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A STUDY OF VIBRATION CONTROL OF BEAMS SUBJECTED TO MOVING LOADS

Zenon J. G. N. del Prado[*1]; Renata M. Soares[1]; Paulo B. Gonçalves[2]

[1]FederalUniversity of Goias
zenon@eec.ufg.br; msrenata@gmail.com

[2] Catholic University of Rio de Janeiro
paulo@puc-rio.br

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Abstract. *The analysis and control of vertical vibrations on bridges due to moving loads is important in project and design then, the correct use and application of vibration absorbers is fundamental to reduce this kind of vibrations. In this work, the vibration control of a simply supported beam subjected to moving vehicles is studied. The linear Euler-Bernoulli theory without considering the effect of rotatory inertia is used to model a simply supported beam subjected to moving loads and, to reduce the vibration of vertical displacements, a discrete damped mass-spring system is applied to the beam. A discrete spring-mass-damper system is used to model the moving vehicles. An expansion of five degrees of freedom is used to model the transversal displacements of the beam and the Galerkin method is used to obtain a set of linear equations of motion which are, in turn, solved by the Runge-Kutta method. Obtained results show the importance of vehicle-bridge interaction and damper position on the vibrations control of beams.*

1 INTRODUCTION

The study of bridge oscillations and control is a problem that has been the object of interest of engineers and scientist over the last century [1,2]. One of the first researches in control vibration was Den Hartog [3], who derived the optimum parameters of the absorber for suppressing the dynamic response of the single degree-of-freedom spring-mass system.

Greco and Santini [4], using an extension of the complex mode superposition method, analyzed the dynamic problem of a continuous beam with two end rotational viscous dampers under a single moving load. They concluded that the damper's effectiveness is strongly dependent on the load velocity and proved that, in the relevant range of velocities, a considerable reduction of the dynamic response of the beam depends on the properly selected damper's constants.

The possibility of reduction of the resonant vibrations of simple supported beams under moving loads, by increasing the structural damping with passive energy dissipation devices, was evaluated by Museros and Martinez-Rodrigo [1]. The authors used a linear viscous damper (FVDs) to connect the main beam, which carries the loads, to an auxiliary beam placed underneath the main one. The results show that the resonant response of the main beam can be drastically reduced with this type of device and the proposed methodology has potential applications for reduction of the response of railway bridges subjected to the transit of high-speed trains.

Yang et al [5] looking to identify the key parameters of vehicle-bridge interaction response, studied the frequencies of vibration of bridges. The vehicle was modeled as a mass-spring system and the bridge as a simply supported beam and, in analysis, only the first mode of vibration was considered. Ferreira [6] developed a parametric analysis of the effects generated by the action of moving loads on bridges due to the vehicle mobility as well as the impact on the road surface irregularities by considering a mathematical model of beams with concentrated masses and the vehicle as a system composed by masses, springs and absorbers.

Chtiba et al [7] propose a novel strategy for the optimal design of supplementary absorbers that warrant confinement with and without suppression of vibrations in flexible structures. The authors assumed that the uncontrolled structure is sensitive to vibrations and that the absorbers are the elements where the vibrational energy is to be transferred. The design of these absorbers is formulated as a dynamic optimization problem in which the objective function is the total energy of the uncontrolled structure. The locations, masses, stiffnesses, and damping coefficients of these absorbers are optimized to minimize the total energy of the structure. To show the viability of the proposed design, the authors considered a simply supported beam with and without external excitations and, in the absence of structural damping, its demonstrate that the beam, subjected to either an initial distributed energy or a harmonic excitation, periodically exchanges the vibration energy with the added absorbers.

In this work, the linear Euler-Bernoulli beam theory is used to study the vibration control of simply supported beams subjected to moving loads and controlled by a fixed absorber. The beam is considered as a linear elastic continuous system and the absorber is described as a linear spring-mass-damper system. A discrete spring-mass-damper system is used to model the moving vehicles. A modal expansion with five modes is used to model the lateral displacements of the beam and the Galerkin method is used to obtain a set of discretized equations of motion which are, in turn, solved by the Runge-Kutta method.

