

MODELLING PEDESTRIAN WALKING LOCOMOTION OVER LIVELY FOOTBRIDGE DECKS

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Abstract. *During walking humans experience side to side, up and down and forward movements characterised with alternating acceleration and deceleration of their body. As a result, people induce a three dimensional dynamic force into the supporting structure, with strongest component of the force acting in the vertical direction. To evaluate the vibration serviceability of footbridge structures against the vertical vibration, most structural engineers use pedestrian force models that are defined for walking on rigid surfaces and that consider the pedestrian as an entity independent from the structural vibration. This approach is no longer applicable for slender, light-weight and low-frequency structures that are prone to perceptible vibrations under walking excitation. To improve the models of pedestrian loading, it is necessary to understand the pedestrian walking locomotion as well as the ways how the locomotion process interacts with the vibrating structure.*

This paper compares three approaches for modelling pedestrian locomotion over lively structures, and it critically evaluates their suitability for modelling the feedback mechanism between the structure and the pedestrian. The models are evaluated with respect to their capability to reproduce human-like motion as well as to replicate the vibration patterns observed on lively bridges. It has been shown that models used in biomechanics are good candidates for applications in the structural engineering context.

1 INTRODUCTION

During walking humans experience side to side, up and down and forward acceleration and decelerations of their body. A simple way to represent the human movement is to observe motion of the body centre of mass (BCoM) in three-dimensional space. While walking, the BCoM is usually located within the pelvis, and it moves in oscillatory fashion a few centimetres in both the vertical and horizontal-lateral directions as shown in Figure 1 [1]. As a result of this movement, people induce a three-dimensional force into the supporting structure, with the strongest component of the force acting in the vertical direction.

An example of the vertical ground reaction force (GRF) generated by a person during different phases of walking [2] as well as the corresponding trajectory of the BCoM in the sagittal plane are shown in Figure 2. The two feet often generate slightly different forces due to slight asymmetry in the human body. As a result, the GRF pattern is near-periodic, resulting in some dissipation of the energy around main harmonics [3].

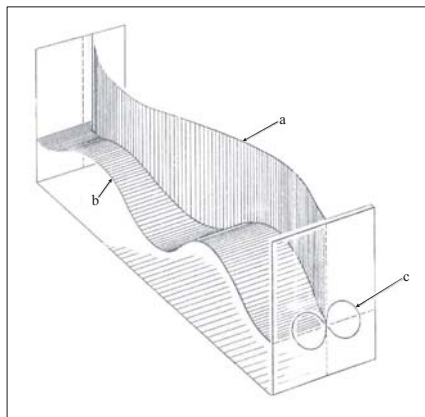


Figure 1: Displacement of BCoM during two walking steps (not to scale). a) Lateral displacement, b) vertical displacement, and c) combined displacements of a and b projected onto a plane perpendicular to the plane of progression (after [1]).

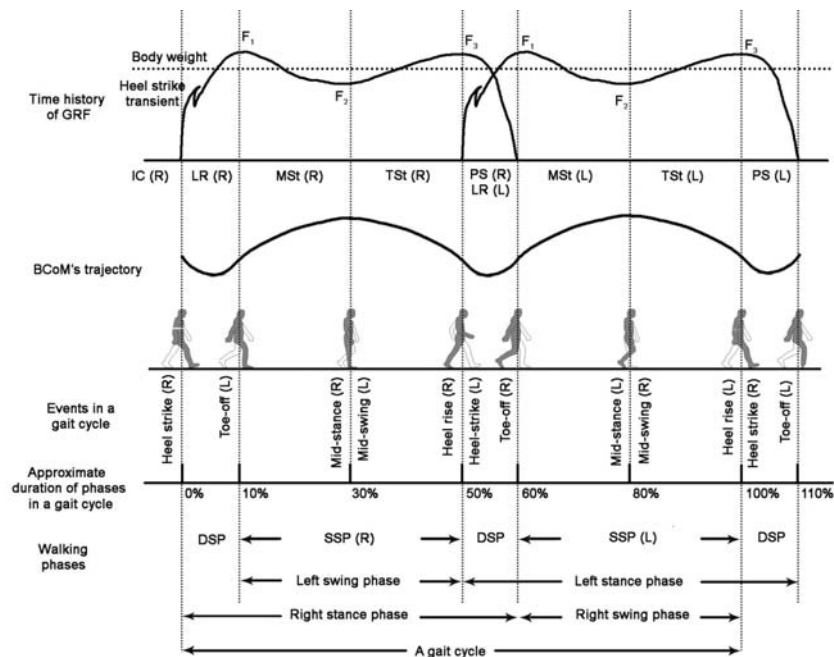


Figure 2: Vertical GRF and BCoM's trajectory during walking (partially recreated from [2]).

