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ESTIMATION OF THE PARAMETERS OF THE DISCRETE MODEL OF A REINFORCED CONCRETE SLAB

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Abstract. *This paper presents parameter estimations of a mathematical model regarding reinforced concrete slab natural vibrations. Parameter estimation was based on the results of experimental research, conducted on a real reinforced concrete slab. Estimated parameters include: substitute longitudinal modulus of elasticity of the reinforced concrete slab, which takes into account longitudinal reinforcement and coefficient of damping. Other parameters of the model were taken from the literature data and from the inventory of the real reinforced concrete slab. Using appropriate criteria during the process of parameter estimation of reinforcement concrete slab models has a great impact on obtaining precise results. The estimation criteria were selected in order to achieve consistency of the natural vibration frequencies, along with vibration modes, measured during experimental with those calculated based on the mathematical model. The model and all the calculations were made using the MATLAB programming environment.*

1 INTRODUCTION

Steel-concrete composite beams are the main focus of the present paper [1, 2]. A composite beam is created as a result of a connection of two or more structural elements made of materials with various properties. An example of such a structure is the steel-concrete composite beam, which consists of a steel I-beam and a concrete slab which rests on it. The present paper focuses on the concrete slab element.

The main topic of the paper is modelling vibration of a concrete slab. A 3D RFE (Rigid Finite Element Method) model is presented. An originally developed algorithm of parameter estimation for the reinforced concrete slab model is also presented. An estimation based on experimental results with FRF (Frequency Response Function) was conducted.

As there is no commercially available finite element - based software, an original computer program was developed in MATLAB environment. The created program can be used to solve the problem of free vibration and to control parameters which can be introduced to describe some selected structural elements.

2 EXPERIMENTAL RESEARCH CONDUCTED ON A REINFORCED CONCRETE SLAB (EXPERIMENTAL RESEARCH OF VIBRATIONS A REINFORCED CONCRETE SLAB)

The experimental stand consisted of two steel bearers at the beam axel spacing of 2 m. The bearers were braced with angle sections. During dynamic tests the beam was suspended on the frames with four steel wire ropes 3 mm in diameter and in this way a free beam scheme was implemented. Bearer (rope) deformability and its effect of obtained results are considered to be negligible in the scheme. Rope deformability was selected so that frequency vibration typical for solid body in motion was beyond the range of the investigated slab’s free vibration. A diagram of the test stand as well as the suspended beam are presented in Figure 1.

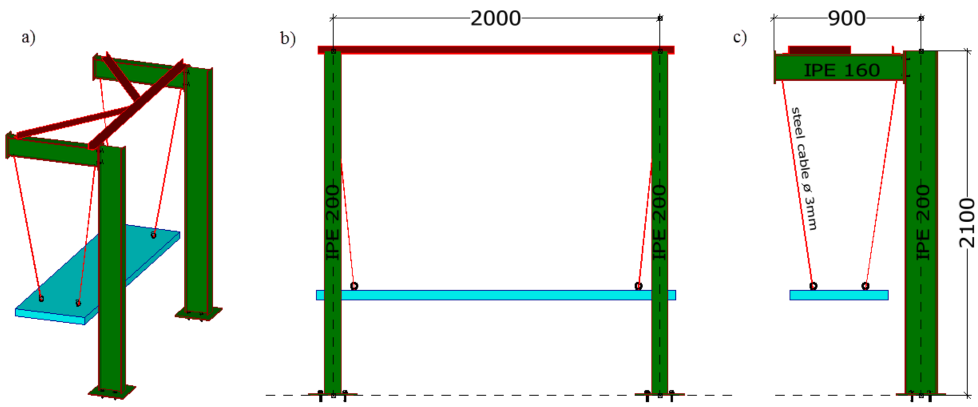

Figure 1: Test stand: a) overview; b) a view from the side; c) a head-on view.

The aim of the conducted tests was to determine fundamental dynamic characteristics. Impulse excitation was used. Vibration acceleration was a measured value which was considered as the response of the system. Acceleration was measured using triaxial piezoelectric sensors. The sensors were attached with wax to circular steel washers, 25 mm in diameter, placed on the reinforced concrete slab Figure 2 a). The washers were fixed with modified epoxy resin.