2 PROBLEM FORMULATION

Consider a simply supported beam with length L, distributed mass m_b, transversal inertia I, Young's modulus E, damping coefficient c_b and subjected to a moving vehicle with constant velocity v_v. The vehicle is described as a damped mass-spring system with mass m_v, stiffness k_v and damping coefficient c_v.

To reduce vibrations, the beam is connected to an absorber composed by a mass m_a, a spring with stiffness k_a and a linear damper with coefficient λ_a. It is supposed that the absorber can be fixed and located at position d as shown in Figure 1.

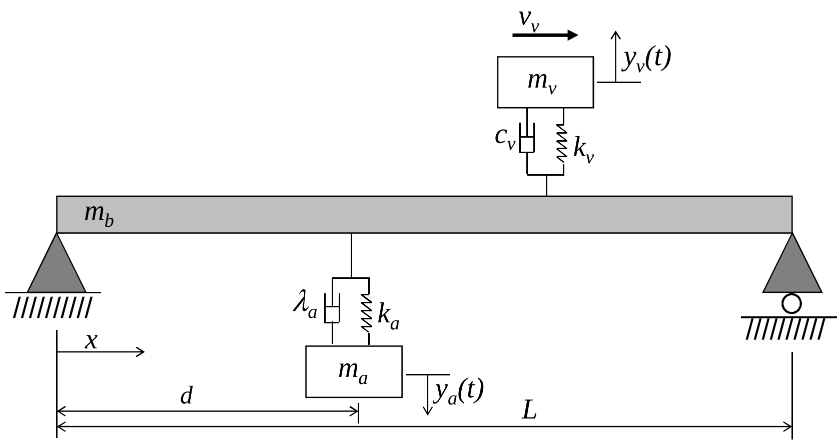


Figure 1 - The controlled beam model.

Using linear Euler-Bernoulli theory to model the beam, the linear equations of motion of the system are given by:

$$m_b \frac{\partial^2 y_b}{\partial t^2} + c_b \frac{\partial y_b}{\partial t} + EI \frac{\partial^4 y_b}{\partial x^4} + \left[k_a \left(y_b(d,t) - y_a(t) \right) + \lambda_a \frac{\partial \left(y_b(d,t) - y_a(t) \right)}{\partial t} \right] \delta(x-d) = \left[k_v \left(y_v(t) - y_b(v_v, t) \right) + c_v \frac{\partial \left(y_v(t) - y_b(v_v, t) \right)}{\partial t} + m_v\, g \right] \delta(x - v_v\, t)\; H\!\left(\frac{L}{v_v} - t \right) ; \quad \textbf{(1)}$$

$$m_a \frac{\partial^2 y_a}{\partial t^2} - k_a \left(y_b(d,t) - y_a(t) \right) - \lambda_a \frac{\partial \left(y_b(d,t) - y_a(t) \right)}{\partial t} = 0 \qquad t > 0\,; \qquad (2)$$

$$m_v \frac{\partial^2 y_v}{\partial t^2} - k_v \left(y_v(t) - y_b(v_v t,\, t) \right) - c_v \frac{\partial \left(y_v(t) - y_b(v_v t,\, t) \right)}{\partial t} = 0 \qquad x \in (0, L) \;\; t > 0\,; \qquad (3)$$

where $y_b(t)$ represents the transversal displacements field of the beam; $y_a(t)$ is the absolute transversal displacement of the mass m_a; $y_v(t)$ is the absolute transversal displacement of the vehicle; g is the acceleration of gravity; δ is the Dirac delta function which defines the location of the absorber while $H(t)$ is the Heaviside function.

The boundary conditions for the simply supported beam and initial conditions for the problem are given by:

$$y_b\,(0,t) = 0; \quad y_b(L,t) = 0; \quad \frac{\partial^2 y_b}{\partial x^2}(0,t) = 0; \quad \frac{\partial^2 y_b}{\partial x^2}(L,t) = 0; \qquad (4)$$

$$y_b\,(x,0) = 0; \quad \frac{\partial y_b}{\partial t}(x,0) = 0; \qquad (5)$$

In this work, the lumped mass m_a of the absorber is assumed to be 5% of the total mass of the beam and the transverse vibrations of the beam can be described as the sum of n eigenfunctions given by [1, 8]:

$$y_b(x,t)=\sum_{r=1}^{\infty}A_r(t)\phi_r(x)=\sum_{r=1}^{\infty}A_r(t)\sin\left(\frac{r\pi x}{L}\right) \tag{6}$$

where the $A_r(t)$ are the unknown functions of time.