Vibration serviceability of modern footbridges under pedestrian traffic is usually of concern if the structure has one of the vibration modes in the frequency range typical of normal human pacing rate, i.e. 1.5-2.4 Hz [4, 5]. Since a single mode is normally of interest, the bridge can be modelled as a single-degree-of-freedom (SDOF) system. Walking at a pacing frequency close to the frequency of the structure increases the incidence of resonance caused by the first harmonic of the walking force, and therefore it can result in development of high-level vibration response. To predict the structural vibration levels reliably, it is necessary to model the first harmonic of the walking force accurately. Many models of the vertical GRF exist in the literature, starting from the design guidelines that account for a dominant frequency line in the spectrum only [6-8] to recently developed models that account for dissipation of the energy [3, 9, 10]. However, all these models assume that the walking takes place on a “rigid” (i.e. imperceptibly vibrating) surface. As a result, these models cannot predict vibration response of slender, perceptibly vibrating, structures on which the pedestrian-structure dynamic interaction takes place [11, 12]. This traditional approach of modelling the pedestrian-induced force independently from the structural vibration is no longer applicable for slender, light-weight and low-frequency structures that are prone to perceptible vibrations under walking excitation. To improve the models for vibration serviceability evaluation of footbridge structures it is necessary to understand the pedestrian walking locomotion as well as the ways how the locomotion process interacts with the vibrating structure.

Several ideas to account for pedestrian structure interaction emerged in last few years motivated by developments in modelling human locomotion in biomechanics, as well as modelling of human-structure interaction for passive (i.e. standing or sitting) humans [13, 14]. These models, however, have to be developed further before being ready for use in the design practice. Critical evaluation of these models for use in the civil engineering context is of particular interest, as well as clear definition of the parameter space relevant for structural engineering applications.

This paper presents key features of the two interaction models and compares their performance against a traditional non-interactive model representing walking on a rigid surface. The three models are critically evaluated with respect to their capability to reproduce kinematic and kinetic features of human walking as well as to replicate the vibration patterns observed on a lively bridge. Only walking at a pacing rate that matches the natural frequency of the structure is investigated, since this situation causes development of strong vibrations which are prerequisite for development of the human-structure interaction.

The paper starts with a description of a lively footbridge and experimental data acquired during single person walking tests. Then the three pedestrian models are described, before proceeding to simulating the structural response and comparing the performance of the models. At the end, conclusions are presented.

2 STRUCTURAL DESCRIPTION AND WALKING TESTS

A light cable-stayed bridge made of fibre reinforced polymer is used to perform simulations in this study. The length of the bridge is 113m, with the main span being 63m long (Figure 3). The bridge weighs 20,000 kg. The fundamental vibration mode at 1.52Hz is extremely responsive when walking at a pacing rate close to the natural frequency. The measured modal mass of the structure is 2,750 kg while the modal damping ratio is 0.42% [15].

The measured modal responses to a pedestrian walking to match a resonance in two tests are shown in Figure 4. The larger response (grey line) was achieved when a pedestrian, with body mass of 86kg and 1.93m tall, walked with help of a metronome. The estimated initial dynamic loading factor (i.e. amplitude of the first forcing harmonic normalised by pedestrian’s weight) and average walking speed in this test are 0.14 and 1.43m/s, respectively [15].

The black line represents the response to walking without help of a metronome by the same pedestrian. The pedestrian parameters in this case were very similar to the previous test, and the difference in the measured response is the consequence of intra-subject variability, i.e. difficulties in repeating the same walking pattern in the two tests by any particular person.

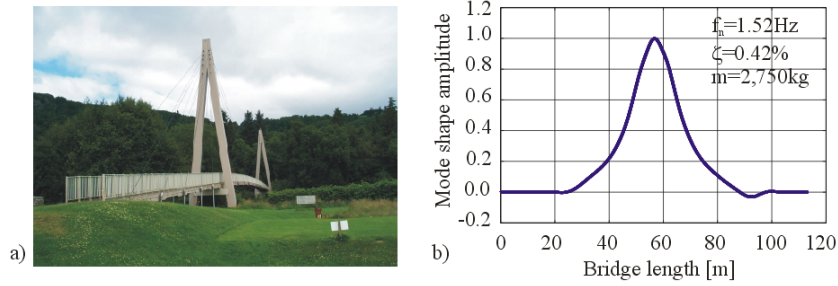


Figure 3: (a) A photo of the cable-stayed footbridge, and (b) fundamental vibration mode.

