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# DYNAMIC INSTABILITY OF CYLINDRICAL SHELLS SUBJECTED TO INTERNAL PULSATING AXIAL FLOW

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**Abstract.** *In this work the dynamic instability of simple supported circular cylindrical shells subjected to internal pulsating harmonic axial flow is studied. The fluid is assumed to be incompressible and non-viscous and the flow to be isentropic and the shell is considered to be made with and elastic, homogeneous isotropic material. To model the shell, the Donnell non-linear shallow shell theory without considering the effect of shear deformation is used. A model with eight degrees of freedom, which satisfies the relevant boundary and continuity conditions, containing the fundamental, companion, gyroscopic and five axi-symmetric modes is used to describe the lateral displacements of the shell and the Galerkin method is applied to derive a set of coupled non-linear ordinary differential equations of motion which are, in turn, solved by the Runge-Kutta method. The effect of pulsating flowing fluid on the non-linear dynamic behavior is observed in detail and for this; the parametric instability boundaries, bifurcations diagrams and time responses are obtained. The obtained results show the great influence of flow velocity and pulsating frequency on the non-linear behavior of circular cylindrical shells.*

## 1 INTRODUCTION

The structural behavior of circular cylindrical shells is an important problem in industrial applications and several engineering areas use these structures extensively such as aeronautics, mechanical, civil, nuclear, among others.

In literature, it is possible to find several studies of non-linear vibrations of cylindrical shells in vacuum [1-4]; fluid-filled or partially filled with fluid [5-7] or shells immersed in fluid [8,9] but, only a few can be found related to the dynamic behavior of shells with internal pulsating fluid.

The fluid-structure interaction problem has several engineering applications, in some applications cylindrical shells are used to hold or transport fluid and usually are subjected to vibrations. Panda and Kar [10] indicate that, for a pipe with internal flow, for a certain value of flow velocity instability can occur by flutter or divergence and are very important to describe instability phenomena and other associated mechanisms. Due to different factors such as parametric excitation, flow velocity, external excitations, boundary conditions, elastic foundations and other non-linear sources the system exhibits several non-linear dynamics phenomena.

The pioneering works related to fluid-structure interaction in shells were due to Weaver and Unny [11] and Weaver and Wyklatun [12]. In the first one, authors used beam modal solutions to describe the shells and, in the second one, the stability of clamped-clamped shells subjected to high internal flow velocities was studied. The potential theory was used to describe the internal fluid flow the Flügge-Kempner shell theory was used to model the shell. In analysis flitter and divergence phenomena were observed.

Amabili et al [13] studied the non-linear dynamics and stability of simply supported cylindrical shells with non-viscous and incompressible internal fluid. To model then shells the Donnell non-linear shallow shell theory was used and the fluid-structure interaction was described by the liner potential theory. It was possible to observe the loss of stability by divergence in the system.

The dynamic instability of cylindrical shells subjected tohot fluid was studied by Ravikiran Kadoli and Ganesan [14] e Ganesan and Ravikiran Kadoli [15]. In both works a finite element formulation was used to describe the fluid-structure interaction together with a thermo mechanical model and the material of the shell was considered as isotropic and composite. The system of equations with periodical terms was obtained by linear transformations and solved by the $4^{th}$ order Runge-Kutta method. The Floquet-Lyapunov theory was used to study the stability the parametrically excited system considering pulsating fluid with varying temperatures.

This work studies the effect of pulsating flowing fluid on the non-linear dynamic behavior of simple supported circular cylindrical shells subjected to internal harmonic pulsating axial flow. The fluid is assumed to be incompressible and non-viscous and the flow to be isentropic and the shell is considered to be made with and elastic, homogeneous isotropic material and the Donnell non-linear shallow shell theory is used. A model with eight degrees of freedom, which satisfies the relevant boundary and continuity conditions, containing the fundamental, companion, gyroscopic and five axi-symmetric modes is used to describe the lateral displacements of the shell and the Galerkin method is applied to derive a set of coupled non-linear ordinary differential equations of motion which are, in turn, solved by the Runge-Kutta method. The non-linear response if the shell is analyzed by obtaining the frequency-amplitude relations, the parametric instability boundaries and bifurcation diagrams. The obtained results display the strong influence of flow on the non-linear response of the shell.

