figure 5: Joint reference velocities $\dot{q}_{rd}$

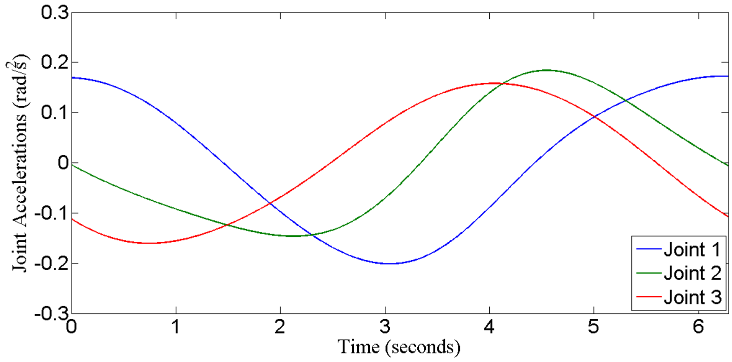

Figure 6: Joint reference accelerations $\ddot{q}_{rd}$

gives uniformly bounded error along the reference trajectory. Where I is a 3×3 identity matrix.

4.3 Results

Figure 7 shows the joint tracking error and Figure 8 shows the endeffector tracking error without payload mass. For the payload mass of 5 Kg, Figure 9 and Figure 10 represent the joint tracking error and the endeffector tracking error, respectively. The joint tracking error and endeffector tracking error for additional payload mass show stable and bounded error along the trajectory.

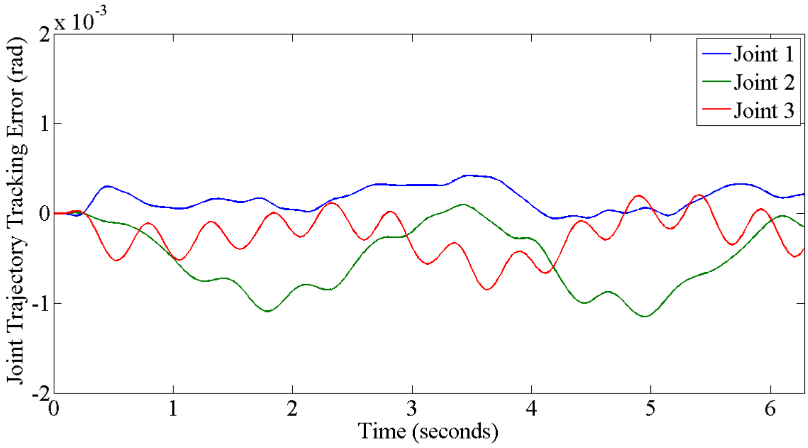

Figure 7: Joint tracking error (Mp=0 Kg)

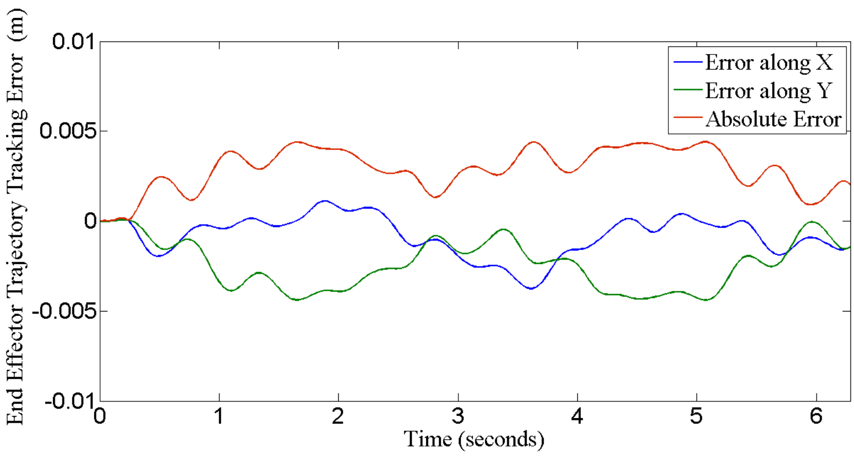

Figure 8: Endeffector tracking error (Mp=0 Kg)

5 CONCLUSIONS

Endeffector trajectory control of flexible manipulators is a complex task due to under actuation and non-minimum phase nature. The flexible manipulators have low stiffness of links, hence they require infinite degrees of freedom to characterize the dynamic behavior. It is not feasible to consider infinite degrees of freedom from the control point of view. Therefore, a finite dimensional dynamic model is derived using Lagrangian formulation along with finite element method. A two stage controller, that allows an endeffector trajectory tracking, is designed and demonstrated using a three links planar flexible manipulator. In stage one, the joint trajectories and bounded elastic deformations are computed using the damped least square method, and the PD type iterative learning method. In stage two, the feed forward compensator and