Substituting Eq. (6) into Eqs. (1) - (3) and applying the Galerkin method, a set of ordinary differential equations of motion are obtained which are, in turn, solved by the Runge-Kutta method. For the linear system, after using the orthonormality conditions, the following equations are obtained:

$$\begin{aligned}&\frac{m_b L}{2}\ddot{A}_p(t)+\xi_p\omega_{bp}m_b L\dot{A}_p(t)+\frac{\omega_{bp}^{2}m_b L}{2}A_p(t)+\\&+\left\{k_a\left[\sum_{r=1}^{\infty}A_r(t)\phi_r(d)-y_a(t)\right]+\lambda_a\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(d)-\dot{y}_a(t)\right]\right\}\phi_p(d)= \qquad ;\\&\left\{k_v\left[y_v(t)-\sum_{r=1}^{\infty}A_r(t)\phi_r(v_v t)\right]+2\,\xi_v m_v\omega_v\left[\dot{y}_v(t)-\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(v_v t)\right]+m_v\,g\right\}H\left(\frac{L}{v_v}-t\right)\end{aligned} \tag{7}$$

$$m_a\ddot{y}_a(t)-k_a\left[\sum_{r=1}^{\infty}A_r(t)\phi_r(d)-y_a(t)\right]-\lambda_a\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(d)-\dot{y}_a(t)\right]=0; \tag{8}$$

$$m_v\ddot{y}_v(t)-k_v\left[y_v(t)-\sum_{r=1}^{\infty}A_r(t)\phi_r(v_v t)\right]-2\,\xi_v m_v\omega_v\left[\dot{y}_v(t)-\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(v_v t)\right]=0 \tag{9}$$

where ω_v is the natural frequency of the vehicle; $\phi_r(x)=\sin(r\pi x/L)$ and $\dot{A}_p(t)=dA_p/dt$.

The natural frequency of the beam for the p^{th} mode is given by:

$$\omega_{bp}=(\pi p)^2\sqrt{\left(\frac{EI}{m_b L^4}\right)} \tag{10}$$

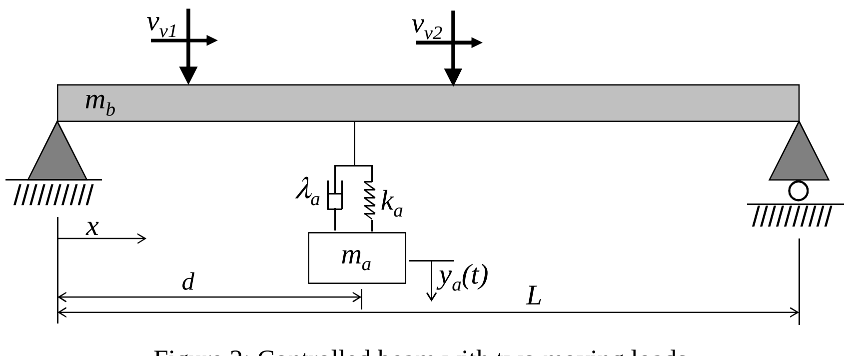


Figure 2: Controlled beam with two moving loads.

Now, considering a beam with two vehicles moving loads with velocities v_{v1} and v_{v2}, as displayed in Figure 2, in this case vehicles are considered as concentrated loads. After applying the Galerking method the obtained system o ordinary differential equation is given by:

$$\begin{aligned}&\frac{m_b L}{2}\ddot{A}_p(t)+\xi_{bp}\omega_{bp}m_b L\dot{A}_p(t)+\frac{\omega_{bp}^{2}m_b L}{2}A_p(t)+\left\{k_a\left[\sum_{r=1}^{\infty}A_r(t)\phi_r(d)-y_a(t)\right]+\right.\\&\left.\lambda_a\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(d)-\dot{y}_a(t)\right]\right\}\phi_p(d)=(m_v\,g)\phi_p(v_{v1}t)H\left(\frac{L}{v_{v1}}-t\right)+(m_v\,g)\phi_p(v_{v2}t)H\left(\frac{L}{v_{v2}}-t\right)\end{aligned} \tag{11}$$

$$m_a\ddot{y}_a(t)-k_a\left[\sum_{r=1}^{\infty}A_r(t)\phi_r(d)-y_a(t)\right]-\lambda_a\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(d)-\dot{y}_a(t)\right]=0; \tag{12}$$