## 2 PROBLEM FORMULATION

Consider a simply supported cylindrical shell with radius $R$, thickness $h$, length $L$, containing an internal flowing fluid with velocity $U$. Figure 1 shows a simply supported cylindrical shell where the axial, circumferential and radial co-ordinates are denoted by $x$, $y = R\theta$ and $z$, respectively. The corresponding displacements of the shell middle surface are denoted by $u$, $v$ and $w$. The shell is assumed to be made of an elastic, homogeneous and isotropic material with Young's modulus $E$, Poisson ratio $\nu$ and mass density $\rho_s$. In this work the mathematical formulation follows that previously presented in references [3, 13, 16]

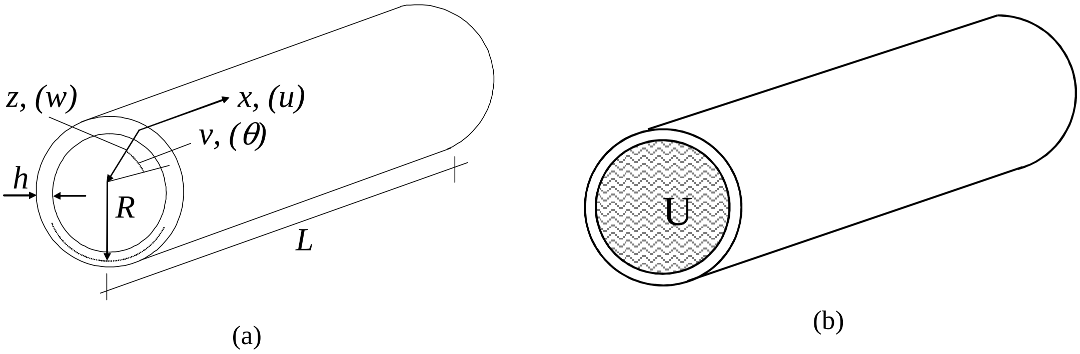



Figure 1 - Shell characteristics. a) Shell geometry; b) Fluid flow.

The nonlinear equation of motion based on the von Kármán-Donnell shallow shell theory, in terms of a stress function $F$ and the lateral displacement $w$, is given by:

$$D\left(\frac{\partial^4 w}{\partial x^4}+\frac{2}{R^2}\frac{\partial^4 w}{\partial \theta^2 \partial x^2}+\frac{1}{R^4}\frac{\partial^4 w}{\partial \theta^4}\right)+c\,h\frac{\partial w}{\partial t}+\rho_s h\frac{\partial^2 w}{\partial t^2}= \\ -P_h+\frac{1}{R}\frac{\partial^2 F}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 F}{\partial \theta^2}\frac{\partial^2 w}{\partial x^2}-\frac{2}{R^2}\frac{\partial^2 F}{\partial \theta \partial x}\frac{\partial^2 w}{\partial \theta \partial x}+\frac{1}{R^2}\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial \theta^2} \tag{1}$$

where $D = E h^3/\left[12(1-\nu^2)\right]$ is the flexural rigidity, $c$ ($kg/m^3\,s$) is the damping coefficient, $P_h$ is the radial pressures applied to the surface of the shell as a consequence of the contained flowing fluid.

The compatibility equation is given by:

$$\frac{\partial^4 F}{\partial x^4}+\frac{2}{R^2}\frac{\partial^4 F}{\partial \theta^2 \partial x^2}+\frac{1}{R^4}\frac{\partial^4 F}{\partial \theta^4}=E\,h\left(-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}+\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta \partial x}\right)^2-\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right) \tag{2}$$

The forces per unit length in the axial and circumferential directions, as well as the shear force are given in terms of the stress function by [17]:

$$N_x=\frac{1}{R^2}\frac{\partial^2 F}{\partial \theta^2}, \qquad N_\theta=\frac{\partial^2 F}{\partial x^2}, \qquad N_{x\theta}=-\frac{1}{R}\frac{\partial^2 F}{\partial x\,\partial \theta} \tag{3}$$