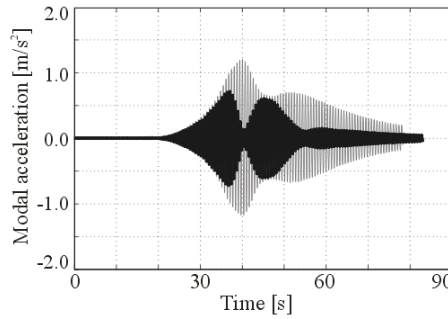


Figure 4: Measured modal responses to a pedestrian walking to match resonance with help of metronome (grey line) and without metronome (black line).

3 PEDESTRIAN-STRUCTURE MODELS

This section describes main features of the three pedestrian-structure models. The models are named according to the strategy used for modelling a pedestrian: 1) moving harmonic force model (MHFM), 2) moving oscillator and actuator model (MOAM), and 3) inverted pendulum model (IPM).

3.1 Moving harmonic force model

The MHFM is a model on which traditional vibration serviceability assessment is based. Pedestrian is represented as a harmonic force crossing the bridge at a constant speed. The force is independent on the vibration of the bridge. The modal force $F(t)$ is obtained after weighting the harmonic force $F_h(t)$ by a mode shape of interest $\phi(x) = \phi(vt)$:

$$F(t) = F_h(t)\phi(vt) = (DLF \cdot W) \sin(2\pi f_p t) \cdot \phi(vt) \quad (1)$$

where W is the pedestrian weight, DLF is the dynamic loading factor, f_p is the pacing frequency, x is the pedestrian's position along the bridge, v is the pedestrian's speed and t is the time elapsed since the pedestrian entered the bridge. To calculate the modal vibration response of the structure at the midspan, the well-known equation of motion has to be solved:

$$m\ddot{y}(t) + 2\zeta m\omega_n \dot{y}(t) + m\omega_n^2 y(t) = F(t) \quad (2)$$

where m , ω_n and ζ are the modal mass, natural frequency and damping ratio of the vibration mode of interest, while $\ddot{y}(t)$, $\dot{y}(t)$ and $y(t)$ are the modal acceleration, velocity and displacement of the structure, respectively.

3.2 Moving oscillator-actuator model

The MOAM represents a pedestrian's body as a dynamic entity (i.e. mass-spring-damper oscillator) crossing the bridge at a constant speed. As a consequence the pedestrian-structure system is represented by a 2DOF system which dynamic properties are changing slightly as the pedestrian is crossing the bridge. To account for the muscle action of the pedestrian, an actuator is introduced. The actuator generates harmonic force acting on both pedestrian and structural DOFs [14]. The system of two differential equations coupling the pedestrian and structural vibration can be written as:

$$\begin{aligned} & \begin{bmatrix} m & 0 \\ 0 & m_p \end{bmatrix} \begin{Bmatrix} \ddot{y}(t) \\ \ddot{y}_p(t) \end{Bmatrix} + \begin{bmatrix} 2\zeta m \omega_n + 2\zeta_p m_p \omega_{np} \Phi^2(x) & -2\zeta_p m_p \omega_{np} \Phi(x) \\ -2\zeta_p m_p \omega_{np} \Phi(x) & 2\zeta_p m_p \omega_{np} \end{bmatrix} \begin{Bmatrix} \dot{y}(t) \\ \dot{y}_p(t) \end{Bmatrix} + \\ & + \begin{bmatrix} m \omega_n^2 + m_p \omega_{np}^2 \Phi^2(x) & -m_p \omega_{np}^2 \Phi(x) \\ -m_p \omega_{np}^2 \Phi(x) & m_p \omega_{np}^2 \end{bmatrix} \begin{Bmatrix} y(t) \\ y_p(t) \end{Bmatrix} = \begin{Bmatrix} F(t) \\ -F_h(t) \end{Bmatrix} \end{aligned} \quad (4)$$

where m_p , ω_{np} and ζ_p are the mass, natural frequency and damping ratio of the pedestrian, while $\ddot{y}_p(t)$, $\dot{y}_p(t)$ and $y_p(t)$ are the acceleration, velocity and displacement of the pedestrian's centre of mass, respectively. The natural frequency in radians/s ω_{np} can be transferred into the natural frequency f_{np} expressed in Hz using the well-known relationship $\omega_{np} = 2\pi f_{np}$. The remaining variables in the equation have been introduced in the previous equations.