2.1 Optimal Absorber Parameters

Considering a principal system with an added classic absorber as shown in Figure 1, the dynamic amplification factor is given by [3]:

$$R = \sqrt{\frac{(\alpha^2 - \beta^2)^2 + (2\xi_2 \alpha \beta)^2}{C_1}} \tag{13}$$

and

$$\begin{aligned} C_1 = &\left[(\alpha^2 - \beta^2)(1-\beta^2) - \mu \alpha^2 \beta^2 - 4\xi_1 \xi_2 \alpha \beta^2\right]^2 \\ &+ \left[2\xi_2 \alpha \beta (1-\beta^2 - \mu \beta^2) + 2\xi_1 \beta (\alpha^2 - \beta^2)\right]^2 \end{aligned} \tag{14}$$

where ξ_1 and ξ_2 are, respectively, the damping ratio of the principal system and absorber; α is the ratio of the natural frequencies of the absorber and the principal system; μ is the ratio of the masses of the absorber and the principal system and β is the ratio of the frequency of the external excitation and the natural frequency of the principal system.

The optimal parameters of the system can be found by applying different control theories; in this work, the Den Hartog [3] theory will be used. Assuming zero damping for the principal system ($\xi_1 = 0$), it is possible to find the classical expressions obtained by Den Hartog [3] for the optimum tuned mass and the optimal damping ratio which are given, respectively by:

$$\alpha_{ot} = \frac{1}{1+\mu} \tag{15}$$

$$\xi_{2ot} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{16}$$

The generalized modal mass and stiffness of the principal system, $\tilde{m}_b$ and $\tilde{k}_b$, necessary for the calculation of the optimal parameters are given by:

$$\tilde{m}_b = \int_0^1 \sin(\pi x)\, m_b \sin(\pi x)\, dx \tag{17}$$

$$\tilde{k}_b = \omega_b^{\,2} \tilde{m}_b \tag{18}$$

where m_b is the mass of the principal system and μ is written as:

$$\mu = \frac{m_a}{\tilde{m}_b} \tag{19}$$

Thus, the absorber stiffness k_a and the damping coefficient of the viscous damper λ_a are, respectively, given by:

$$k_a = \alpha^2 \omega_b^{\,2} m_a = \alpha_{ot}^{\,2} \frac{\tilde{k}_b}{\tilde{m}_b} m_a \tag{20}$$

$$\lambda_a = 2\xi_2 m_a \sqrt{\frac{k_a}{m_a}} = 2\xi_{2ot} \sqrt{k_a m_a} \tag{21}$$

3 NUMERICAL RESULTS

For the numerical analysis, consider a simply supported beam of length L = 25 m with Young's modulus E = 27.5 GPa, mass per unit length m_b = 4800 Kg/m, cross-sectional area $A = 2$ m^2, moment of inertia $I = 0.12$ m^4, first natural frequency ω_{b1} = 13.09 rad/s [5] and damping coefficient $\xi_p = 0.03$ (p = 1, 2, …, n) [6]. The adopted data for the vehicle are: mass m_v = 1200 kg, spring stiffness k_v = 500 kN/m, frequency ω_v = 20.42 rad/s [5] and damping coefficient $\xi_v = 0.1$ [6]. As indicated, the optimal parameters were found using the Den Hartog's theory [3] then, the ratio between the mass of the absorber and the principal system is $\mu = 0.1$, the optimum tuned mass is $\alpha_o = 0.909$ and the optimal damping ratio is found to be $\xi_{2ot} = 0.18464$. The absorber mass is $m_a = 6\ 10^3$ kg, absorber stiffness $k_a = 849.604$ kN/m and the damping coefficient of the viscous damper $\lambda_a = 26.366$ kNs/m.

