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APPLICATION OF A DIRECT MULTIVARIATE SUBSPACE ALGORITHM IN THE MODAL IDENTIFICATION OF AN AERONAUTICAL PANEL

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Abstract., *The paper is an application of a time domain, multi-input multi-output deterministic parameter identification algorithm. The method is used to estimate the natural frequencies, damping factors and mode shapes of an aeronautical panel assemblage, comparing the results with previously known identification experiments in order to evaluate the method´s capabilities.*

1 INTRODUCTION

The direct multivariate subspace time identification scheme (MUST) [1] is based on multi-input multi-output measurements from the dynamic test of structures. The method is formulated from deterministic auto-regressive-exogenous relations. Deterministic identification methods use the least-squares (LS) minimization technique for coefficients estimation. When the analysis data are corrupted by noise, the LS approach leads to biased parameter estimations. To minimize such a problem, a large overestimation of the order of the model is usually required [2]. This brings the extra difficulty of separating the true system parameters from the computational ones, introduced by the overestimation of the model´s order. Recursive multi-input-multi-output identification algorithms of the auto-regressive moving average with exogenous excitation type (VARMAX) achieve more accurate parameter estimates from noise contaminated data, through the use of statistic maximum likelihood techniques, [3] and [4]. The drawback to such methods is the intense computational effort required.

Subspace-based state space system identification methods (4SID) [5] and [6] have been used in the analysis of noise contaminated data, as an alternative to overcome the drawbacks of the recursive techniques such as the VARMAX. The computational effort is relatively smaller in such methods, since non-linear optimization schemes are not used. The system model order in subspace methods can also be kept to a minimum size, contrary to the LS deterministic solution methods.

The present work consists of an application of the multivariate subspace time identification scheme in the determination of the dynamic characteristics of an aeronautical panel structure. Identification results from the MuST technique are compared to those from the frequency domain POLYMAX algorithm [7] of the LMS Test.Lab analysis software.

Section 2 of the present paper describes the rationale and implementation of the MUST technique. Section 3 Describes the experimental measurement environment. Section 4 presents the identification results and their comparisons. Section 5 presents the main conclusions of the work.

2 IMPLEMENTATION OF THE MUST ALGORITHM

The time space differential equation of vibration of a discrete dynamic system with n degrees of freedom is given by:

$$\mathbf{M}\ddot{\boldsymbol{u}}(t) + \mathbf{C}\dot{\boldsymbol{u}}(t) + \mathbf{K}\boldsymbol{u}(t) = \boldsymbol{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ square, positive definite matrices of mass, damping and stiffness, respectively, all of dimension $n \times n$ .Vectors $\boldsymbol{u}(t)$ and $\boldsymbol{f}(t)$ of dimension $n \times 1$ are functions of time, representing the degrees of freedom of response and excitation to the system, respectively.

In practical vibration measuring applications only a selected number m of output degrees of freedom are measured, as well as, only a number p of input forces are exerted at some degrees of freedom of to the system. In this way, the measured, and forced degrees of freedom of the system, relate to the global response and excitation vectors through the output and exerted force indication matrices, respectively denoted by $\mathbf{O}$ and $\mathbf{E}$, yielding:

$$\bar{\boldsymbol{u}} = \mathbf{O}\boldsymbol{u} \tag{2}$$

and

$$\bar{\boldsymbol{f}} = \mathbf{E}\boldsymbol{f}, \tag{3}$$

where $\bar{\boldsymbol{u}}$ is the vector of measured response degrees of freedom, of dimension m, and $\bar{\boldsymbol{f}}$ is the vector of measured degrees of freedom forces, of dimension p.

