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EVALUATION OF UNCERTAINTY INTRODUCED BY FOUNDATION VIBRATION OF PRECISION CENTRIFUGE

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Abstract. *In order to assess the influence of foundation vibration on precision centrifuge accuracy, the uncertainty of centripetal acceleration introduced by foundation vibration is analyzed and evaluated. First of all, the characteristics of foundation vibration is analyzed, and the result shows that the foundation vibration signal is composed of random and sine signal. Then, the relationship of centripetal acceleration and foundation vibration is analyzed based on kinematics. Further, the mathematical model of uncertainty is established, in which the random vibration and multiple sine vibrations are considered. The uncertainty evaluated based on theory analysis and measurement data is obtained, which indicates that the relative standard uncertainty gradually decreases with acceleration increasing, and the uncertainty introduced by random vibration plays a leading role in total uncertainty. At last, the uncertainty evaluated from two different measurement way is compared, which shows that the uncertainty evaluated with multiple measurement is significantly smaller than the result evaluated with single measurement.*

1 INTRODUCTION

The precision centrifuge in this presentation is used to calibrate linear accelerometer. With the improvement of accelerometer accuracy, the requirement for centrifuge accuracy is also increasing accordingly. The U.S. Central Inertial Guidance Laboratory has already owned several precision centrifuges [1], of which the relative standard uncertainty is about 2×10^{-6}.The precision centrifuge with the uncertainty better than 1×10^{-5} is also being developed in China. The amplitude of foundation vibration should be less than the uncertainty of acceleration at 1 g level, which is mandatory requirement in JJG1066-2011” Verification regulation of precision centrifuge” [2].So, the lower uncertainty, the smaller acceleration of foundation vibration. Literature [3] points out that the influence of foundation vibration to precision centrifuge accuracy should be considered carefully if the relative standard uncertainty is between 1×10^{-5} and 1×10^{-4}. It is necessary to study effects of foundation vibration on precision centrifuge, and to analyze the uncertainty introduced by foundation vibration further.

At present, the error analysis and uncertainty evaluation of precision centrifuge have been study in some literatures, and the main error source or uncertainty source are considered in these papers including angular velocity, static radius, dynamic radius, misalignment angle, and spindle rotation error etc.[4-6]. However, there is no literature to discuss the problem of uncertainty evaluation introduced by foundation vibration. And there is only a few literatures to analyze the influence of foundation vibration on precision centrifuge centripetal acceleration. The literature [1] pointed out that the error introduced by foundation vibration will be reduced significantly after the datas from several circles are averaged when foundation vibration is treated as random vibration. The effect of vibration on calibration result of micro-accelerometer is analyzed in paper[7] ,and a method to eliminate foundation vibration noise is proposed, but that is not suitable for precision centrifuge. The foundation vibration is measured and analyzed in literature[8],which indicate that the foundation vibration is composed of random signal and periodic signal, and the periodic vibration is introduced by the rotation of precision centrifuge and other machine nearby. As to random vibration and sine vibration,the relationship of centripetal acceleration and foundation vibration is analyzed in paper[9],which indicates that the error introduced by foundation vibration will be reduced partly after averaging by theory analysis and numerical simulation, but the error introduced by precision centrifuge rotation will not be reduced by averaging ,because the error introduced is system error.

The uncertainty introduced by foundation vibration is analyzed and evaluated in this paper, which is the further study based on literatures [8-9]. The results in this paper is maybe useful to uncertainty evaluation, error analysis and foundation design of precision centrifuge.

2 ANALYSIS OF CHARACTERISTICS OF FOUNDATION VIBRATION SIGNAL

2.1 Characteristics of foundation vibration signal

In order to analyze the uncertainty introduced by the foundation vibration, we need to understand the characteristics of the vibration at first. The main vibration source of precision centrifuge foundation are considered including the microtremor, precision centrifuge rotation and external machine rotation nearby such as air compressor, air conditioning, dehumidifiers, etc. And the foundation vibration caused by the microtremor is usually random vibration, but the vibration caused by precision centrifuge and external machine rotation is sine vibration generally [8]. The time domain curves and spectrum curves of foundation vibration acceleration measured when precision centrifuge rotated at speed of 300 r/min are shown in Figure 1.The upper two are time domain curves of tangential and radial acceleration, and the lower two are spectrum curves corresponding. From Figure 1, we can see that the foundation vibration signal is composed of random signal and multiple sine signals, and the one sine vibration with frequency 5Hz is caused by precision centrifuge rotation, another with frequency 15Hz is caused by air compressor nearby whose rotation speed is 900 r/min.