Figure 3 displays the maximum displacement at the mid-span of the beam for increasing values of the load velocity considering the system of equations (Eq. 1-3) with and without absorber and with and without the vehicle model.

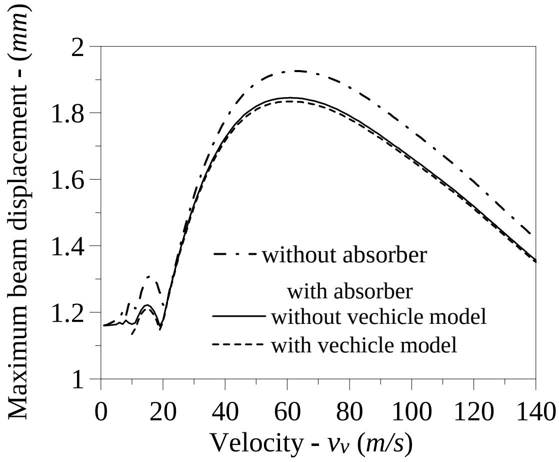


Figure 3 – Maximum displacement at mid-span of the beam ($x = 0.5L$) versus load velocity.

As can be observed, for the model without considering the absorber, the maximum displacement occurs for a velocity $v_v = 60.9$ *m/s*. The inclusion of the absorber reduces the maximum displacement of the beam up to 5% however, the consideration of the vehicle as a spring-mass system and as a moving load generate results rather similar. The optimization of the absorber is focused on the minimization of the maximum beam displacement. Then, the stiffness, the viscous damping coefficient and the location of the absorber can be varied to find the optimum values.

The absorber position was varied considering the load velocity that generates the maximum displacements at the beam ($v_v = 60.9$ m/s) and these results are shown in Figure 4. The best absorber position is located close to the mid-span of the beam and the inclusion of the vehicle model reduces the maximum displacement of the beam up to 0.5%

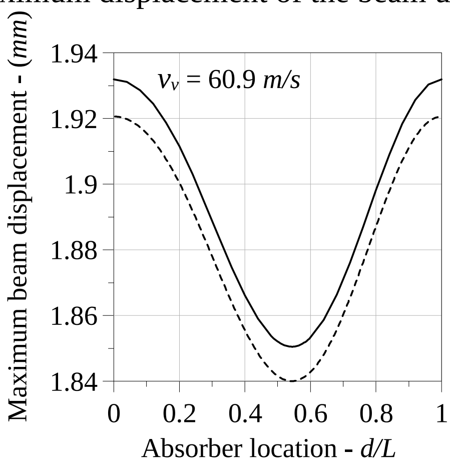


Figure 4 – Maximum beam displacement with varying absorber position.

without vehicle model ($v_{v2} = 0$) with vehicle model.

Now, the influence of considering a model vehicle as a spring-mass system (Eq. 7-9) or a concentrated load (Eq. 11-12) will be analyzed. Figure 5 shows the comparison of the maximum displacement at the mid-span of the beam, with a fixed absorber at $d = 0.5L$, vehicle velocity $v_v = 28$ *m/s*, with and without consideration of the vehicle model and different beam lengths. As can be observed, the displacement curves with and without ($v_{v2} = 0$) vehicle model are very similar which means that the considerations of the vehicle in the present problem does not affect the global results of the beam.