Impulse excitation was performed using a modal hammer KISLER 9726A20000 (500 g) (see Figure 2 b). LMS SCADAS III analyser connected to the work station fitted with computer aided system and *Test Lab* package manufactured by LMS was used to record signals. *Impact Testing* module of the *Test Lab* package was used for impulse tests. During each cycle of measurements, acceleration in nine measurement points was recorded. Ten excitation cycles were performed in a predefined spot of the beam. Signal averaging was conducted automatically according to the algorithm implemented in *Impact Testing* module.

a) b)

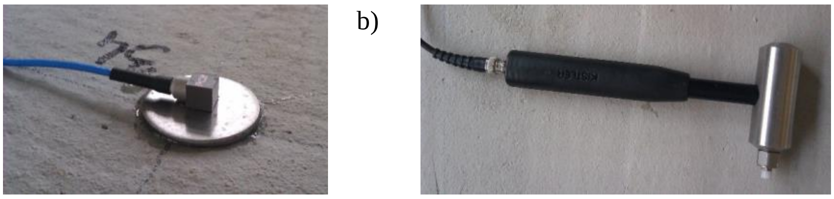

Figure 2: Measurement instruments: a) triaxial piezoelectric acceleration sensor; b) modal hammer.

The obtained characteristics of Frequency Response Functions were determined as the ratio of vibration acceleration (transform response) to the force (excitation signal transform). Frequency response functions were used to determine the so-called modal model using *Modal Analysis* module of the *Test Lab* system. Stability analysis method, using *Polymax* algorithms, was used for parameter estimation of the modal model.

3 THE ANALYSED REINFORCED CONCRETE SLAB

The reinforced concrete slab, 60 mm thick, 600 mm wide and 2200 mm long, was made of C25/30 concrete. Concrete mix was purchased from a local concrete producer. A sulphate resistant and low-alkali Portland cement CEM I 42.5 N-HSR was used. The maximum size of aggregate was reduced to 8 mm owing to a relatively small size of the investigated elements.

Ribbed steel bars, 6 mm in diameter made from A-I steel, were used as concrete reinforcement. Longitudinal reinforcement was placed every 75 mm while transverse reinforcement every 150 mm. Reinforcing fabric from top and bottom was used.

Impulse excitation was applied to the slab at three points: 2-Z - vertical impact at the slab axis performed with modal hammer, 1-Z - vertical impact at the edge of slab performed with modal hammer, 2+Z - horizontal impact at the face of slab performed with modal hammer (see Figure 3 a). Various excitation points aimed at producing different vibration forms of the slab which are presented in Table 1. Measurement points were defined at the upper area of the slab, spaced in three rows (9 points in each row) which gave a total of 27 measurement points (see Figure 3 b).

a) b)

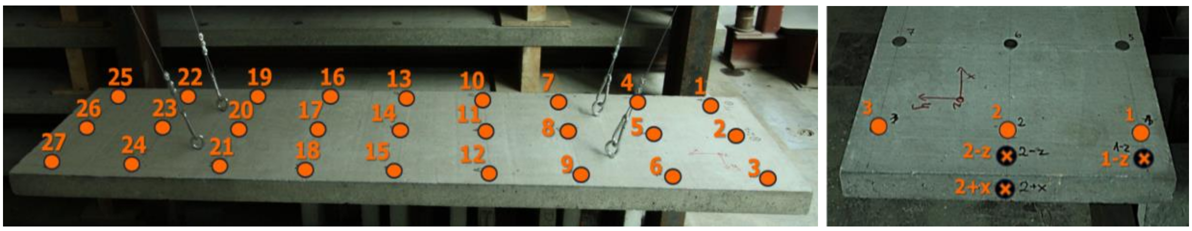

Figure 3: Investigated slab: a) excitation points, b) measurement point grid.

Excitation		Excited vibration forms
Point	**Direction**	
1	-Z	transverse and torsional
2	-Z	transverse
2	+X	axial

Table 1: Excitation points and directions.

