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A SIMPLE EVALUATION TECHNIQUE FOR ENVIRONMENTAL VIBRATION OF A RIGID EMBEDDED FOUNDATION REFLECTING THE DYNAMIC SOIL-STRUCTURE INTERACTION

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Abstract. *A laboratory building for electron microscopes, a factory producing electronic chips, and etc. is designed to avoid excessive vibration. Performance of the vibration-proof design is to be evaluated by a vibration measurement after its construction; however, it is desirable to assess the performance before the construction by means of in-situ measurement rather than an intricate computation based on many assumptions.*

We have proposed a simple technique to evaluate a structural response to environmental vibration which takes the dynamic soil-structure interaction into account. Conventional techniques assume an equivalent force to reproduce measured vibration at a site before construction, and then apply the force to evaluate vibration of a structure to be built at the site. The concept of the equivalent force is ambiguous since disturbance in reality usually occurs along a stretching line or in an expanse of area which may not be stationary in time and space. Our proposal does not require the evaluation of the equivalent force; it is based on sub-structure modelling of dynamic soil-structure interaction where an input motion is evaluated by averaging free field motion.

In the proposed method, free field motion is measured on the ground at Gauss's points of foundation area and integration is carried out by Gauss integral with measured data. For a surface foundation, the input motion is literally obtained by weighted average of the free field ground motion, but for an embedded foundation the free field underground motion is required. The simple evaluation technique is introduced for an embedded foundation by applying extrapolation into underground by FEM with measured data on the ground and utilizing foundation impedance by a method such as FEM [1]. The applicability of the method is verified by comparing with conventional method. This simple technique enables us to assess the performance of a vibration-proof measure with ease and directness.

1 INTRODUCTION

A laboratory building for electron microscopes, a factory producing electronic chips, and etc. is designed to avoid excessive vibration which may disturb its activity. Performance of the vibration-proof design is to be evaluated by a vibration measurement after its construction; however, it is desirable to assess the performance before the construction by means of in-situ measurement rather than an intricate computation based on many assumptions.

We have proposed a simple technique to evaluate a structural response to environmental vibration which properly reflects the dynamic soil-structure interaction. Conventional techniques assume an equivalent force to reproduce measured vibration at a site before construction, and then apply the force to evaluate vibration of a structure to be built at the site as shown in Figure 1. The concept of the equivalent force is ambiguous since disturbance in reality usually occurs along a stretching line or in an expanse of area which may not be stationary in time and space. Our proposal does not need the equivalent force; it is based on substructure modelling of dynamic soil-structure interaction as shown in Figure 1 where a foundation input motion is evaluated by averaging free field motion [2].

In this paper, we will demonstrate the effectiveness of our method for an embedded foundation with regard to traffic excitation. Measurements are carried out on the ground at Gauss's integral points of a designed foundation area. For a surface foundation, the input motion is literally obtained by weighted average of the free field ground motion, but it is not the case for an embedded foundation since it requires underground free field motion. Applying extrapolation into underground by FEM on measured data on the ground and utilizing foundation impedance of the embedded foundation by FEM, we have introduced the simple technique for an embedded foundation and verified its applicability by comparing with the conventional method. This simple technique enables us to assess the performance of a vibration-proof measure with ease and directness.