Coupling between the two DOFs in this equation is a means of accounting for interaction between the pedestrian and structural vibration. A particular challenge when using this model is the choice of the pedestrian parameters; these parameters are yet to be calibrated against the experimental data. In the absence of more definitive guidance, some data from the literature related to bouncing activity will be used in the context of this paper.

3.3 Inverted pendulum model

The IPM is the simplest bipedal walking model of a pedestrian. The model consists of a point mass at the top of two rigid and massless legs [16]. The point mass m_p represents the mass of the pedestrian, while l is the distance between the BCoM and the foot (Figure 5). The model is set in motion by specifying the angle of attack θ_0 (Figure 5) and the initial forward speed of the point mass. After a step is completed, an upwards impulse has to be applied to prevent the mass of falling [13]. To simplify the modelling, the initial conditions are reset at the beginning of a new step.

The system of differential equations describing the structural and pedestrian movement can be derived using Lagrangian approach:

$$\ddot{\theta}(t) = \frac{\cos \theta(t)}{l} (g - \phi(x) \ddot{y}(t)) \quad (5)$$

$$(m + m_p \phi^2(x) \sin^2 \theta(t)) \ddot{y}(t) + 2\zeta m (2\pi f_n) \dot{y}(t) + m (2\pi f_n)^2 y(t) = \phi(x) m_p (g \sin^2 \theta(t) + l \dot{\theta}^2(t) \sin \theta(t)) \quad (6)$$

where g is the acceleration of gravity, $\theta, \dot{\theta}$ and $\ddot{\theta}$ are the leg angle measured from the horizontal reference, angular speed and acceleration, respectively, while all other parameters have been described previously. Differently from MOAM where the interaction is accounted for by modelling the pedestrian body dynamics, the IPM accounts for interaction through timing of the step that is influenced by the structural vibration.

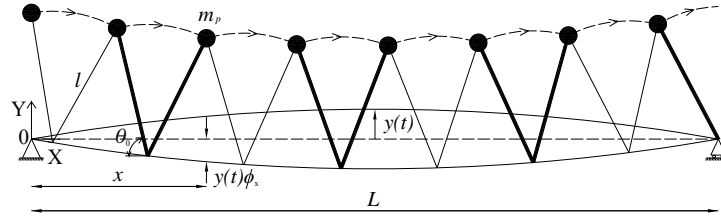


Figure 5: Inverted pendulum model walking across a structure.

4 SIMULATIONS ON LIVELY FOOTBRIDGE

This section compares the force and trajectory of BCoM as generated by different models while walking over the bridge shown in Figure 3. In addition, the response of the structure calculated in each model is compared with the experimental values (Figure 4). Before performing simulations, the choice of parameters to be used in each model is first explained.

4.1 Choice of input parameters

The input parameters for modelling the bridge DOF are based on experimentally acquired properties of the first vibration mode. These are natural frequency of 1.52Hz, damping ratio of 0.42%, modal mass of 2,750kg and mode shape represented in Figure 3b.

The input parameters related to pedestrian in the MHFM are also straight forward, since they are based on the experimental data explained in Section 2: $DLF=0.14$, $W=86\text{kg}$, pedestrian's average speed $v=1.43\text{m/s}$, and pacing frequency $f_p=1.52\text{Hz}$.

For the MOAM, two choices of the oscillator properties and the dynamic loading factor of the actuator force DLF_a are made. First choice is based on studies of dynamic properties of bouncing people [17]. The pedestrian natural frequency equal to that of an average bouncing person ($f_{np}=2.3\text{Hz}$) is chosen, while damping and DLF_a identified for bouncing people (approximately 25% and 0.3, respectively) are reduced three times (to 8% and 0.1, respectively) to account for the fact that the walking activity results in shorter contact time and less severe dynamic loading compared with the bouncing. Given that the natural frequency of the pedestrian is likely to have strong influence on the results, similarly to the effects of tuned mass dampers, second choice of parameters is made so that the pedestrian frequency matches the structural frequency (i.e. $f_{np}=1.52\text{Hz}$) while the damping ratio and DLF_a all kept as before.

The two sets of pedestrian parameters will be referred to as $MOAM_1$ and $MOAM_2$.