The boundary conditions for the simply supported shell with axial loads at $x=0$ and $x=L$ are:

$$w=0, \quad M_x=\frac{\partial^2 w}{\partial x^2}=0, v=0 \tag{4}$$

### 2.1 Expansion for the lateral displacement

From previous investigations on modal solutions for the nonlinear analysis of cylindrical shells under axial fluid flow, a suitable expansion for present problem is as follows [13, 16]:

$$w(x,\theta,t) = \xi_{11}(t)\, h\, \sin(q)\cos(n\theta) + \xi_{11c}(t)\, h\, \sin(q)\sin(n\theta) + \xi_{11g}(t)\, h\, \sin(2q)\cos(n\theta)$$
$$+ \xi_{12g}(t)\, h\, \sin(2q)\sin(n\theta) + \xi_{01}(t) h\, \sin(q) + \xi_{03}(t) h\, \sin(3q) \tag{5}$$
$$+ \xi_{05}(t) h\, \sin(5q) + \xi_{07}(t) h\, \sin(7q)$$

where $\xi_{11}(t)$, $\xi_{11c}(t)$, $\xi_{11g}(t)$, $\xi_{12c}(t)$, $\xi_{01}(t)$, $\xi_{03}(t)$, $\xi_{05}(t)$ and $\xi_{07}(t)$ are the time-dependent non-dimensional modal amplitudes, $q = m\pi x / L$ and $m$ and $n$ are, respectively, the number of half-waves in the axial direction and the number of waves in the circumferential direction. This modal expansion satisfies the out-of-plane boundary conditions (4) and includes the basic vibration (driven) mode, the companion mode, gyroscopic modes and four axi-symmetric modes and has been thoroughly tested in Amabili et al [13] and Amabili [4]. The choice of these modes is based on previous investigations on modal solutions for the non-linear analysis of cylindrical shells (3, 7, 16, 18). Using these modes it is possible to take into account all relevant non-linear modal interactions observed in the past in the non-linear vibrations of cylindrical shells with and without flow.

The solution for the stress function may be written as $F = F_h + F_p$, where $F_h$ is the homogeneous solution and $F_p$, the particular solution. The particular solution $F_p$ is obtained analytically by substituting the assumed form of the lateral deflection, $w$ Eq. (5), on the right-hand side of the compatibility equation, Eq. (2), and by solving the resulting linear partial differential equation together with the relevant boundary and continuity conditions.

The homogeneous part of the stress function can be written as:

$$F_h = \frac{1}{2}\overline{N}_{xx} R^2 \theta^2 + \frac{1}{2}x^2 \left\{ \overline{N}_{\theta\theta} - \frac{1}{2\pi R L}\int_0^{2\pi}\int_0^{L} \frac{\partial^2 F_p}{\partial x^2} R\, d\theta\, dx \right\} - \overline{N}_{x\theta} R\theta \tag{6}$$

where $\overline{N}_{xx}$, $\overline{N}_{\theta\theta}$ and $\overline{N}_{x\theta}$ are the average in-plane restraint stresses generated at the ends of the shell. This solution enables one to satisfy the in-plane boundary conditions on the average [4].

Upon substituting the modal expressions for $F$ and $w$ into the equation of motion (1) and applying the Galerkin method, a set of eight non-linear ordinary differential equations is obtained in terms of the time-dependent modal amplitudes, $\xi(t)_{ij}$.

## 2.2 Modeling of fluid-structure interaction

The fluid is assumed to be incompressible and non-viscous and the flow to be isentropic and irrotational. To determine the perturbation pressure on the shell wall the Païdoussis [6] model is adopted. In this model, the linear potential theory is used to describe the effect of the internal axially flowing fluid. The irrotationality property is the condition for the existence of a scalar potential function $\psi$, from which the velocity may be written as:

$$V = -\nabla\psi \tag{7}$$

Following the procedure presented in previous studies Païdoussis [6], the perturbation pressure on the shell wall is found to be:

$$P_h = -\rho_f \frac{In(qx)}{qIn'(qx)} \left( \frac{\partial^2 w}{\partial t^2} + 2U\frac{\partial^2 w}{\partial x \partial t} + U^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial U}{\partial t}\frac{\partial w}{\partial x} \right) \tag{8}$$

where $\rho_f$ is the fluid density, $I_n$ is the $n$th order modified Bessel function of the first kind, $I_n$'is the derivative with respect to its argument and $U$ is the flow velocity.