The Z-domain relationship between applied forces and responses is governed by the equation:

$$\bar{\boldsymbol{U}}(Z) = \bar{\mathbf{H}}(Z)\bar{\boldsymbol{F}}(Z), \tag{4}$$

where $\bar{\boldsymbol{U}}(Z)$ and $\bar{\boldsymbol{F}}(Z)$ represent the Z-transformed time vectors $\bar{\boldsymbol{u}}(t)$ and $\bar{\boldsymbol{f}}(t)$, respectively, and $\bar{\mathbf{H}}(t)$ is the identifiable transfer matrix defined as:

$$\bar{\mathbf{H}}(t) = \mathbf{O}\mathbf{H}(Z)\mathbf{E}^T. \tag{5}$$

Matrix $\bar{\mathbf{H}}(t)$ contains the product of response measured mode shapes, $\boldsymbol{\phi}_{kO}$, of dimension $m \times 1$, and forcing measured mode shapes, $\boldsymbol{\phi}_{k\mathrm{E}}$ of dimension $p \times 1$, that is,

$$\bar{\mathbf{H}}(z) = \sum_{k=1}^{2n} \frac{Z\bar{\boldsymbol{\phi}}_{k\mathrm{O}}\bar{\boldsymbol{\phi}}_{k\mathrm{E}}^T}{Z-Z_k} \tag{6}$$

where,

$$\bar{\boldsymbol{\phi}}_{k\mathrm{O}} = \mathbf{O}\boldsymbol{\phi}_k \tag{7}$$

and

$$\bar{\boldsymbol{\phi}}_{k\mathrm{E}} = \mathbf{E}\boldsymbol{\phi}_k. \tag{8}$$

The subspace identification solution is implemented firstly by defining an extended coefficients matrix $\widehat{\mathbf{A}}$, comprised of matrix $\mathbf{A}$ and vector $\boldsymbol{x}$:

$$\widehat{\mathbf{A}}=[\mathbf{A}\ \ \boldsymbol{x}] \tag{9}$$

where the minimum order coefficients matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = [\mathbf{F}^T \otimes \mathbf{I}_\mathrm{m} \quad \mathbf{U}], \tag{10}$$

$$\boldsymbol{x} = \begin{bmatrix} \bar{\boldsymbol{u}}(2n) \\ \bar{\boldsymbol{u}}(2n+1) \\ \vdots \\ \bar{\boldsymbol{u}}(M-1) \end{bmatrix}, \tag{11}$$

$$\mathbf{U} = \begin{bmatrix} \bar{\boldsymbol{u}}(0) & \bar{\boldsymbol{u}}(1) & \dots & \bar{\boldsymbol{u}}(2n-1) \\ \bar{\boldsymbol{u}}(1) & \bar{\boldsymbol{u}}(2) & \dots & \bar{\boldsymbol{u}}(2n) \\ \vdots & \vdots & \dots & \vdots \\ \bar{\boldsymbol{u}}(M-2n-1) & \bar{\boldsymbol{u}}(M-2n) & \dots & \bar{\boldsymbol{u}}(M-2) \end{bmatrix} \tag{12}$$

and

$$\mathrm{F} = \begin{bmatrix} \bar{\mathrm{f}}(1) & \bar{\mathrm{f}}(2) & \dots & \bar{\mathrm{f}}(\mathrm{M}-2\mathrm{n}) \\ \bar{\mathrm{f}}(2) & \bar{\mathrm{f}}(3) & \dots & \bar{\mathrm{f}}(\mathrm{M}-2\mathrm{n}+1) \\ \vdots & \vdots & \dots & \vdots \\ \bar{\mathrm{f}}(2\mathrm{n}) & \bar{\mathrm{f}}(2\mathrm{n}+1) & \dots & \bar{\mathrm{f}}(\mathrm{M}-1) \end{bmatrix}. \tag{13}$$

Symbol ”$\otimes$ “ of Eq. (10) indicates the Kronecker product operation [8], $\mathbf{I}_m$ is the identity matrix of dimension $m \times m$. Matrices $\mathbf{F}^T \otimes \mathbf{I}_\mathrm{m}$ and $\mathbf{U}$ are of dimensions $r_\mathrm{FI} \times C_\mathrm{FI}$ and $r_\mathrm{U} \times C_\mathrm{U}$ respectively, where,

$$r_U = m(M-2n), \tag{14}$$

$$C_U = 2n, \tag{15}$$

$$r_{FI} = m(M-2n), \tag{16}$$

and

$$C_{FI} = 2n.m.p. \tag{17}$$

The number of data samples, M, is assumed to have a value that satisfies the condition $r_{FI} > (C_{FI} + 2n)$. This is equivalent to say, according to Eqs. (16) and (17), that