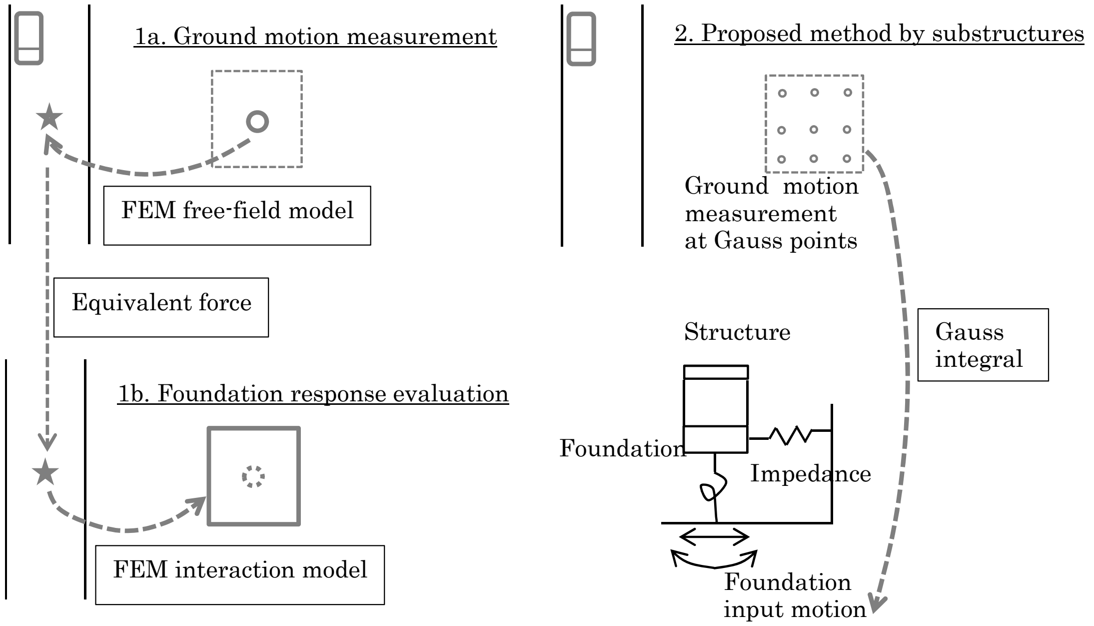


Figure 1: A conventional method (left) and the proposed simple evaluation technique (right).

2 FOUNDATION INPUT MOTION

Figure 1 exemplifies a measurement configuration to evaluate environmental vibration due to traffic. Observation points are placed at Gauss integral points in the area indicated by dotted line where a foundation will be later constructed. For the source of the vibration is on or very close to the ground surface, surface waves including Rayleigh waves will mostly contribute to the foundation input motion. In this section, we will show the formulation of the averaging method for the foundation input motion with Gauss integral of measured data. Effectiveness and limitation of Gauss integral is shown for a surface foundation and an embedded foundation subjected to notable Rayleigh waves.

2.1 Averaging method

It is well known that the foundation input motion for a surface foundation is approximated by averaging free field displacements under the designed foundation. For an embedded foundation, the foundation input motion vector U^* can be approximated with an additional integral of free field tractions at fictitious interfaces of the designed foundation as shown in the second term of Eq. (1). The foundation input motion comprises of translations Δ^* with a suffix indicating directions and rotations ϕ^* with a suffix indicating the reference axis. U^f : free field displacement vector with three components, T^f : free field traction vector with three components, K : impedance matrix for excavated soil, A :a matrix defined by coordinates on interfaces, H : a matrix defined by area integral of $A^T A$, the origin is located at the centre of lower interface of the base where the foundation input motion is referenced.

$$U^* = H^{-1}\iint_S A^T U^f dS - K^{-1}\iint_S A^T T^f dS \tag{1}$$

$$U^* = \{\Delta_x^* \quad \Delta_y^* \quad \Delta_z^* \quad \phi_x^* \quad \phi_y^* \quad \phi_z^*\}^T$$

$$U^f = \{u_x^f \quad u_y^f \quad u_z^f\}^T, \; T^f = \{t_x^f \quad t_y^f \quad t_z^f\}^T$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}$$

$$H = \iint_S A^T A dS$$

$$K = \begin{bmatrix} K_{HH} & 0 & 0 & 0 & K_{HR} & 0 \\ 0 & K_{HH} & 0 & -K_{HR} & 0 & 0 \\ 0 & 0 & K_{VV} & 0 & 0 & 0 \\ 0 & -K_{HR} & 0 & K_{RR} & 0 & 0 \\ K_{HR} & 0 & 0 & 0 & K_{RR} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{TT} \end{bmatrix}$$

for a square foundation.

In practice, it is difficult to carry out the integral of the 2nd term in Eq. (1), since underground stress distribution is hard to obtain by measurements. Kurimoto, et al. [3] showed an alternative formulation of Eq. (1) by replacing traction with inertia force and converting area integral to volume integral as shown in Eq. (2), where ρ is mass density of soil to be replaced by the foundation.

$$U^{*} = H^{-1}\iint_{S} A^{T}U^{f}dS - K^{-1}\iiint_{V}\left[-\rho A^{T}U^{f}\right]dV \tag{2}$$

Eq. (2) is used to evaluate the foundation input motion for earthquake ground motion by interpolating recorded earthquake ground motion at six of eight apex locations of the designed foundation; two of them were placed underground. It is costly to install seismometers underground and is not desirable especially for temporary purpose such as the evaluation of vibration-proof design.