4 3D COMPUTATIONAL MODEL OF THE REINFORCED CONCRETE SLAB

A discrete computational model of the beam was developed in the convention of Rigid Finite Element Method - RFE model. The method consists in dividing a real system into rigid finite elements, which are connected with spring-damping elements (SDEs). While rigid finite elements (RFEs) are characterized by masses and mass moments of inertia , spring-damping elements (SDEs) are defined by stiffness and damping coefficients. This RFE method was developed by Kruszewski et al. [3 - 5].

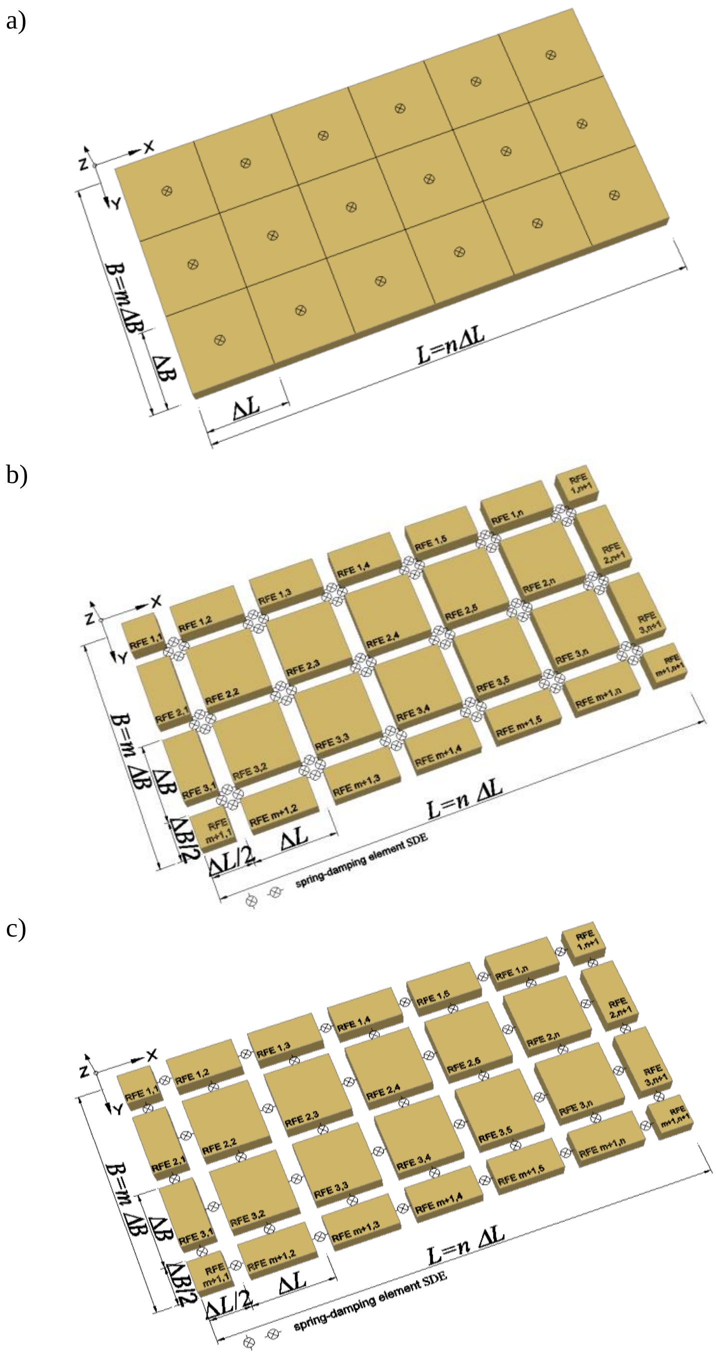

Figure 4: 3D finite element method: a) primary segmentation;
b) secondary segmentation - classic positioning of SDEs;
c) secondary segmentation - modified positioning of SDEs.