$$M > 2n(p + \frac{1}{m} + 1). \tag{18}$$

Matrix $\mathbf{A}$ and vector $\boldsymbol{x}$ can be extracted from $\widehat{\mathbf{A}}$ by means of the column exclusion matrix $\bar{\mathbf{I}}_x$ and column inclusion vector $\boldsymbol{i}_x$, both comprised of elements with values 0 or 1. That is:

$$\mathbf{A} = \widehat{\mathbf{A}}\, \bar{\mathbf{I}}_{\boldsymbol{x}} \tag{19}$$

and

$$\boldsymbol{x} = \widehat{\mathbf{A}}\, \boldsymbol{i}_x. \tag{20}$$

The singular value decomposition of the extended matrix $\widehat{\mathbf{A}}$ can be written as:

$$\widehat{\mathbf{A}} = \boldsymbol{\mathcal{U}} \boldsymbol{\Sigma} \boldsymbol{\mathcal{V}}^T \tag{21}$$

where matrices $\boldsymbol{\mathcal{U}}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{\mathcal{V}}$ are of respective dimensions $m(M-2n)\times 2n(pm+1)$, $2n(pm+1)\times 2n(pm+1)$ and $2n(pm+1)+1\times 2n(pm+1)$. Note that although matrix $\widehat{\mathbf{A}}$ has a number of $2n(pm+1)+1$ columns, its rank is only $2n(pm+1)$, since it is an extension of the minimum order coefficients matrix $\mathbf{A}$, appended with vector $\boldsymbol{x}$. Matrix $\mathbf{A}$ and vector $\boldsymbol{x}$, are both constructed from the same combination of the basic minimum order set of dynamic parameters, that is, $2n$ poles and $2n$ matrix residues of dimension $m\times p$.

The solution vector, $\widehat{\boldsymbol{v}}$ is calculated from the subspace of singular vectors of $\widehat{\mathbf{A}}$ as

$$\hat{\mathbf{v}} = (\boldsymbol{\mathcal{V}}^T\bar{\mathbf{i}}_x)^+(\boldsymbol{\mathcal{V}}^T\boldsymbol{i}_x). \tag{22}$$

The subspace solution depends only on the singular vectors of the extended system coefficients matrix, having a statistically low sensitivity to variations in the individual elements of matrix $\widehat{\mathbf{A}}$, caused by random noise contamination.

Solution vector $\hat{\mathbf{v}}$ is comprised of the exogenous and auto-regressive parameter vectors, $\boldsymbol{b}$ and $\boldsymbol{a}$, respectively, in such a way that

$$\widehat{\boldsymbol{v}} = \begin{bmatrix}\boldsymbol{b}\\ \boldsymbol{a}\end{bmatrix}. \tag{23}$$

Vector $\boldsymbol{a}$ contains the coefficients of the system's characteristic polynomial,

$$A(Z) = Z^{2n} + a_{2n-1}Z^{2n-1} + \cdots + a_0, \tag{24}$$

with

$$\boldsymbol{a}^T = [-a_0 \quad -a_1 \quad \dots \quad -a_{2n-1}]. \tag{25}$$

Roots of the characteristic polynomial are the Z plan poles defined as Z_k, $k = 1,2,\dots,2n$. The system poles, s_k, , that contain the parameters of modal damping ratio, ξ_k and natural frequency, ω_k, are defined as,

$$s_k = \omega_k\left(-\xi_k + j\sqrt{1-\xi^2}\right),\ k = 1,2,\dots,2n. \tag{26}$$