Our method utilizes seismometers on the ground surface only; they are placed at the Gauss integral points for a rectangular or a circular shape which is equivalent to an arbitrary shape of the designed foundation in terms of area. For an embedded foundation, measured ground motion is extrapolated into the ground by vertical distribution of displacements calculated by FEM; the extrapolation enables us to use Gauss integral for the area integral at the fictitious interfaces as well as volume integral for the excavated soil for the designed foundation. By this approach, we can fully utilize Gauss numerical integration which is efficient and orderwise accurate.

2.2 Applicability of extrapolation and Gauss integration

Accuracy of the proposed method is checked by a square foundation embedded in a half-space where Rayleigh waves are propagating in x direction. The half-space is characterized by Poisson's ratio of 0.25, and an embedment ratio is defined by the ratio of the foundation depth to the radii of the equivalent circle of the square foundation with regard to their area. The foundation impedance in the second term in Eq. (2) is computed by axis-symmetric FEM with base-damper and energy transmitting boundary. The results, for the embedment ratio of zero (surface foundation) and one, are compared with the approximate solution with analytical integration of the Rayleigh wave field [4]. The Rayleigh waves have particle motion invariant in y direction, so that the foundation input motion has translations in x and z direction and rocking motion around y axis as shown in Eq. (3). For this reason, we use one point Gauss integral in y direction for area integrals and volume integral.

$$U^{*} = \left\{\Delta_{x}^{*} \quad 0 \quad \Delta_{z}^{*} \quad 0 \quad \phi_{y}^{*} \quad 0\right\}^{T} \tag{3}$$

Since a surface foundation does not require z integral, we take the number of integral points in x direction as a parameter and use three point integral in z direction for the embedded foundation. Figure 3 compares foundation input motion normalized by a relevant component of the free field motion at the centre of the designed foundation on the ground. The rocking component is measured by the vertical motion at the foundation edge and normalized by the free field horizontal component. The abscissa is the dimensionless frequency a_0 with regard to half width of the foundation. The numerical results with three point integral show better fit with analytical ones than two point integral up to higher dimensionless frequencies; up to $a_0 = 3$ for three points and to $a_0 = 2$ for two points in translations and up to $a_0 = 2$ for three points and to $a_0 = 1$ for two points in rocking motion.

Higher frequency implies shorter wave length or fluctuated displacement distribution in certain distance, which needs increased number of the integral points for better accuracy. Then increased number of the integral points can accommodate better accuracy for higher polynomial. Rocking component requires higher order integration than translation by one order as shown in Eq. (1) and it is reasonable to see poorer accuracy in the rocking component compared to the translational components with the same number of integral points. Since environmental vibration contains large portion of surface waves, it is advisable to use good number of integral points in propagating direction for better accuracy, provided the source location is known beforehand.

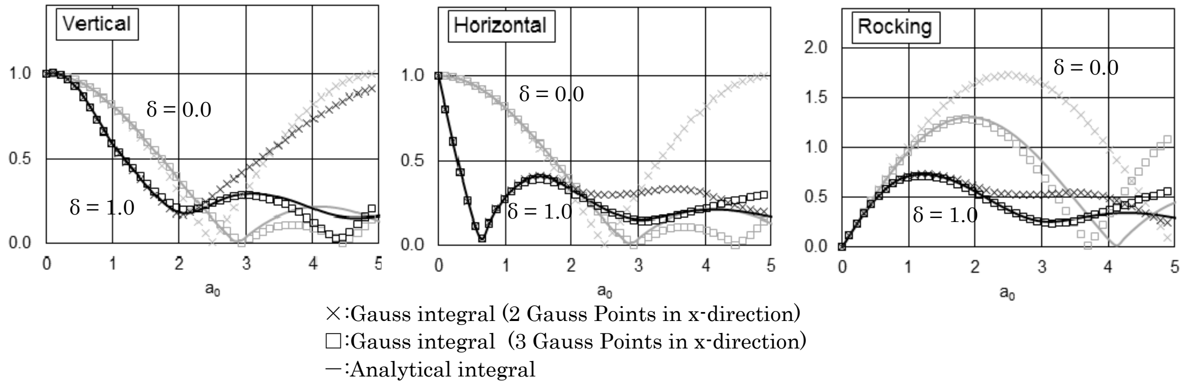


Figure 2: Foundation input motion of a foundation subjected to Rayleigh waves.