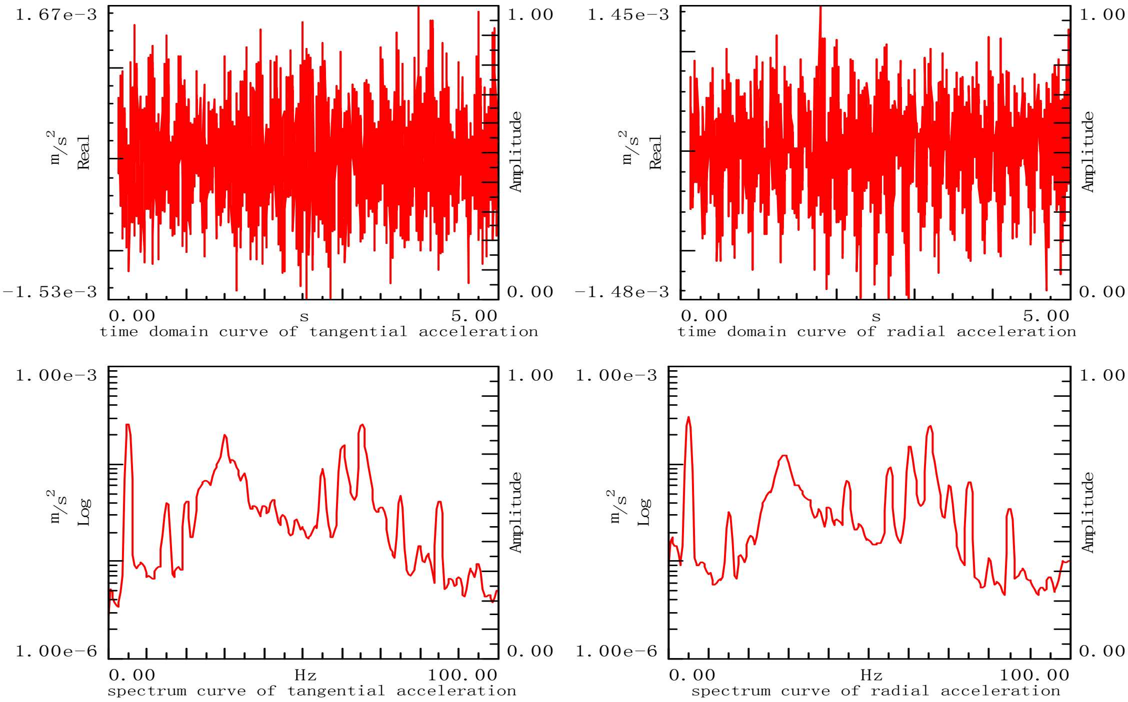

Figure 1: Time domain and spectrum curves of foundation vibration acceleration

2.2 Decomposition of foundation vibration signal

The purpose of signal decomposition is to obtain the root-mean-square (RMS) value of each component of vibration signal, which is required for uncertainty evaluation. According to Section 2.1, the foundation vibration acceleration signal can be decomposed into random signal and multiple sine signals. Assuming the acceleration signal $a(t)$ is composed of one random component $a_1(t)$ and one sine component $a_2(t)$, $a(t)$ can be expressed as $a(t) = a_1(t) + a_2(t)$. According to the definition, the RMS value of $a(t)$ is write as Eq. (1).

$$a(t)_{rms} = \sqrt{\frac{1}{T}\int_0^T (a_1(t) + a_2(t))^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T a_1(t)^2 dt + \frac{1}{T}\int_0^T a_2(t)^2 dt + \frac{1}{T}\int_0^T 2a_1(t) \cdot a_2(t) dt} \tag{1}$$

Because $a_1(t)$ and $a_2(t)$ are independent and irrelevant, and the both mathematical expectation is zero, Eq. (1) can be rewrite as

$$a(t)_{rms} = \sqrt{a_1(t)_{rms} + a_2(t)_{rms}} \tag{2}$$

The RMS value of each component can be obtained step by step as follows. First, the total RMS value of vibration is calculated with statistical method, then the RMS value of each sine signal component is get according to the acceleration spectrum. last, the RMS value of the random signal component can be obtained according to Eq. (2) .