System poles are calculated from the Z plan poles such as that,

$$s_k = \frac{\ln(Z_k)}{\Delta t},\quad k = 1,2,\dots,2n. \tag{27}$$

Vector $\boldsymbol{b}$ contains the elements of the columns of matrices $\overline{\mathbf{B}}_k$ which are the coefficients of a polynomial defined as

$$\overline{\mathbf{B}}(Z) = \overline{\mathbf{B}}_{2n-1}Z^{2n} + \overline{\mathbf{B}}_{2n-2}Z^{2n-1} + \cdots + \overline{\mathbf{B}}_0, \tag{28}$$

with

$$\boldsymbol{b}^T = [\ {}_0\overline{\boldsymbol{b}}_1^T \quad {}_0\overline{\boldsymbol{b}}_2^T \quad \dots \quad {}_0\overline{\boldsymbol{b}}_p^T \quad {}_1\overline{\boldsymbol{b}}_1^T \quad {}_1\overline{\boldsymbol{b}}_2^T \quad \dots \quad {}_1\overline{\boldsymbol{b}}_p^T \quad \dots \quad {}_{2n-1}\overline{\boldsymbol{b}}_1^T \quad {}_{2n-1}\overline{\boldsymbol{b}}_2^T \quad \dots \quad {}_{2n-1}\overline{\boldsymbol{b}}_p^T]. \tag{29}$$

and ${}_k\overline{\boldsymbol{b}}_j$, $j = 1,2,\ldots,p$ and $k = 0,1,\ldots,2n-1$ are generic columns of matrices $\overline{\mathbf{B}}_k$.

The complex product that contains the response measured mode shapes, $\overline{\boldsymbol{\phi}}_{kO}$, and forcing measured mode shapes, $\overline{\boldsymbol{\phi}}_{k\mathrm{E}}$ are calculated from the residues of the ratio of the matrix and characteristic coefficients polynomials such as,

$$\overline{\boldsymbol{\phi}}_{kO}\overline{\boldsymbol{\phi}}_{k\mathrm{E}}^T = \lim_{Z\to Z_k}\frac{(Z-Z_k)\overline{\mathbf{B}}(Z)}{Z\,A(Z)}. \qquad . \tag{30}$$

3 MEASUREMENT SETUP

The aeronautical aluminium panel of dimensions $1.2m$ (width) and $0.8m$ (height) is hung vertically and excited at the back by two shakers. The panel is comprised of three larger vertical stiffening elements and five smaller vertical stiffeners. Two transducers attached to the shakers are used to measure the driving force exerted on the structure. Seventeen small mass accelerometers are attached to the front of the panel to pick up the movement of the structure. Such an arrangement is shown in Figure 1.

The shakers were driven with two independent random excitation signals. A number $M = 2^{13}$ of time data samples from the force excitation and acceleration response signals were taken with a sampling frequency $f_s = 2560\ Hz$. No window was applied to the time sampled signals.

The MUST identification algorithm was used to identify natural frequencies and natural modeshapes. The relevant parameters used in the identification scheme are listed on table 1 below.

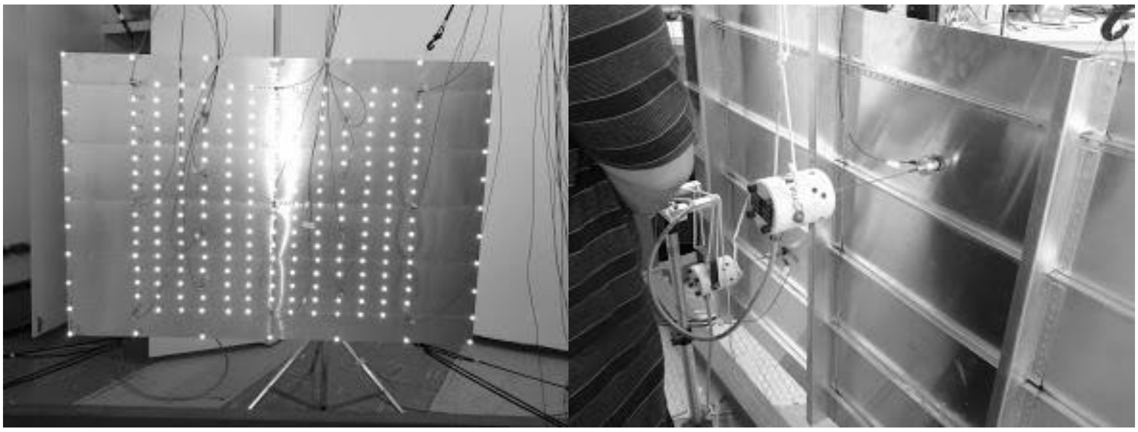

Figure 1: Testing panel.