The flow velocity is assumed to be composed by a constant flow ($U_E$) superposed by a harmonic part given by:

$$U = U_E + U_D \cos(\Omega_f t) \tag{9}$$

where $U_E$ is the flow velocity, $U_D$ represents the amplitude of perturbation and $\Omega_f$ is the pulsating frequency of flow.

## 3 NUMERICAL RESULTS

For the numerical analysis, consider a simply supported cylindrical shell of radius $R = 1.6\ m$ and thickness $h = 0.002\ m$ ($h/R = 800$) with Young's modulus $E = 210$ G$Pa$, density $\rho_s = 7850\ Kg/m^3$, Poisson's ratio $\nu = 0.3$ and density of fluid $\rho_f = 1000\ Kg/m^3$. The damping coefficient is defined as $c = 2\zeta\rho_s\omega_o$ where $\zeta$ is the viscous damping factor. In the present analysis, the adopted viscous damping factor is $\zeta = 0.02$.

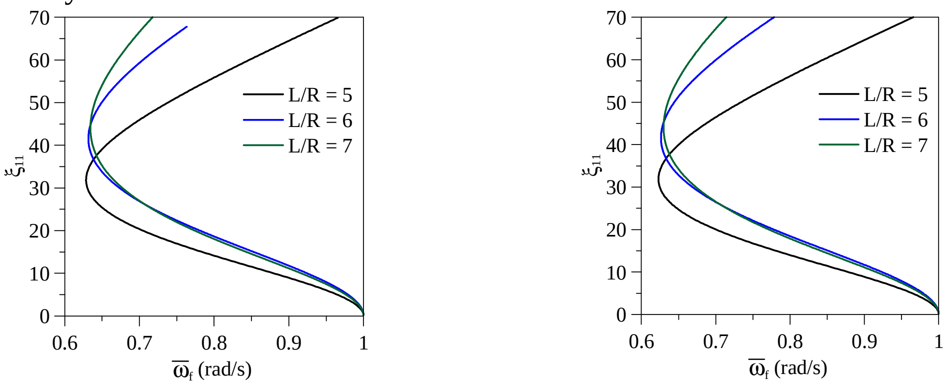
To study the influence of geometry on the response of the shell, three different ($L/R$) relations were adopted. Table 1 displays the $L/R$ relation used, its corresponding axial half-wave number ($m$), circumferential wavenumber ($n$), natural frequency of both empty ($\omega_o$) and fluid filled ($\omega_{of}$) shell and critical flow velocity ($U_{cr}$). As observed, low $L/R$ relation will generate higher natural frequencies and critical flow velocities than high $L/R$ ratios.

Table 1 – Geometric relations, natural frequencies and critical flow velocity.

| $L/R$ | $m$ | $n$ | $\omega_o$ (rad/s) | $\omega_{of}$ (rad/s) | $U_{cr}$ (m/s) |
|---|---|---|---|---|---|
| 5 | 1 | 6 | 56.66 | 13.39 | 35.09 |
| 6 | 1 | 5 | 46.74 | 10.13 | 31.72 |
| 7 | 1 | 5 | 40.21 | 8.09 | 29.47 |

Figure 2 displays the frequency-amplitude relation of the cylindrical shell for the three adopted geometric relations for increasing values of flow velocity. In this Figure 2the natural frequencies ( $\overline{\omega}_f$ ) are parameterized in relation to the natural frequency of the fluid-filled shell ($\omega_{of}$) considering only permanent flow ($U_D$=0).

As can be observed, the non-linearity of the shell depends directly on both geometric relatons and internal flow velocity. All curves display softening behavior but shell with lower $L/R$ ratio depicts higher non-linearity than shells with high $L/R$ ratio; it is possible to that, flow velocity affects the non-linear response of the shell and, as the flow velocity is increased the non-linearity of the shell is also increased.