Modelling of continuous elements in finite element method starts from primary segmentation. For a slab, segmentation must be conducted in two directions, i.e. in longitudinal direction along *ΔL* of elements and in transverse direction along *ΔB* of elements. A model is divided into segments of equal or comparable length. The primary segmentation of the present study is shown in Figure 4 a. Then, at the center of gravity of each element, an SDE (spring-damping element) is placed which focuses spring and damping properties of that element. Each SDE is broken down into four smaller SDEs so that it is possible to connect the corners of four adjacent finite elements - this is secondary segmentation. In every set of four SDEs, two of them are parallel to the main axis X and the other two are parallel to the main axis Y. In the classic approach [3, 4] spring properties of respective elements are reflected by SDEs spaced as shown in Figure 4 b.

In the classic approach, elements have 5 degrees of freedom. By placing an SDE at the corners, it is possible to neglect rotation in the axes perpendicular to the area of a primary element. The proposed model attempts to define the slab with 6 degrees of freedom (three translational and three rotational displacements). In order to limit element rotation in the axis perpendicular to the area of a primary element, SDEs must be moved from corners to the centers of finite elements as shown in Figure 4 c. A similar approach was used in [6].

Each finite element has its own independent coordinate system $X_{SES}^{(i)}, Y_{SES}^{(i)}, Z_{SES}^{(i)}$ which is selected so that it overlaps with the principal central axes of inertia of a given RFE (rigid finite element). Given this assumption, mass and moments of inertia are the only parameters necessary to describe any RFE. These quantities can be given in a form of a diagonal mass matrix:

$$\mathbf{M}^{(i)} = diag\left[m^{(i)}, m^{(i)}, m^{(i)}, J_X^{(i)}, J_Y^{(i)}, J_Z^{(i)} \right] \tag{1}$$

The first three terms of the matrix are equal to the mass of an RFE, while the other three are RFE mass moments of inertia relative to the axes $X_{SES}^{(i)}, Y_{SES}^{(i)}, Z_{SES}^{(i)}$. The values of diagonal mass matrix $\mathbf{M}^{(i)}$ coefficients for RFEs modelling the inside part of the slab were determined in the following way:

$$m^{(i)} = h_c \cdot \Delta L \cdot \Delta B \cdot \rho_c \tag{2}$$

where:
h_c – thickness of reinforced concrete slab,
ρ_c – mass density of RFE material.

$$J_X^{(i)} = \frac{m^{(i)}}{12}\left(\Delta L^2 + h_c{}^2 \right) \tag{3}$$

$$J_Y^{(i)} = \frac{m^{(i)}}{12}\left(\Delta B^2 + h_c{}^2 \right) \tag{4}$$

$$J_Z^{(i)} = \frac{m^{(i)}}{12}\left(\Delta L^2 + \Delta B^2 \right) \tag{5}$$

Every SDE of k number has its own independent coordinate system of the main axes $X_{EST}^{(k)}, Y_{EST}^{(k)}, Z_{EST}^{(k)}$. The main axes of an SDE have this property that forces acting on an SDE in a direction compatible with these axes result in its translational deformations, which occur only in the direction where these forces are applied. The main parameters which describe an SDE of k number are coefficients defining its spring and damping properties. Spring properties are described by means of two matrices: a matrix of translational stiffness coefficients $\mathbf{K}_T^{(k)}$ and a matrix of rotational stiffness coefficients $\mathbf{K}_R^{(k)}$. Both matrices are diagonal and they are 3x3 in size.

$$\mathbf{K}_T^{(k)} = diag\left[k_{T,X}^{(k)}, k_{T,Y}^{(k)}, k_{T,Z}^{(k)} \right] \tag{6}$$

$$\mathbf{K}_R^{(k)} = diag\left[k_{R,X}^{(k)}, k_{R,Y}^{(k)}, k_{R,Z}^{(k)} \right] \tag{7}$$