Finally, for implementation of the IPM the leg length is chosen as $l=1.216\text{m}$ (this corresponds to the pedestrian height of 1.93m). The initial conditions for the pedestrian (attack angle θ_0 and initial forward speed) are chosen so to produce best match of pedestrian parameters (pacing rate, DLF and walking speed) seen in the experiment on the bridge. These three parameters are mutually dependent, so it is not possible to match all three at the same time. The priority is given to the pacing rate, since the structural response is known to be very sensitive to this parameter. Several combinations of initial conditions are possible to choose to match the pacing rate. Only two combinations are chosen for further presentation: 1) the com-

bination that produces the closest match of the actual DLF ($DLF=0.147$ and $v = 0.956$ m/s, achieved for $\theta_0=75^\circ$ and initial forward speed =1.19m/s) and 2) the combination that produces the closest match of the actual pedestrian speed ($DLF=0.341$ and $v = 1.438$ m/s, achieved for $\theta_0=67.1^\circ$ and initial forward speed=1.73m/s). These will be referred to as IPM_1 and IPM_2 .

4.2 Motion of pedestrian's BCoM

MHFM does not provide insight into pedestrian kinematics. Therefore, displacement of pedestrian's BCoM can be compared between MOAM and IPM only. Figure 6a shows the BCoM trajectory for pedestrian having natural frequency either 2.30Hz (black line) or 1.52Hz (grey line). Peak to peak displacement of BCoM (i.e. vertical excursion of BCoM) when walking on rigid surface is usually around 20-60mm, with the excursion increasing with increase in walking speed [18]. For $MOAM_1$ simulation the BCoM movement has been reduced to 16mm peak to peak, while for $MOAM_2$ it has been amplified to 134mm peak to peak. In the latter case, when the pedestrian is exposed to strong vibration in the midspan the interaction between 2DOFs develops resulting in a dramatic reduction of pedestrian movement (see the time history between 30 and 60s in Figure 6a). Variations in case of $MOAM_1$ are much less dramatic. In both cases, however, the BCoM excursion seems to be outside the range recorded on rigid surfaces, even when pedestrian does not perceive strong vibrations, such as when crossing the initial and the final third of the bridge where mode shape amplitude is small (Figure 3b). This suggests that the MOAM is unlikely to genuinely represent the BCoM movement. Figure 6b shows the results for IPM. The black line represents the BCoM elevation during crossing the bridge at slow speed (IPM_1). In this case peak to peak movement of BCoM is mainly about 40mm, increasing to 70mm at the midspan when the higher vibrations cause the pedestrian to change their walking style. If the pedestrian is walking faster (IPM_2) the BCOM initially exhibits movement of 96mm peak to peak, and it increases to 145mm at the midspan. The movement in IPM_1 seems to be more realistic than in IPM_2 .

While the trajectory of BCoM in MOAM is a smooth harmonic function, in the IPM case the trajectory is arch-like (see insert in Figure 6b showing 3s of the time history for IPM_1). The BCoM trajectory shown in Figure 2 has been recorded on rigid surfaces, and this shape is also expected to be seen for lower levels of vibrations on bridges. It seems that none of the two models is perfect represent of the actual trajectory, albeit it can be said that the IPM represents well the single support phase. This is expected given that the IPM is built on the assumption that the double support phase is ignored.

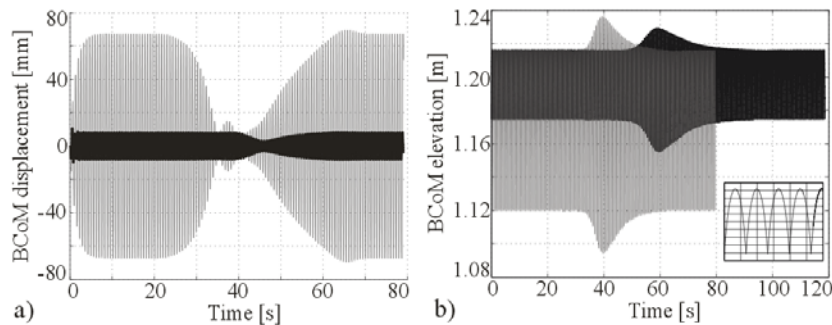


Figure 6: BCoM displacement in (a) $MOAM_1$ (black) and $MOAM_2$ (grey), and (b) IPM_1 (black) and IPM_2 (grey).