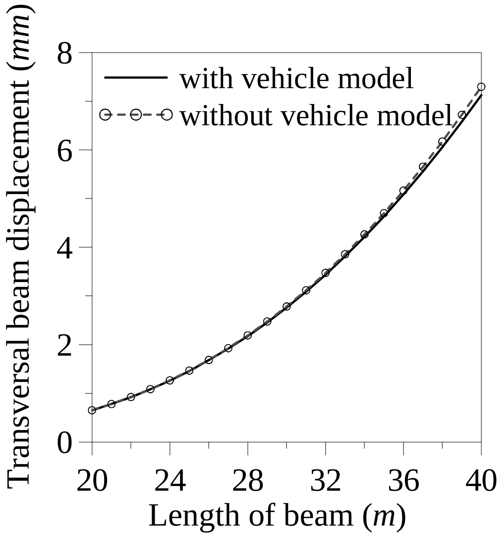
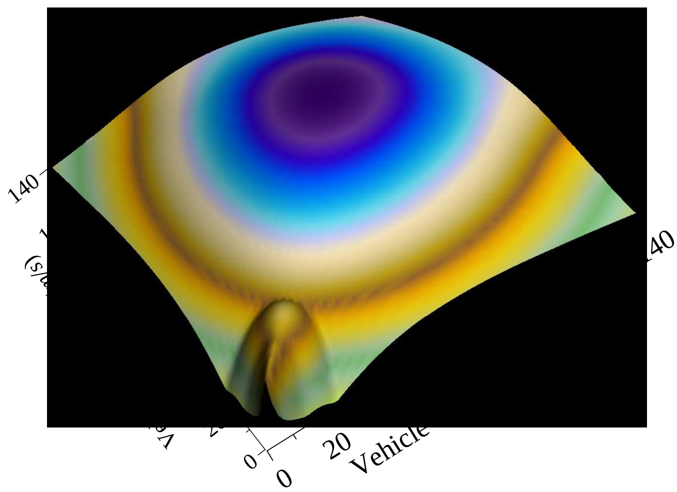


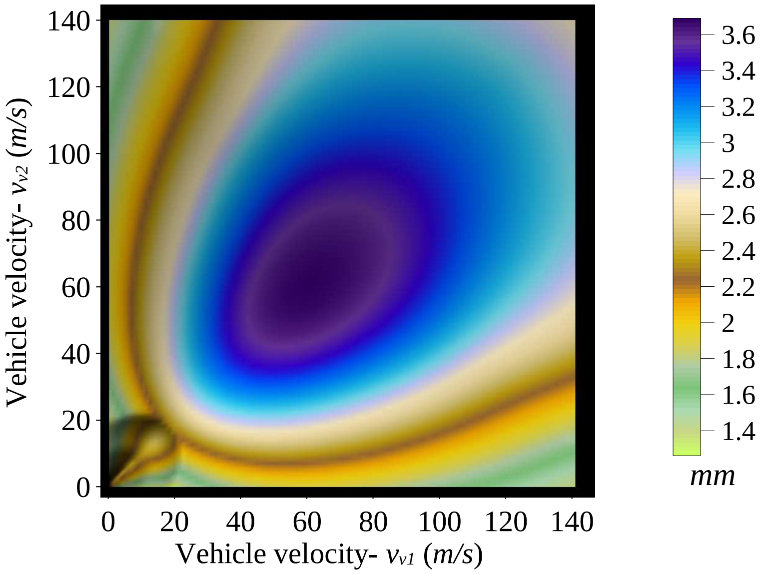
Figure 5 – Transversal beam displacement in function of length of beam with and without ($v_{v2} = 0$) consideration of the vehicle model.

Consider now the case where two vehicles passing through the beam at velocities v_{v1} and v_{v2}, respectively. Figure shows the maximum displacement in a color scale at mid-span of the beam ($x = 0.5L$) with absorber position in $d = 0.5L$ as a function of vehicles velocities v_{v1} and v_{v2}. Figure (b) is the projection of **Erro! Fonte de referência não encontrada.**(a) in the plane of vehicles velocities. It is observed that the maximum displacements occur when both velocities are $v_{v1} = v_{v2} = 60.9$ *m/s*.

Figure shows the maximum normalized transversal displacement of the beam as a function of absorber position for different vehicles velocities. As can be observed, for the studied vehicle velocities, the Best absorber position is located close to the midspan of the beam varying at $0.4 < d/L < 0.7$.



(a) Continue



(b)

Figure 6 - Maximum displacement in a color scale at mid-span of the beams (x = 0.5L) with absorber position in d = 0.5L versus loads velocities (v_{v1}; v_{v2}).