3 APPLICATION TO TRAFFIC VIBRATION

We measured acceleration on the ground caused by a series of bangs of a minivan passing a step on the road as shown in Figure 3. The foundation input motion is computed for a flat foundation and for an embedded foundation by the proposed method. The results are compared with the conventional ones with an equivalent force to show the applicability and efficiency of the proposed method.

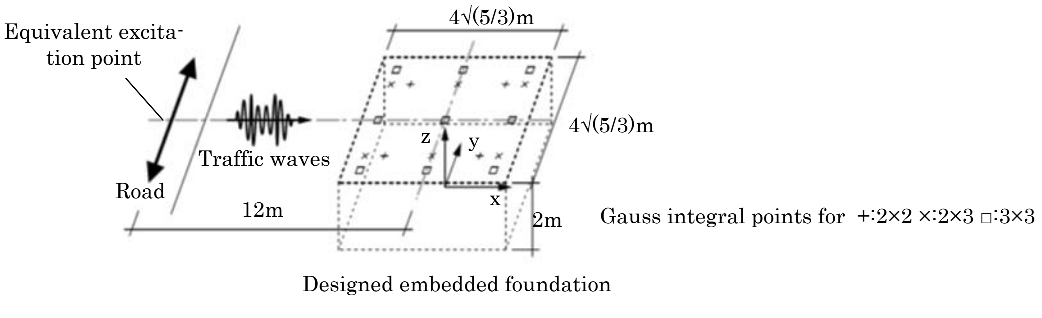


Figure 3: Configuration of measurements for the designed foundation.

3.1 Measurement

Translations in x and z and rocking around y axis, the components shown in Eq. (4), are our main concern according to the configuration in Figure 3. A designed square foundation is $4\sqrt{5/3} \cong 5.16\,\mathrm{m}$ wide and 2m thick based on Gauss integral point location as shown in Figure 4. The centre is located 12 m from the centreline of the road. Measurement points are located at Gauss three integral points as shown in Figure 4. Two servo type accelerometers, one for vertical and the other for horizontal x component, were placed at each measurement point and at the centre. Eighteen times of measurements were carried out with 16 seconds for each bang. Figure 5 shows an example acceleration Fourier amplitude in x and z directions at the centre. Both components exhibit similar acceleration level and several common peaks between 2 Hz and 25 Hz. Frequency components higher than 25 Hz are neglected according to the maximum frequency of the FEM model. PS logging data at the site is used to model the subsurface ground as shown in Table. 1.

No.	Thickness [m]	Vs [m/s]	Vp [m/s]	ρ [kg/m^3]	damping
1	2	120	330	1,440	0.02
2	7	60	170	1,480	0.02
3	8	130	360	1,610	0.02
4	13	290	800	1,910	0.02

Table 1: Subsurface layer model.

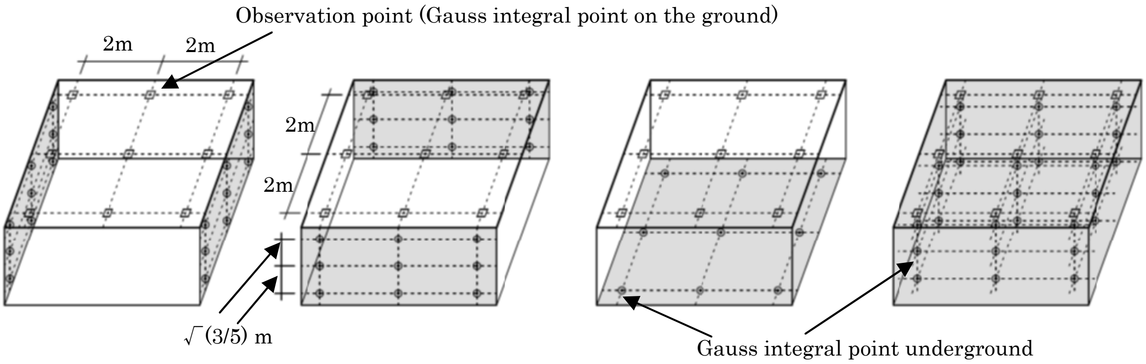


Figure 4: Acceleration measurements and 3 * 3 Gauss integral points.