For random signal with zero mathematical expectation, the RMS value and standard deviation is numerically equal, and the standard uncertainty component can be evaluated futher based on standard deviation. For sine signal, the confidence interval with probability of 100% can be determined based on RMS value, and then the standard uncertainty component can evaluated with the method as shown in GJB [10].

3 RELATIONSHIP OF CENTRIPETAL ACCELERATION AND RANDOM VIBRATION

The radial vibration at the position of the effective center of mass(ECM) [1] is also random vibration when the the foundation vibration is random. If only the influence of random vibration is considered based on kinematics, the centripetal acceleration at the position of the ECM can be expressed as Eq.(3).

$$a(t) = \omega_0^2 R_0 + a_b(t) \tag{3}$$

Where

$a(t)$ is the centripetal acceleration at the position of the ECM, m/s^2;

ω_0 is the angular velocity, rad/s;

R_0 is the effective radius, m;

$a_b(t)$ is foundation vibration acceleration along the turntable radial direction, m/s^2.

The $a_b(t)$ in Eq. (3) is also the component of centripetal acceleration caused by foundation random vibration, and the key point of uncertainty analysis is to obtain the standard deviation of the term $a_b(t)$ with statistical method.

4 RELATIONSHIP OF CENTRIPETAL ACCELERATION AND SINE IBRATION

The relationship of centripetal acceleration and foundation sine vibration along single direction has been established in paper [9], from which a good research base for uncertainty analysis is provided.The mathematical model of centripetal acceleration is proposed as below, in which the sine vibration along two orthogonal directions is considered.

Assuming the foundation vibrates in a horizontal plane, the movement of foundation in the earth coordinate system xoy can be expressed as Eq.(4).

$$\begin{cases} x_b(t) = d_b \cos(\omega_b t + \theta) \\ y_b(t) = d_b \sin(\omega_b t + \theta) \end{cases} \tag{4}$$

Where

ω_b is the angular velocity of foundation vibration, rad/s;

d_b is the displacement amplitude of foundation vibration, m;

θ is the initial phase angle of foundation vibration, rad.

The movement of the ECM in the coordinate system $x'o'y'$ (shown in Figure 2) can be described as Eq. (5).

$$\begin{cases} x'(t) = R_0 \cos(\omega_0 t + \alpha) \\ y'(t) = R_0 \sin(\omega_0 t + \alpha) \end{cases} \tag{5}$$

Where

R_0 is the effective radius,m;

ω_0 is the angular velocity of centrifuge rotation,rad/s;

α is the initial phase angle of centrifuge rotation,rad.

x'

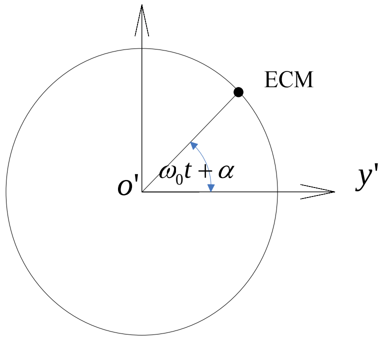

Figure 2: Diagram of centrifuge rotation

So, the movement of the ECM in the earth coordinate system xoy can be expressed as Eq.(6) according to Eqs.(4-5).

$$\begin{cases} x(t)=d_b\cos(\omega_b t+\theta)+R_0\cos(\omega_0 t+\alpha) \\ y(t)=d_b\sin(\omega_b t+\theta)+R_0\sin(\omega_0 t+\alpha) \end{cases} \tag{6}$$

The centripetal acceleration of the ECM is

$$a(t)=\ddot{x}(t)\cos(\omega_0 t+\alpha)+\ddot{y}(t)\sin(\omega_0 t+\alpha) \tag{7}$$

Substituting Eq. (6) into Eq. (7), we have

$$a(t)=\omega_0^2 R_0+\omega_b^2 d_b\cos\left[(\omega_b-\omega_0)t+\theta-\alpha\right] \tag{8}$$

As shown in Eq. (8), the centripetal acceleration is composed of two items, the first is the constant acceleration introduced by centrifuge rotation, and the second is the variable acceleration introduced by foundation vibration. The key point of uncertainty analysis is to obtain the standard deviation of the second term with statistical method. The foundation vibration signal often contains multiple sine signals, each one should be considered in the uncertainty evaluation.