The values of translational and rotational stiffness coefficients were determined according to the following rules:

a) for SDEs parallel to the main axis X

$$k_{T,X-X}^{(k)} = \frac{E_c \cdot h_c \cdot \Delta B}{\Delta L} \tag{8}$$

$$k_{T,Y-X}^{(k)} = \frac{G_c \cdot h_c \cdot \Delta B}{\Delta L} \tag{9}$$

$$k_{T,Z-X}^{(k)} = \frac{G_c \cdot h_c \cdot \Delta B}{\Delta L} \tag{10}$$

$$k_{R,X-X}^{(k)} = \frac{G_c \cdot h_c^{\ 3} \cdot \Delta B}{6 \cdot \Delta L} \tag{11}$$

$$k_{R,Y-X}^{(k)} = \frac{E_c \cdot h_c^{\ 3} \cdot \Delta B}{12 \cdot \left(1 - \nu_c^{\ 2}\right) \cdot \Delta L} \tag{12}$$

$$k_{T,Z-X}^{(k)} = \frac{E_c \cdot h_c \cdot \Delta B^3}{12 \cdot \Delta L} \tag{13}$$

b) for SDEs parallel to the main axis Y

$$k_{T,X-Y}^{(k)} = \frac{G_c \cdot h_c \cdot \Delta L}{\Delta B} \tag{14}$$

$$k_{T,Y-Y}^{(k)} = \frac{E_c \cdot h_c \cdot \Delta L}{\Delta B} \tag{15}$$

$$k_{T,Z-Y}^{(k)} = \frac{G_c \cdot h_c \cdot \Delta L}{\Delta B} \tag{16}$$

$$k_{R,X-Y}^{(k)} = \frac{E_c \cdot h_c^{\ 3} \cdot \Delta L}{12 \cdot \left(1 - \nu_c^{\ 2}\right) \cdot \Delta B} \tag{17}$$

$$k_{R,Y-Y}^{(k)} = \frac{G_c \cdot h_c^{\ 3} \cdot \Delta L}{6 \cdot \Delta B} \tag{18}$$

$$k_{T,Z-Y}^{(k)} = \frac{E_c \cdot h_c \cdot \Delta L^3}{12 \cdot \Delta B} \tag{19}$$

where:

E_c – substitute dynamic longitudinal modulus of elasticity of the reinforced concrete slab (which takes into account the effect of reinforcement used),

G_c – substitute dynamic transverse modulus of elasticity of the reinforced concrete slab (which takes into account the effect of reinforcement used),

ν_c – the Poisson's ratio value of concrete.

Dumping properties are described by means of two matrices of dumping coefficients: $\mathbf{C}_T^{(k)}$ and $\mathbf{C}_R^{(k)}$. Both matrices are diagonal and they are 3x3 in size. The relation between equivalent stiffness $k_{i,j}^{(k)}$ and damping coefficients $c_{i,j}^{(k)}$ can be given by:

$$\begin{aligned} & c_{i,j}^{(k)} = \frac{\eta}{\omega} k_{i,j}^{(k)} \\ & i = T, R \\ & j = X, Y, Z \end{aligned} \tag{20}$$

where:

η – loss ratio,

ω – vibration frequency.

5 PARAMETER ESTIMATION FOR THE MODEL

While working on the finite element method model of the slab, an assumption was made that the substitute dynamic longitudinal modulus of elasticity of the reinforced concrete slab E_c (which takes into account the effect of reinforcement used) and loss ratio of concrete would be determined based on identification. The value of loss ratio is dependent on frequency, temperature and other factors.

The loss ratio of concrete varies in a range $(2 \div 6)10^{-4}$ according to [7]. High diversity of concrete types results in very different range of concrete damping values. Damping depends on concrete density, amount of cement slurry, load history, intensity of stress, etc. Other parameters used for the identification of the computational model were taken from the literature or from the inventory of the analysed slab.