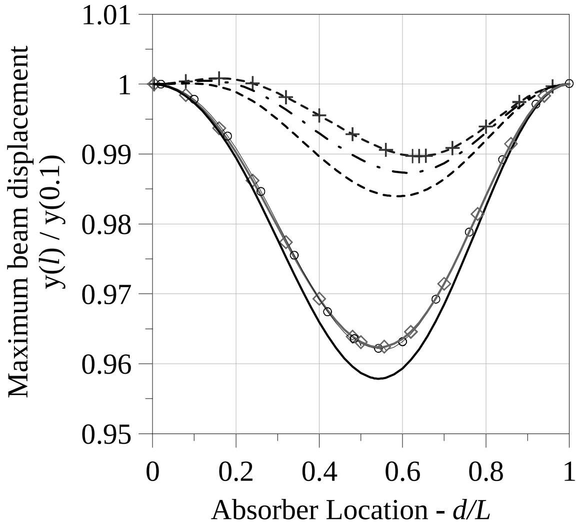


Figure 7 – Maximum beam displacement with varying absorber position.

v_{v1} = 17; v_{v2} = 28 m/s v_{v1} = 20; v_{v2} = 60.9 m/s v_{v1} = 40; v_{v2} = 60.9 m/s
v_{v1} = 23; v_{v2} = 28 m/s v_{v1} = 28; v_{v2} = 28 m/s v_{v1} = 60.9; v_{v2} = 60.9 m/s

Figure shows the maximum transversal displacement (l_{max}) of the beam for varying vehicles velocities. It is possible to observe that, depending on the velocities v_{v1} and v_{v2}, the maximum transveral displacement occurs close to the midspan of the beam at 0.44 < x/L < 0.56.

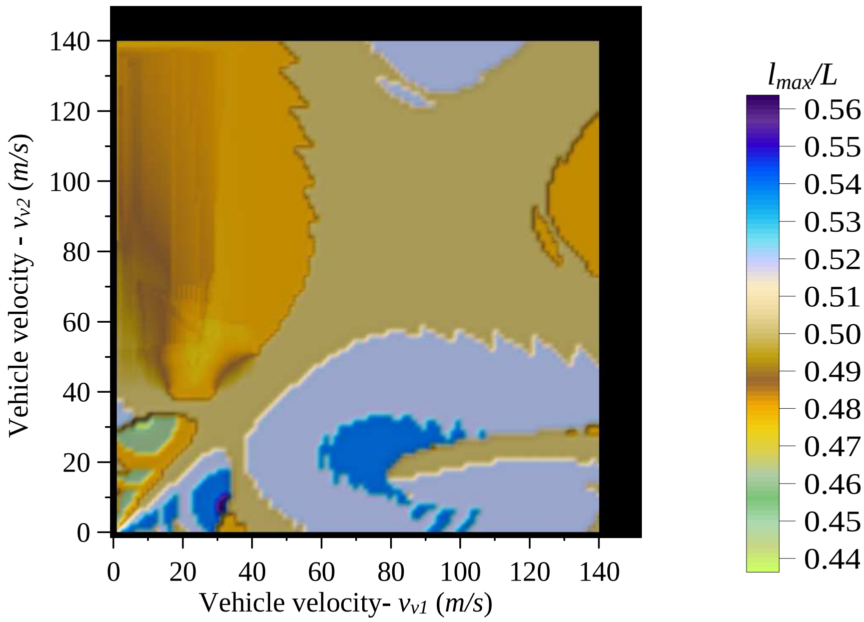


Figure 8 – Maximum displacement location (l_{max}/L) in a color scale versus loads velocities (v_{v1}; v_{v2}).

4 CONCLUSIONS

In this work, the linear Euler-Bernoulli beam theory was used to study the vibrations control of a simply supported beam subjected to moving loads and controlled by a fixed absorber. The beam is considered as an elastic continuous system, the absorber is described as a linear mass-spring-damper system and the vehicle is modeled as a lumped mass-spring system or concentrated load moving with a defined velocity along the beam.

These initial results show the influence of the absorber position on the beam vibrations and the maximum vibration amplitudes of the beam depend on both the load velocity and absorber position. Depending on the load velocity, the maximum beam displacement occurs very close to the mid-span. The results show that the inclusion of the vehicle mass-spring model in the system of equations does not modify significantly the beam displacements.

Moreover, for fixed absorber, the obtained results show the influence of absorber position on the reduction of the maximum beam displacement. Results show that, when the absorber position is close to the beam mid-span, there is a maximum transversal displacement reduction.

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