5 EVALUATION OF UNCERTAINTY INTRODUCED BY FOUNDATION VIBRATION

5.1 Mathematical model of uncertainty

The uncertainty analysis is a rigorous mathematical process, the most importance is to establish an appropriate mathematical model of uncertainty according to measurement method. The uncertainty model is proposed as Eq. (9) based on the analysis in Section 3 and Section 4 above, in which both random vibration and sine vibration are consiered.

$$\begin{aligned} a(t)&=a_1(t)+a_2(t)+a_3(t) \\ a_2(t)&=A_{s1}\cos(\theta-\alpha) \\ a_3(t)&=A_{s2}\cos\left[(\omega_{s2}-\omega_0)t+\beta-\alpha\right] \end{aligned} \tag{9}$$

Where

$a(t)$ is the centripetal acceleration introduced by foundation vibration, m/s^2;

$a_1(t)$ is the component of centripetal acceleration introduced by random vibration, m/s^2;

$a_2(t)$ is the component of centripetal acceleration introduced by centrifuge rotation, m/s^2;

$a_3(t)$ is the component of centripetal acceleration introduced by external machine rotation, m/s^2 ;

A_{s1} is the acceleration amplitude of foundation sine vibration caused by centrifuge rotation, m/s^2;

θ is the initial phase angle of foundation sine vibration caused by centrifuge rotation, rad;

α is the initial phase angle of centrifuge rotation rad;

A_{s2} is the acceleration amplitude of foundation sine vibration caused by external machine rotation,m/s^2;

ω_{s2} is the angular velocity of external machine rotation, rad/s;

ω_0 is the angular velocity of centrifuge rotation, rad/s;

β is the initial phase angle of foundation sine vibration caused by external machine rotation, rad;

5.2 Uncertainty evaluation

Because $a_1(t)$, $a_2(t)$ and $a_3(t)$ are independent and irrelevant, standard uncertainty introduced by foundation vibration can be write as[10]

$$u_c = \sqrt{u^2{}_{a1} + u^2{}_{a2} + u^2{}_{a3}} \tag{10}$$

Where

u_c is the standard uncertainty introduced by foundation vibration,m/s^2;

u_{a1} is the component of standard uncertainty introduced by random vibration,m/s^2;

u_{a2} is the component of standard uncertainty introduced by centrifuge rotation,m/s^2;

u_{a3} is the component of standard uncertainty introduced by external machine rotation,m/s^2.

And the relative standard uncertainty is expressed as

$$u_r = u_c / a_0 \tag{11}$$

Where

u_r is the relative standard uncertainty introduced by foundation vibration,m/s^2;

a_0 is the nominal centripetal acceleration,m/s^2.

The standard uncertainty component, standard uncertainty and relative standard uncertainty evaluated at different centripetal acceleration are shown in Table 1, which indicates that the relative standard uncertainty gradually decreases with acceleration increasing, and the uncertainty introduced by random vibration plays a leading role in total uncertainty.

a_0 m/s^2	u_{a1} $\times 10^{-5}$m/s^2	u_{a2} $\times 10^{-5}$m/s^2	u_{a3} $\times 10^{-5}$m/s^2	u_c $\times 10^{-5}$m/s^2	u_r $\times 10^{-6}$
9.9	5.72	0.07	2.10	6.09	6.2
39.4	7.21	0.17	2.09	7.51	1.9
88.7	8.62	0.45	2.20	8.91	1.0
157.8	17.05	0.99	2.13	17.21	1.1
354.9	26.08	3.26	2.15	26.37	0.7
631.0	34.08	8.79	2.20	35.27	0.6
986.0	39.21	25.11	2.23	46.61	0.5