Parameter estimation was conducted by fitting frequency response functions calculated with the finite element method model to characteristics obtained in experimental research. A system of differential equations defining the oscillating motion with damping can be given by:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{f}(t) \tag{21}$$

where:
q – vector of generalised displacement,
M, **C**, **K** – inertia, dumping and stiffness matrices,
f – vector of generalised forces.
Vectors **q** and system response **f** are functions dependent on time *t*. The above system of differential equations can be solved, depending on the form of excitation signal, using either Fourier or Laplace integral transform [8]. By using Laplace transform, it is possible to move from time domain over to the domain of complex frequency s. Given zero initial conditions, while performing Laplace transform, the system of equations (21) takes the following form:

$$(\mathbf{M}s^2+\mathbf{C}s+\mathbf{K})\cdot\mathbf{q}(s)=\mathbf{f}(s) \tag{22}$$

A consequence of using Laplace transform is algebraisation of the system of equations (26). While solving a system of linear algebraic equations, we assume that the matrix $(\mathbf{M}s^2+\mathbf{C}s+\mathbf{K})$ is not singular, i.e. that there is a matrix inverse to it. As a result, Equation (22) takes the form:

$$\mathbf{q}(s)=(\mathbf{M}s^2+\mathbf{C}s+\mathbf{K})^{-1}\cdot\mathbf{f}(s) \tag{23}$$

To find a solution in the frequency form, if excitation applied to a system is periodic (solution for a steady state), Fourier transform can be used. A solution is found directly in Laplace solution by substituting it to Equation (23) $s=j\omega$, where $j=\sqrt{-1}$.

$$\mathbf{q}(j\omega)=(\mathbf{K}-\omega^2\cdot\mathbf{M}+j\omega\cdot\mathbf{C})^{-1}\cdot\mathbf{f}(j\omega) \tag{24}$$

where:

$$\mathbf{A}(j\omega)=\mathbf{K}-\omega^2\cdot\mathbf{M}+j\omega\cdot\mathbf{C} \tag{25}$$

is referred to as dynamic stiffness matrix, while

$$\mathbf{W}(j\omega)=\mathbf{A}^{-1}(j\omega)=(\mathbf{K}-\omega^2\cdot\mathbf{M}+j\omega\cdot\mathbf{C})^{-1} \tag{26}$$

is dynamic flexibility matrix.

Dynamic flexibility is a characteristic obtained on the premise that the system's input is force and its output is displacement. During experimental research, acceleration was measured. A characteristic found given the condition that the system's input is force and its output is displacement is called inertance $\mathbf{G}(j\omega)$. Both dynamic flexibility and inertance are frequency characteristics defined for the steady motion and they are therefore closely interrelated [9].

$$\left|\mathbf{G}(j\omega)\right|=\omega^2\left|\mathbf{W}(j\omega)\right| \tag{27}$$

To find the inertance of a system based on a finite element model, it is necessary to know the stiffness matrix **K**, inertia matrix **M** and dumping matrix C. The methods were described in-depth elsewhere in the literature [2 - 4].

To solve the problem, an optimisation procedure implemented in *Optimization Toolbox* package which is part of MATLAB was used. Large- and medium-scale optimisation algorithms are used in the procedure. Medium-scale algorithms use SQP (*Sequential Quadratic*

Programming), i.e. which performs successive series of minimisation of a quadratic approximation of an objective function.

Our identification criterion was a minimisation of coefficient J_{FRF} (28) which is a double sum for *m-th* measurement points, a sum of relative quadratic deviation of the first *n-th* measurement points for a given FRF amplitude to the same amplitude determined in experimental research. While determining loss ratios, an attempt was made to fit the calculated amplitudes with those determined experimentally.

$$J_{FRF} = \sum_{i=1}^{m} \sum_{i=1}^{n} \left(\frac{FRF_{i,amp}^{num} - FRF_{i,amp}^{\exp}}{FRF_{i,amp}^{\exp}} \right)^2 \tag{28}$$

The above algorithm allowed to select an analysed point and as a result it was possible to choose vibration forms (transverse and torsional vibration) and vibration modes (1, 2 ... n). Both transverse and torsional vibration forms were taken into account in the estimation procedure. Results of analysis are presented in Table 2.

$E_{c,x}$	$E_{c,y}$	η_c
[N/m^2]	[N/m^2]	-
3,54E+10	4,25E+10	0,0107

Table 2: Results of estimated parameters during optimisation.