The force (driving) and acceleration measuring points are displayed in Figure 2.

Parameter	Nomenclature	Value
Number of outputs	m	17
Number of input	p	2
Identification order	n	90
Number of samples	M	8192

Table 1: Parameters of the MUST identification scheme .

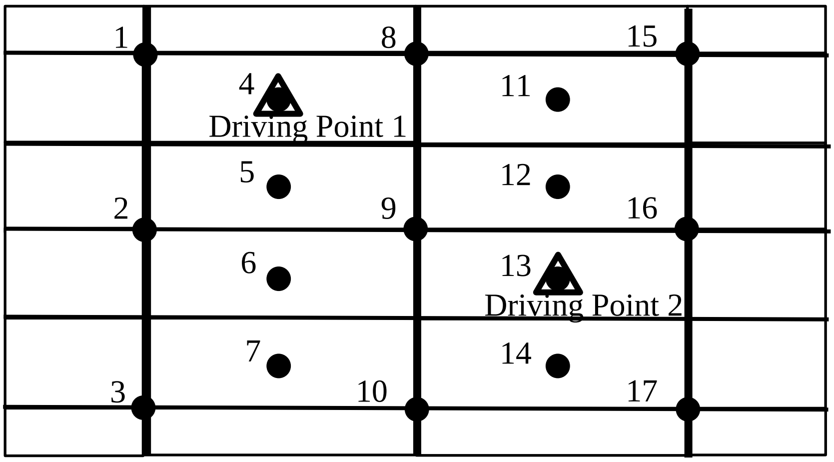

Figure 2: Measuring points.

Results from identification of the structure using the MUST algorithm and those from the frequency domain POLYMAX algorithm [14] of the LMS Test.Lab analysis software are presented in the next section.

4 IDENTIFICATION RESULTS AND DISCUSSIONS

Three of the most important low frequency modes are identified using the POLYMAX and MUST algorithm, yielding the following identification results:

Mode #	MUST Freq. (Hz)	POLYMAX Freq. (Hz)
2	26.9	27.2
3	30.1	30.4
5	75.5	74.9

Table 1: Parameters of the MUST identification scheme .

Mode-shapes associated with the frequencies are presented in Figs. 3, 4 and 6 5 below.

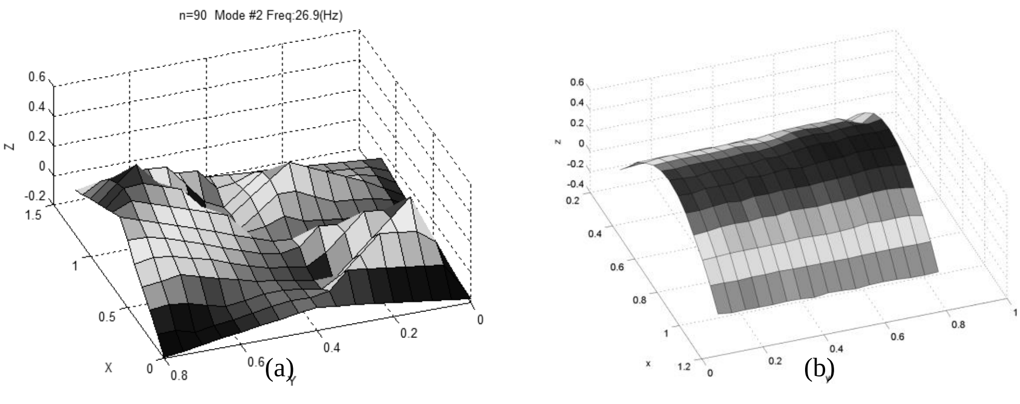

Figure 3: Mode-shape frequency 27Hz. (a) MUST, (b) POLYMAX.