(a) (b)

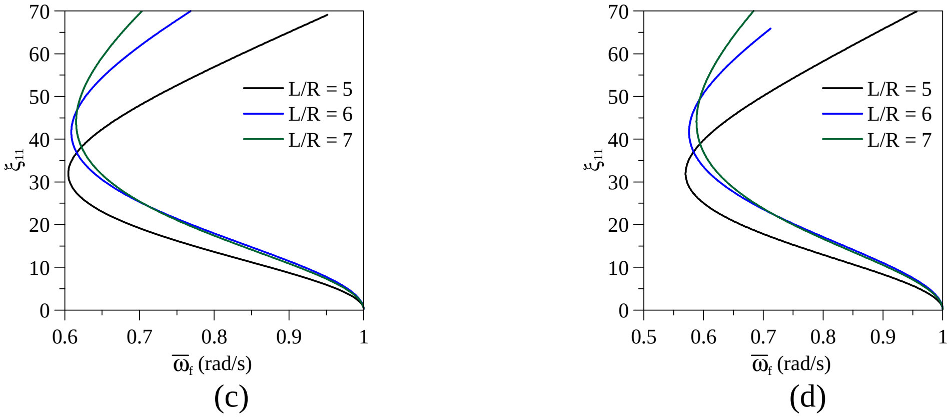



Figure 2 – Influence of flow velocity and geometry on the frequency-amplitude relations. (a) $U_E = 0.0\ U_{cr}$; (b) $U_E = 0.1\ U_{cr}$; (c) $U_E = 0.2\ U_{cr}$; (d) $U_E = 0.3\ U_{cr}$

Figure 3 shows the effect of the flow velocity $U_E$ on the frequency-amplitude relations of the shell for the three selected geometries and increasing values of flow velocity. As can be seen, the flow velocity affects the non-linear behavior of the shell and, as the flow is increased, the non-linearity is highly enhanced.

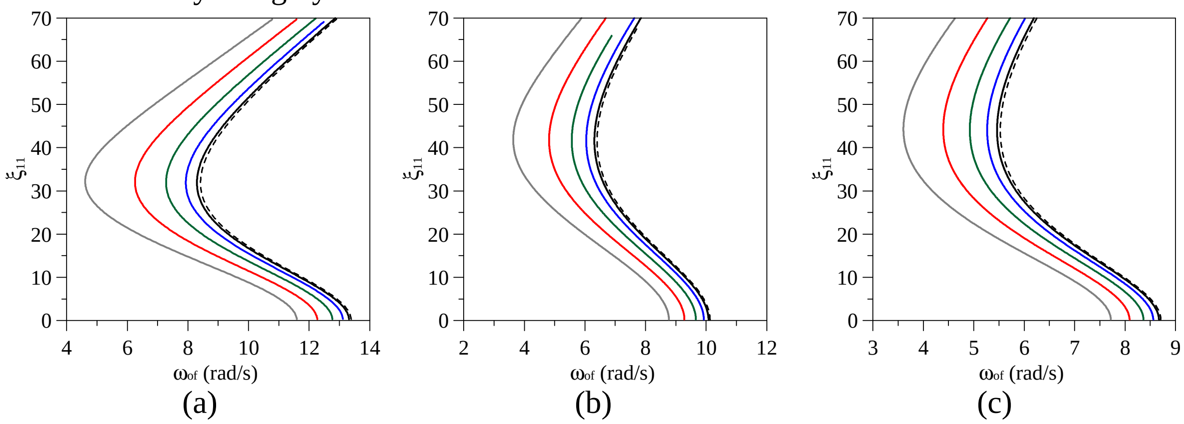



Figure 3 – Frequency-amplitude relations for increasing values of flow velocity. (a) L/R = 5; (b) L/R = 6; (c) L/R = 7. $\xi_{11}$. --- $U_E$ = 0.0 Ucr ; —— $U_E$ = 0.1 Ucr ; —— $U_E$ = 0.2 Ucr ; —— $U_E$ = 0.3 Ucr ; —— $U_E$ = 0.4 Ucr ; —— $U_E$ = 0.5 Ucr

Now, let's consider internal pulsating fluid acting in the shell. Figure 4 displays the parametric instability boundaries of the shell considering the three selected geometries for increasing values of the amplitude of the pulsating flow $U_D$ and varying the harmonic flow frequency, $\Omega_f$. It was considered three values for permanent flow $U_E = 0$, $U_E = 0.1\ U_{cr}$ and $U_E = 0.2\ U_{cr}$. These boundaries are obtained by increasing slowly the excitation amplitude of $U_D$ while holding the frequency constant and are is composed of various curves, each one associated with a particular bifurcation event. It is possible to observe that the amplitude of the pulsating fluid has the effect of shifting the instability boundaries to the left. Also, for low values of $U_E$ it is possible to observe the appearance of well between the principal and secondary regions of parametric instability. When $U_E$ is increased, the boundaries are displaced to a lower level which means that hidro-dynamic pressure due to fluid affects strongly the instability of cylindrical shells.