Table 1: Uncertainty evaluation at different centripetal acceleration

5.3 Uncertainty evaluation in multiple measurement

In fact, we always using the average centripetal acceleration obtained by multiple measurement to calibrate linear accelerometer, so it is necessary to evaluate the uncertainty based on multiple measurement. The mathematical model of uncertainty can be expressed as

$$\bar{a} = \bar{a}_1 + \bar{a}_2 + \bar{a}_3 = \sum_{k=1}^{n} a_1(t_k)/n + A_{s1}\cos(\theta - \alpha) + \sum_{k=1}^{n} a_3(t_k)/n \tag{12}$$

According to the uncertainty evaluation methods [10], the relationship of uncertainty obtained by single measurement and multiple measurement is expressed as $u_{\bar{a}1} = u_{a1}/\sqrt{n}$, $u_{\bar{a}2} = u_{a2}$, $u_{\bar{a}3} = u_{a3}/\sqrt{n}$. The results of uncertainty evaluation by the same methods in Section 5.2 is list in Table 2 (n=10).

a_0 m/s^2	$u_{\bar{a}1}$ $\times 10^{-5}$m/s^2	$u_{\bar{a}2}$ $\times 10^{-5}$m/s^2	$u_{\bar{a}3}$ $\times 10^{-5}$m/s^2	$u_{c'}$ $\times 10^{-5}$m/s^2	$u_{r'}$ $\times 10^{-6}$
9.9	1.81	0.07	0.66	1.93	2.0
39.4	2.28	0.17	0.66	2.38	0.6
88.7	2.73	0.45	0.70	2.85	0.3
157.8	5.39	0.99	0.67	5.52	0.4
354.9	8.25	3.26	0.68	8.89	0.3
631.0	10.78	8.79	0.70	13.92	0.2
986.0	12.40	25.11	0.71	28.01	0.3

Table 2: Uncertainty evaluation at different centripetal acceleration (multiple measurement)

As can be seen from Table 2, the relative standard uncertainty evaluated with multiple measurement is significantly smaller than that evaluated with single measurement. The uncertainty components introduced by random vibration and external machine rotation will decrease after averaging, but that introduced by centrifuge rotation will not be, because the error caused by centrifuge rotation is system error.

6 CONCLUSION

The mathematical model of the uncertainty introduced by foundation vibration is established considering the characteristics of vibration, and the uncertainty is evaluated based on theory analysis and measurement data. The conclusions are list as below.

- Foundation vibration signal is composed of random signal and sine signal.
- The relative standard uncertainty gradually decreases with acceleration increasing.
- Uncertainty introduced by random vibration plays a leading role in total uncertainty.
- The uncertainty evaluated with multiple measurement is significantly smaller than the result evaluated with single measurement.

REFERENCES

[1] IEEE recommended practice for precision centrifuge testing of linear accelerometers. IEEE Std 836-2009,2009.

[2] Verification Regulation of Precision Centrifuge. JJG 1066-2011,2011.

[3] LONG Zu-hong, XIONG Lei, XING Xin-ting. LXJ-70 precision centrifuge Aviation Metrology and Measurement Technology, 2001,21(5):25-29.

[4] XIONG Lei, HE Yi-cai, LONG Zu-hong. Uncertainty Analysis and Application of Precise Centrifuge. Aviation Metrology and Measurement Technology, 2003,23(6):36-37.

[5] Yang Yafei. The influence of accelerometer's fixing attitude errors and main-axis' attitude errors on accelerometer's test on the precision centrifuge. Journal of Astronautic Metrology and Measurement, 2002,22(4):31-35.

[6] REN Shun-qing, CHEN Yan, ZHAO Zhen-hao. Influence of rotary errors of centrifuge spindle on accelerometer input accuracy. Journal of Chinese Inertial Technology, 2007,15(1):116-119.

[7] ZHAO Jun-zhe, YANG Guo-jun, MENG Fang. Foundation Vibration Influences and Makes Micro Accelerometer Static Mathematics Model to Distinguish. Journal of Metrology and Measurement, 2006,26(2):21-23.

[8] Feng Li, Qiang Du, Jia-quan Feng. Measurement and analysis of microtremor on construction of precision centrifuge. Journal of Metrology and Measurement, 2012,26(2):21-23.

[9] Feng Li, Qiang Du, Jia-quan Feng. Influence of foundation vibration on average centripetal acceleration of precision centrifuge.Sound and vibration control, 2013,33(4) (In press).

[10] Expression and evaluation of uncertainty in measurement. GJB3756-99,1999.