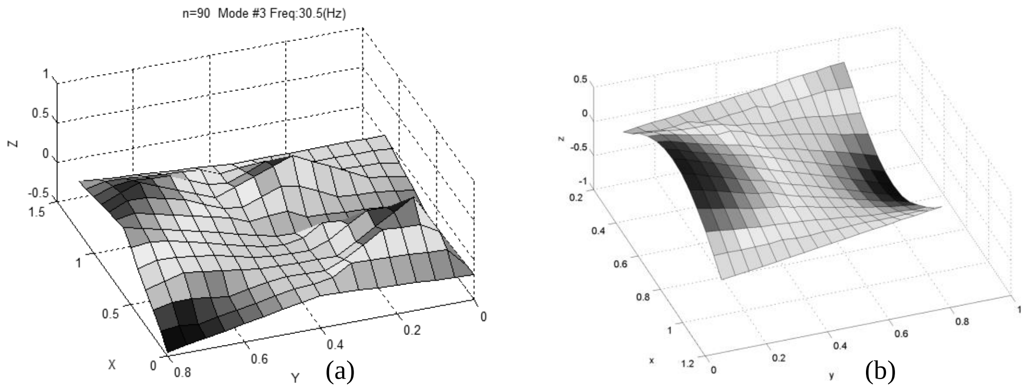

Figure 4: Mode-shape frequency 31Hz. (a) MUST, (b) POLYMAX.

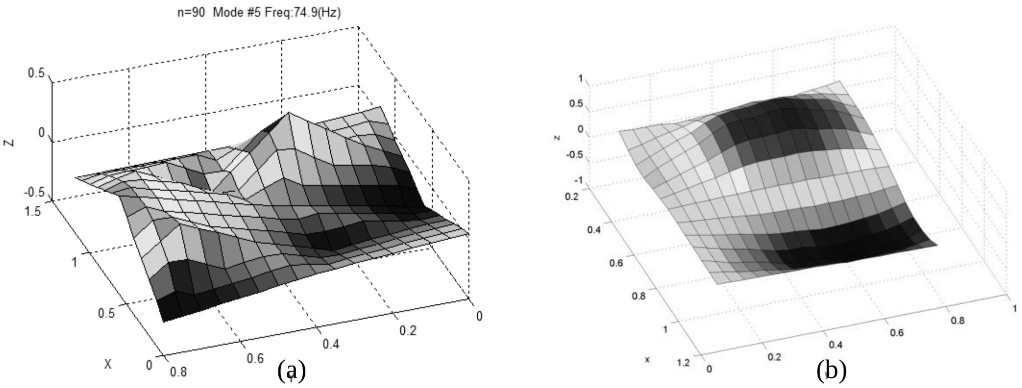

Figure 4: Mode-shape frequency 75Hz. (a) MUST, (b) POLYMAX.

The natural frequencies calculated in both methods are in good agreement, as seen in Table 2. The mode-shapes displayed in Figures 3-5, however, showed some differences between the two methods. The MUST algorithm provided modal displacement information with higher amplitudes at the internal degrees of freedom of the panel. It can be seen in the scheme of measuring positions, Figure 2, that points 4,5 6, 7, 11, 12,13 and 14 are located at the centre of the unstiffened aluminium plate sections, which justify the greater mobility of such regions. The MUST algorithm processes simultaneously all displacement and force signals, yielding a single set of natural frequencies and two sets of equivalent mode-shapes for each natural frequency. The minimum order for stabilization of the identification algorithm is proportional to the number of simultaneously measured inputs and outputs. In the present case of 2 inputs and 17 outputs, stabilization of the identified parameters start at an identification order $n > 80$. An order $n = 90$ was used to yield the present identified parameters. Higher identification orders were also tested, without significant changes to the values of the relevant natural frequencies or shapes of associated vibration modes.

5 CONCLUSIONS

- The MUST identification algorithm has correctly identified the natural frequencies of the dynamic system with 17 simultaneous outputs and 2 input signals. .

- The identified shapes were most likely influenced by the greater flexibility of the internal measuring points, yielding a somewhat less smooth representation of the vibration modes from those obtained in the LMS/POLYMAX scheme.

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