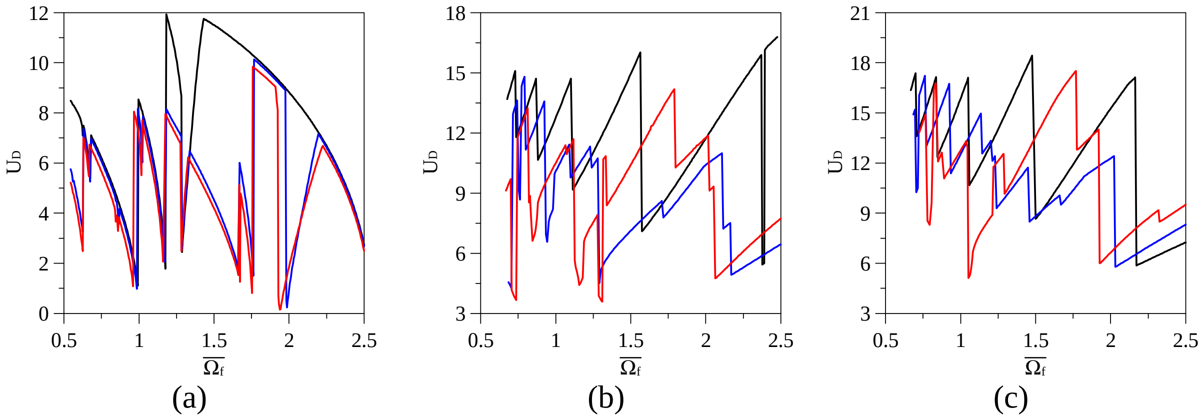



Figure 4 – Parametric instability boundaries. (a) *L/R* = 5 (b) (c) *L/R* = 6 (d) *L/R* = 7. ——— $U_E = 0.0$ ; ——— $U_E = 0.1$ ; ——— $U_E = 0.2$