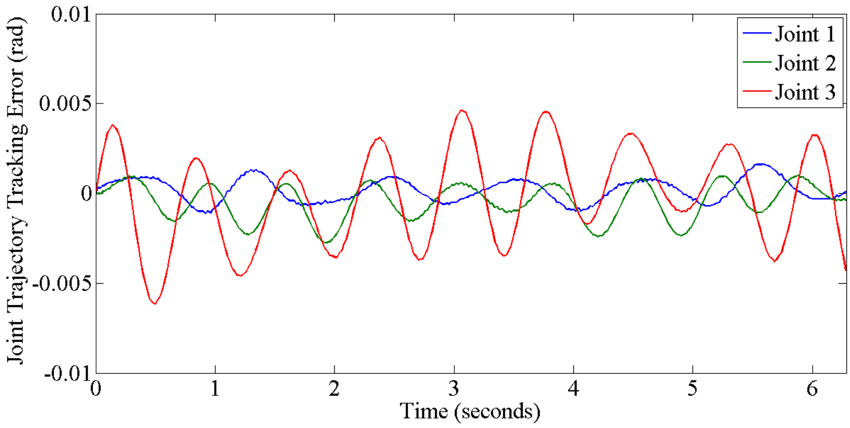

Figure 9: Joint tracking error (Mp=5 Kg)

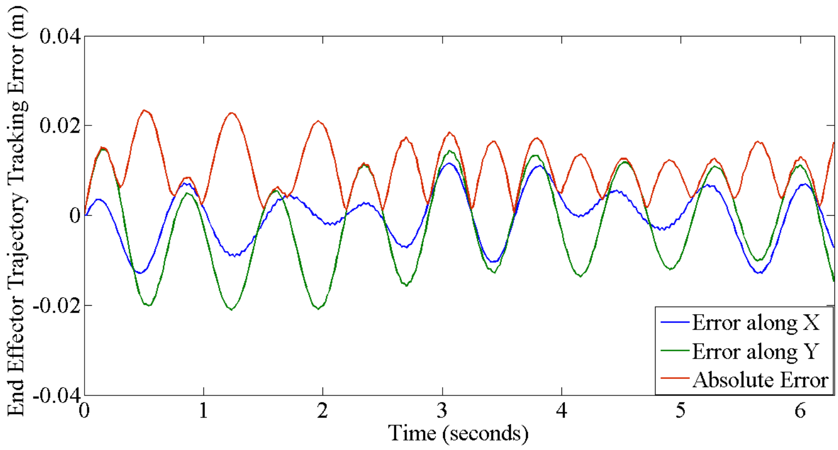

Figure 10: Endeffector tracking error (Mp=5 Kg)

a robust feedback control ensure the trajectory tracking of manipulator joints. The feed forward compensator is derived using stable inversion model and robustness of feedback loop is designed using Lyapunov function. The proposed controller is demonstrated on a three links planar flexible manipulator. The robustness of control is verified in the presence of additional payload mass on the endeffector. With increase in payload mass M_p, there is a slight increase in the joint trajectory tracking error and endeffector tracking error, but the accuracy of endeffector and stability of the system is maintained. The advantage of this approach is that it considers the endeffector trajectory as input and converts non-minimum phase system into minimum phase system using the damped least square method and the PD type Iterative learning method. Then, the robust feedback control applied at joint space ensures the stability of the system for unmodeled payload mass.

REFERENCES

[1] A.De. Luca, et al, Control problems in underactuated manipulators, *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, vol. 2, 855-861, 2001.

[2] B. Siciliano and W.J. Book, A Singular Perturbation Approach to Control of Lightweight Flexible Manipulators, *The Int. J. of Robotics Research*, vol. 7, no. 4, 79-90, 1988.

[3] B. Subudhi and A.S. Morris, Singular Perturbation Approach to trajectory tracking of flexible robot with joint elasticity, *International Journal of Systems Science*, vol. 34, no. 3, 167-179, 2003.

[4] A. De. Luca, Trajectory control of flexible manipulators, *Lecture Notes in Control and Information Sciences*, Vol. 230, 83-104, 1998.

[5] A. De. Luca, P. Lucibello, and G. Ulivi, Inversion techniques for trajectory control of flexible robot arms, *Journal of Robotic Systems*, Vol. 6, no. 4, 325-344, 1989.

[6] R. Seifried, M. Burkhardt, and A. Held, Trajectory Control of Flexible Manipulators using Model Inversion, *ECCOMAS Thematic Conference on Multibody Dynamics*, Belgium, 2011.

[7] H. Zhao and D. Chen, Tip Trajectory Tracking for Multilink Flexible Manipulators using Stable Inversion, *Journal of Guidance Control and Dynamics*, vol. 21, no. 2, 314-320, 1998.

[8] A. Shabana, Dynamics of Multibody Systems, *Cambridge University Press*, Cambridge, 2005.

[9] S.R. Buss and J.S. Kim, Selectively damped least squares for inverse kinematics, *Journal of Graphics Tools*, vol. 10, no. 3, 37-49, 2005.

[10] F.L. Lewis, D.M. Dawson and C.T. Abdallah, Robot Manipulator Control: Theory and Practice, Second Edition, ISBN: 0-8247-4072-6.