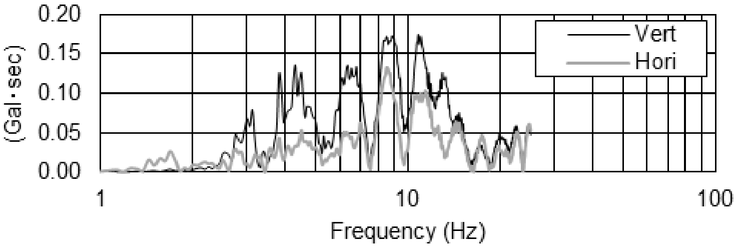
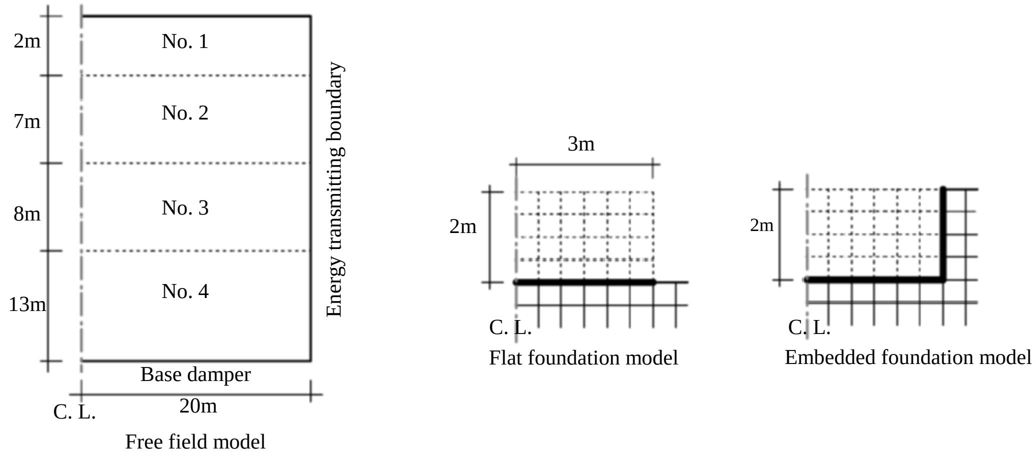


Figure 5: Example acceleration Fourier amplitude at the centre of the square.

3.2 FEM modelling

A FEM model shown in Figure 6 is an axis-symmetric model equipped with base damper and energy transmitting boundary at the boundaries. The extent of the model is 20m radius and 30m depth with four layers according to Table. 1. An element size of 0.5 m can accommodate maximum frequency of 25 Hz in terms of five elements in the smallest wave length. The designed foundation is converted to the equivalent circular foundation of radius 3m. Impedance matrix shown in Figure 7 is computed by constraining displacements at nodes on interfaces between the foundation and the excavated soil as shown in Figure 6.

Arrangements of Gauss integral points in the ground for the side walls and interior domain of the designed foundation are shown in Figure 4. Three point integral in z direction is used according to the results in the section 2. Acceleration at the Gauss integral points underground are extrapolated by the response ratio to the nearest surface measurement obtained by FEM free field model in Figure 6. The free field motion is calculated for each extrapolation by using the reciprocal theorem; source point is located at the centre of the FEM model and the response point is at the reversed location of the real excitation to the measurement point. FEM model is also used to calculate the reference foundation input motion and the reference foundation response in Figures 8 and9. The reciprocal theorem is also used with FEM soil-foundation system, where a massless rigid foundation is inserted at the top centre of the free field FEM.



Free field model

Flat foundation model

Embedded foundation model

Figure 6: FEM models.

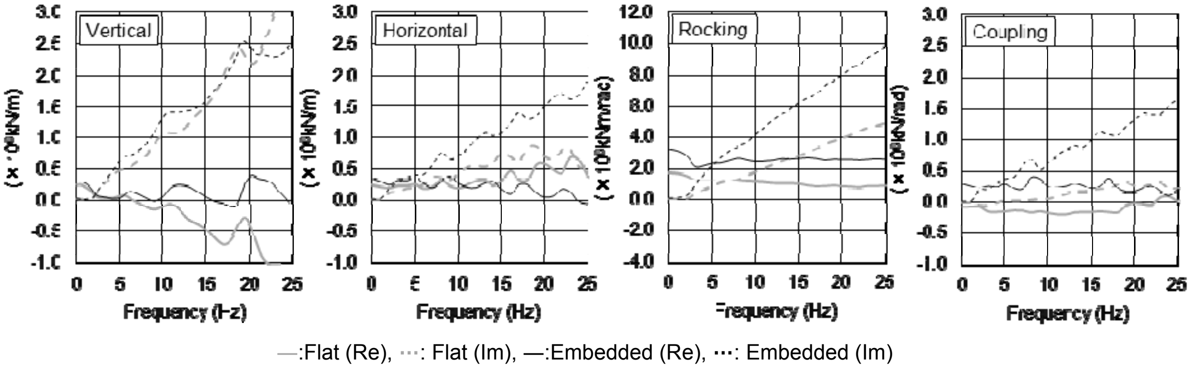


Figure 7: Foundation impedance matrix components.