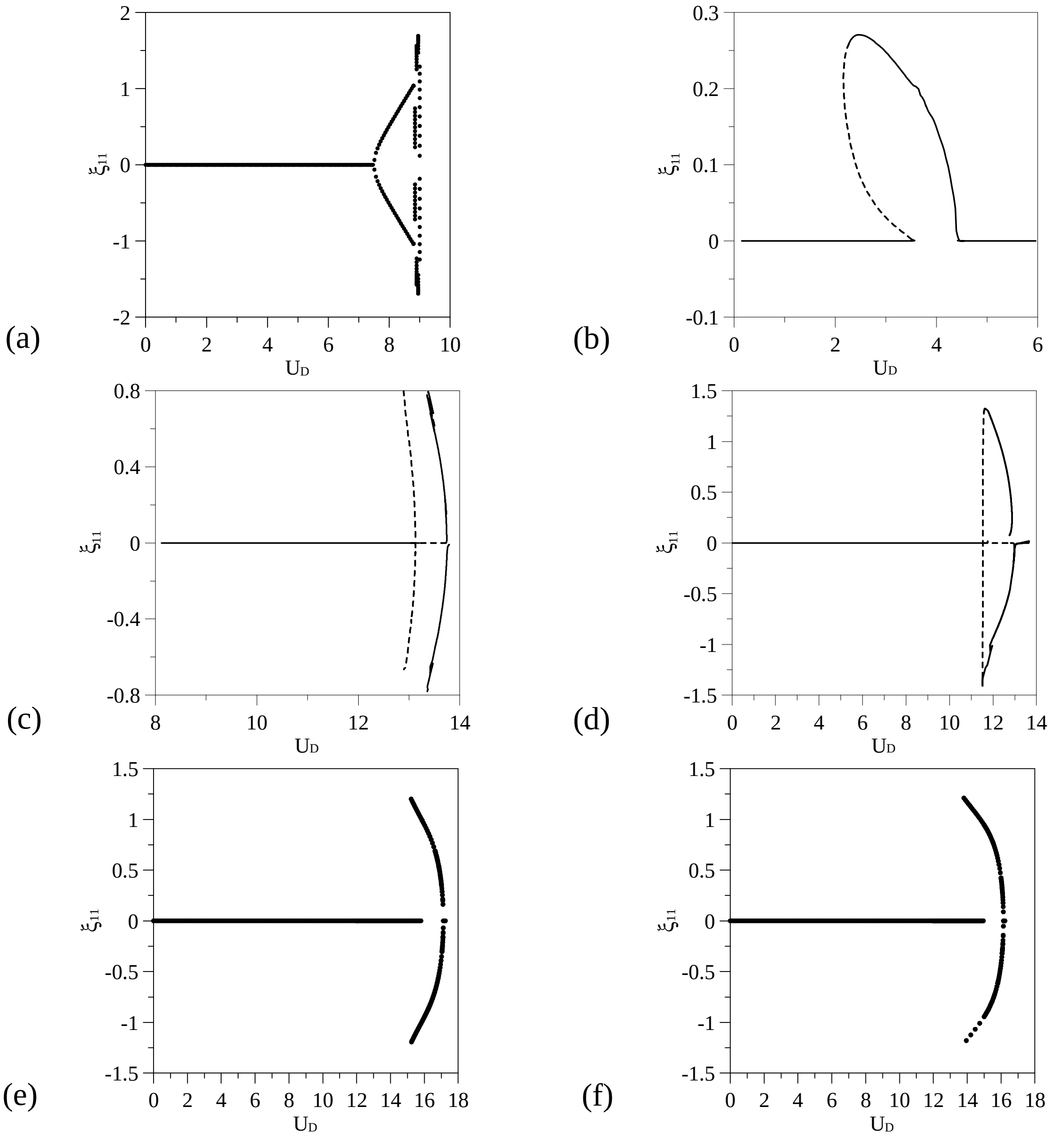
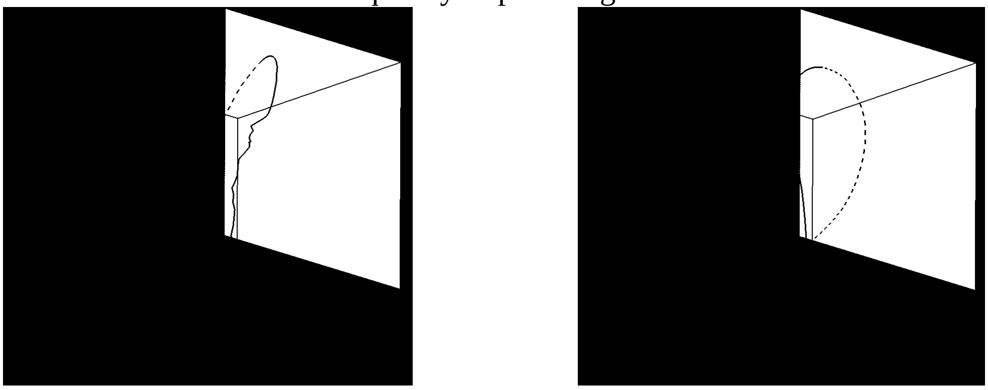
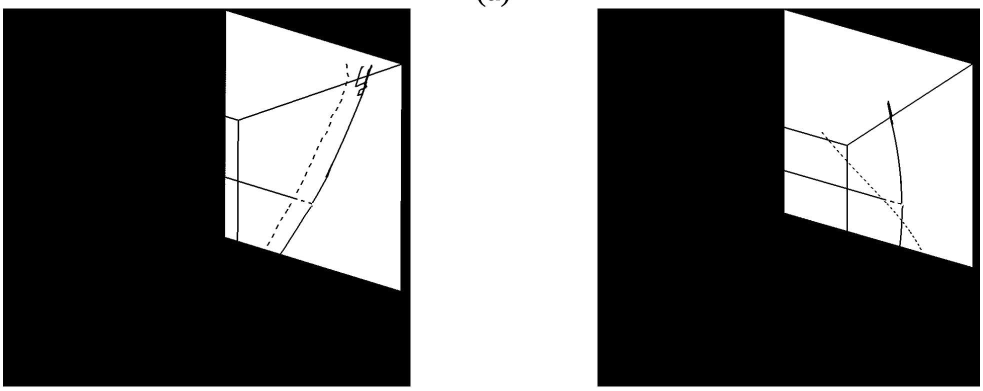