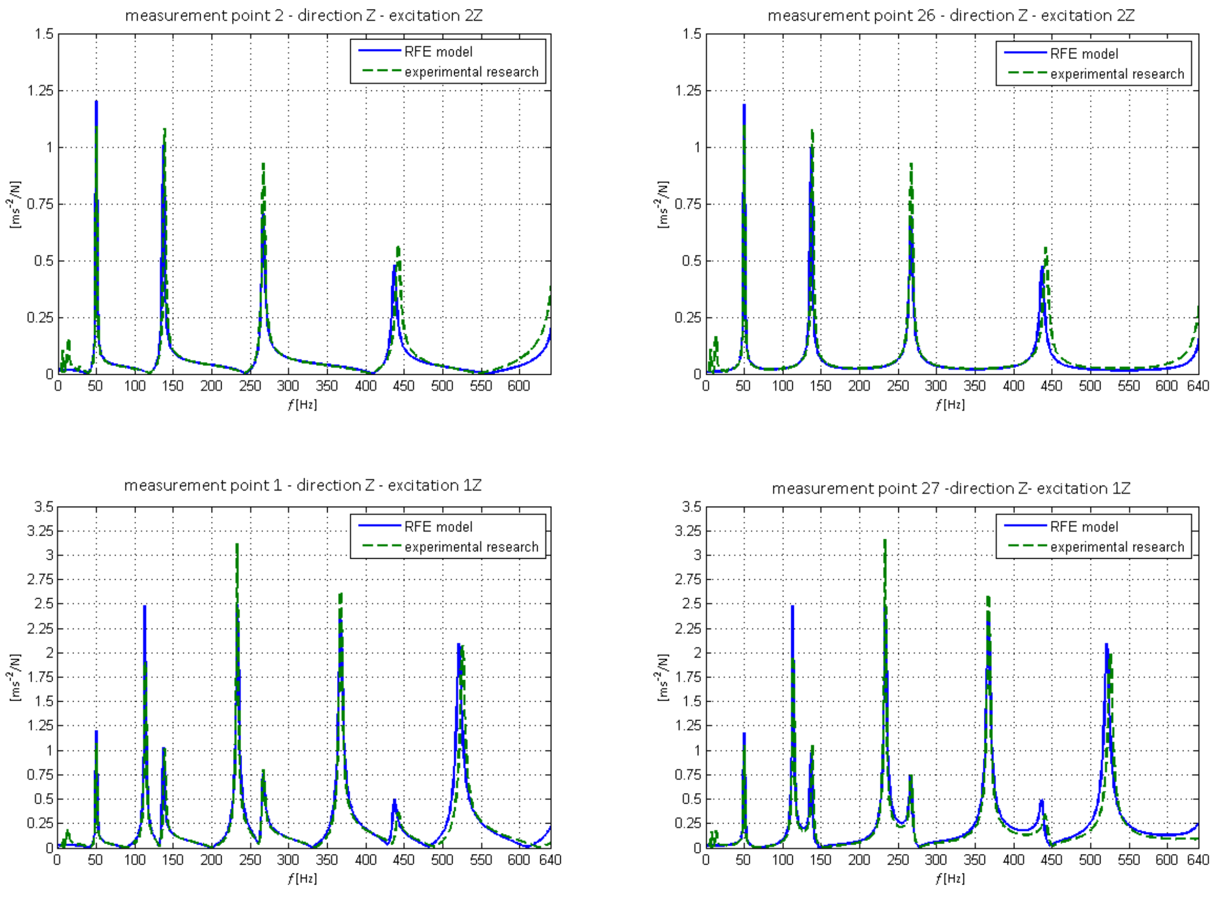

Figure 5. A comparison of FRF runs for the reinforced concrete slab.

A comparison of FRF runs determined using the above estimated parameters with those determined experimentally is presented in Figure 5. As the consistency of runs was high, it is fair to state that the identified parameters were determined correctly.

6 CONCLUSIONS

- A very good fit of FRF (frequency response function) calculated using the model with that obtained in experimental research was achieved.
- The originally developed 3D finite element model allows to take into account both transverse and torsional vibration forms.
- This well-developed 3D model of the reinforced concrete slab is going to allow the authors to develop a model of the steel-concrete composite beam which has been the focus of their research attempts.

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