3.3 Foundation input motion and foundation response

Foundation input motion is evaluated for the designed foundation by the proposed method with the measured data. They are normalized by the free field motion at the centre as before and compared in Figure 8. The lines indicated by FEM are evaluated by the reciprocal theorem with the FEM soil-foundation model and marks indicated by AVG are the ones by our proposed method. Though the results of FEM are influenced by the model and are not necessarily exact, tendency must be preserved and comparison will give some measure of the proposed method. The vertical translation shows uniform decrement up to around 15 Hz regardless of embedment. The results of the proposed method and FEM are in good agreement. For the horizontal translation, the results of the embedded foundation show fluctuations with frequency and the results of the proposed method can express those fluctuations. The horizontal translations of the embedded foundation compare better than those of the flat foundation. Rocking component shows the most fluctuations in those three components and an agreement between the proposed method and FEM is not good as those for horizontal translation.

Foundation response is evaluated by a lumped mass model with the foundation input motion evaluated by the proposed method. The designed RC foundation has mass density of 2,400 kg/m^3 and Young's modulus of 20,500N/mm^2. Translation and rocking motion are normalized as before and shown in Figure 9. The vertical translation and the rocking motion are similar to the corresponding component of foundation input motion in Figure 8, but horizontal translation is distinctively different from that of the foundation input motion; the flat foundation shows much larger response below 10 Hz and the embedded foundation shows much less response above 10 Hz.

In Figure 10, transfer functions of the responses at the top of the foundation to the foundation input motion are plotted. Vertical translation response is computed from the vertical input motion and designated as uncoupled transfer relation which has three indefinite peaks around 2.5 Hz, 8 Hz, and 15 Hz. The vertical foundation response is similar to the foundation input motion due to large damping. Horizontal translation and rocking motion are coupled with each other. The coupled transfer relation plays an important role in the response of a flat foundation, i.e. unlike earthquake ground motion, traffic excitation induces large amount of rocking motion to excite horizontal response of a foundation depending on wavelength and foundation size.

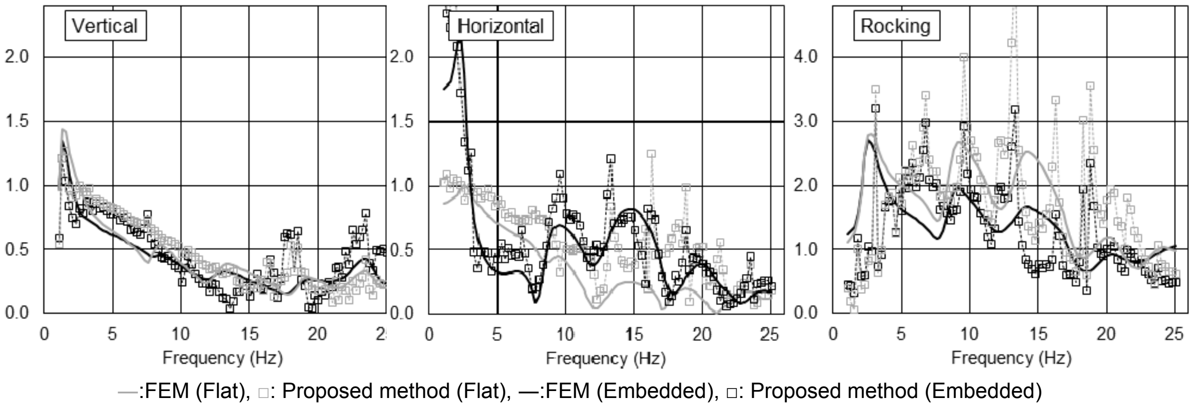


Figure 8: Foundation input motion.