Figure 5 – Typical bifurcations diagrams. (a) *L/R* = 5: $U_E = 0.0$; $\Omega = 1.1$; (b) *L/R* = 5: $U_E = 0.0$; $\Omega = 1.3$, (c) *L/R* = 6: $U_E = 0.0$; $\Omega = 0.8$; (d) *L/R* = 6: $U_E = 0.0$; $\Omega = 1.3$; (e) *L/R* = 7: $U_E = 0.0$; $\Omega = 1.0$; (f) *L/R* = 7: $U_E = 0.0$; $\Omega = 1.3$. _________ stable points; - - - -unstable points.

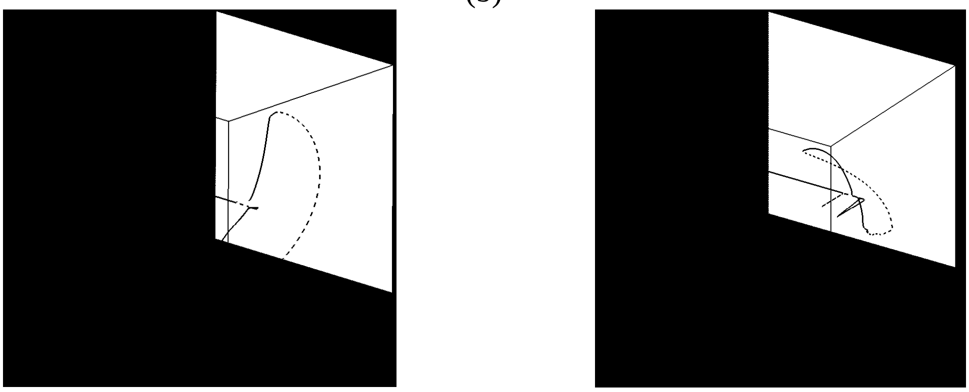
Figure 5 shows typical bifurcation diagrams associated with the instability regions due to the variation of the pulsating fluid flow, for different values of the excitation frequency and velocity of fluid. These bifurcation diagrams were obtained by using the brute-force method, together with continuation techniques by increasing the amplitude of flow velocity. In all diagrams for low values of flow velocity, the shell displays trivial solutions and, at a critical point the shell looses stability showing supercritical and subcritical bifurcations depending on both the value of *L/R* relation and frequency of pulsating fluid.



(a)



(b)



(c)

Figure 6 – 3D bifurcations diagrams. (a) *L/R* = 5: $U_E = 0.0$; $\Omega = 1.3$; (b) *L/R* = 6: $U_E = 0.0$; $\Omega = 0.8$, (c) *L/R* = 6: $U_E = 0.0$; $\Omega = 1.3$; ________ stable points; - - - -unstable points.

Finally, Figure 6 depicts the bifurcations diagrams in 3D space driven mode ($\xi_{11}$), companion mode ($\xi_{11c}$) and gyroscopic mode ($\xi_{11g}$) as a function of amplitude of amplitude of pulsating flow. Through these diagrams it is possible to have a whole vision of the non-linear behavior of the shell as well as the behavior of both companion and gyroscopic modes. As can be seen, depending on the geometric relation and on the frequency of the pulsating flow, the shell displays softening behavior and the companion and gyroscopic modes will have very different vibrations with co-existence of stable and unstable solutions.

## 4 CONCLUSIONS

- In this work the influence of geometry on the linear and non-linear vibrations of simply supported isotropic cylindrical shells subjected to internal harmonic pulsating fluid, too model the shell, the Donnell's non-linear shallow shell theory is used.
- Results show that the geometry relations *L/R* and *R/h* ratios and internal flow have strong influence on the critical flow velocity and natural frequencies.
- When a pulsating fluid flow is applied to the shell, the parametric instability boundaries are strongly affected with new sub-harmonic wells and the shell displays both stable and unstable paths.

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