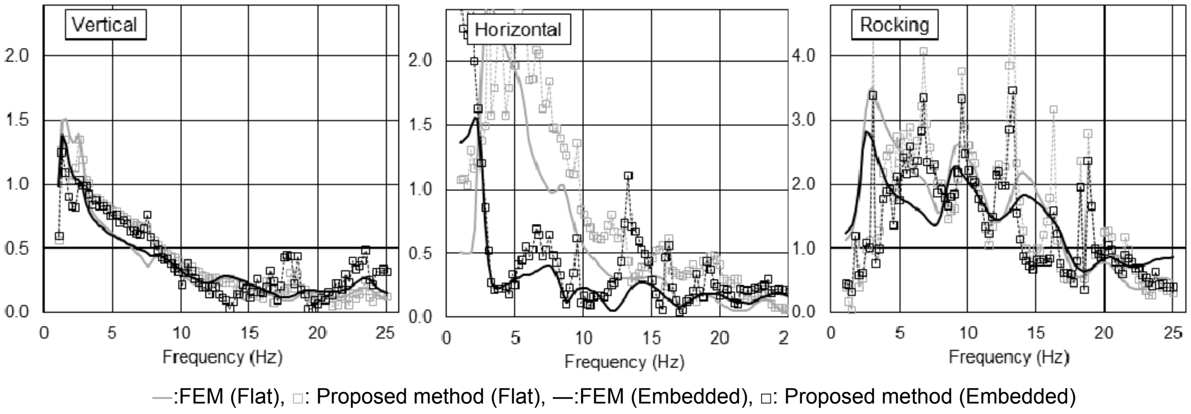


Figure 9: Foundation response motion.

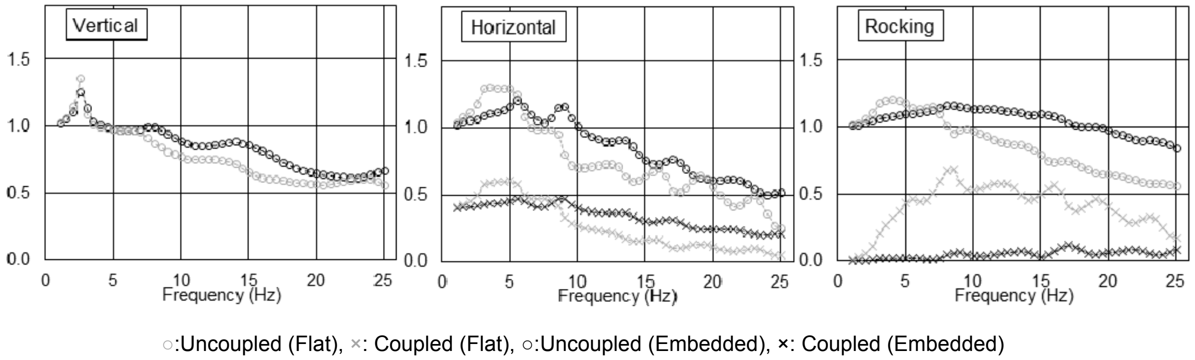


Figure 10: Transfer functions of foundation response to foundation input motion.

4 CONCLUSIONS

We have proposed a simple evaluation technique which is based on a substructure system with the foundation input motion directly evaluated by the averaging method. In the measurements, several sensors are placed at Gauss integral points on free surface where a foundation will be constructed. The averaging method is carried out by summation of acceleration times Gauss integral weight. Unlike a flat foundation, an embedded foundation requires integration of underground motion, which is dealt with FEM based extrapolation on the measurement closest to the underground Gauss integral point. This technique is verified with a numerical test of an embedded foundation subjected to Rayleigh wave excitation and the results are in good agreement with analytically integrated results. With more integral points, the accuracy is improved for higher frequency and rotational components show poorer results than translational components due to one order higher polynomial dependence of the integrand.

This simple technique is applied to the measured data obtained by traffic excitation. Subsurface model is constructed by referring to PS logging and the foundation impedance matrix is evaluated by axis-symmetric FEM. The foundation input motion obtained by this technique is compared with those by the axis-symmetric FEM with consideration on the reciprocal theorem. Though the results of FEM depend on the subsurface model and are not necessarily regarded as exact, tendency must be preserved and their comparativeness will give some measure of the proposed method. Then the foundation response is evaluated by sub-structured lumped mass system. Since the traffic excitation exerts dominant rocking input motion, it may lead to large horizontal response through a coupled term in transfer functions.

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