4.3 Pedestrian-induced force

Pedestrian force induced by MHFM is a harmonic force with amplitude of 118N. By the nature of MOAM, the force applied to the bridge is equal to the product between the mass and

the acceleration of pedestrian's DOF. Therefore, the time history of the force resembles that of BCoM trajectory, as it can be seen in Figure 7a. In both cases (MOAM₁ and MOAM₂) the force is dominated by frequency line at 1.52Hz (Figure 7b).

The forces generated by the IPM₁ and IPM₂ are shown in Figure 8a. It is interesting that in both cases the spectra contain an additional component at a frequency of 1.49Hz. This occurs due to interaction of the bipedal model with the high vibration levels of the structure, causing the IPM to reduce the pacing rate from the initial 1.52Hz to 1.49Hz at these vibration levels. The change in the initial pacing rate due to pedestrian exposure to the high vibration levels is a possible consequence of the interaction [15]. The IPM model seems to have potential to capture this phenomenon. However, it is also important to notice that this model results in unrealistically high higher harmonics (the second harmonic is shown in Figure 7b only), and therefore the model should not be used for structures which have lively modes which are excitable by higher harmonics.

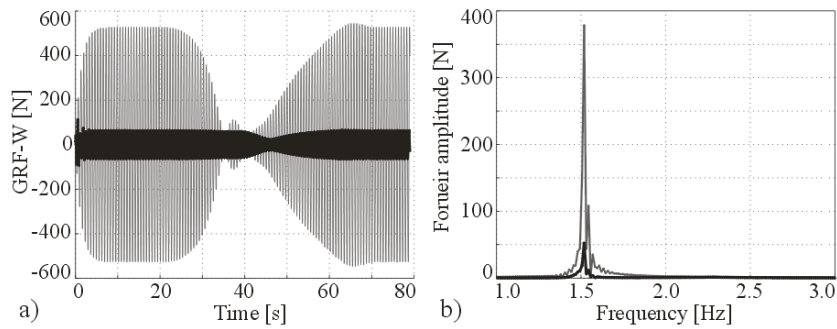


Figure 7: GRF in MOAM₁ (black) and MOAM₂ (grey) in (a) time domain and (b) frequency domain.

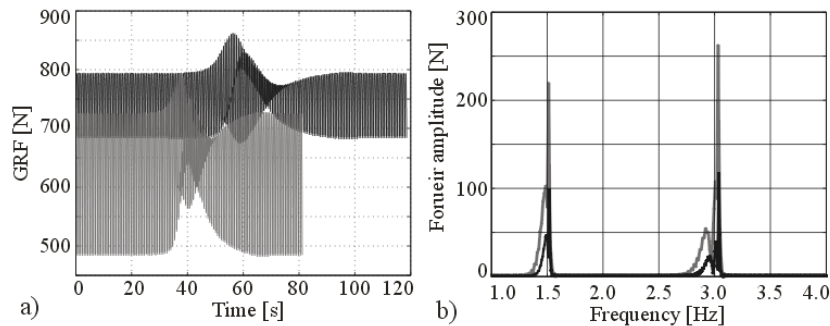


Figure 8: GRF in IPM₁ (black) and IPM₂ (grey) in (a) time domain and (b) frequency domain. Impulses are omitted from Figure 8a for clarity.

4.4 Structural response

Traditional pedestrian modelling in form of MHFM generates a peak vibration of 2m/s^2 on the structure investigated. This response is presented as an envelope in Figure 9 for comparison purposes. Therefore, use of MHFM substantially overestimates the two experimentally recorded responses having peak values of 1.17m/s^2 and 0.73m/s^2 (Figure 4). As for MOAM, it is expected that the dynamics of the pedestrian is an important factor that determines the extent of the interaction. This can be seen in Figure 9a in which two sets of parameters used in the model produce quite different peak responses of 0.70m/s^2 and 1.72m/s^2 . Similarly, the response to IPM is sensitive to the input parameters, with two simulations resulting in peak responses of 1.44m/s^2 and 2.07m/s^2 (Figure 9b). Although both IPM simulations overestimate the measured responses, slight variation of the input parameters can produce a range of possi-

ble solutions. For example, if the target pacing rate is set to $1.52\text{Hz} \pm 0.03\text{Hz}$, and all combinations of initial conditions that generate DLF within $\pm 5\%$ of the target value of 0.14 are considered, the peak response would range between 0.7m/s^2 and 2.7m/s^2 .

These results show that the two models have potential to represent a range of pedestrians under a range of different conditions. However, before being able to do modelling with confidence, it is necessary to calibrate the models against an extensive set of experimental data. Unfortunately, this data set is currently not available and should be subject of future research.

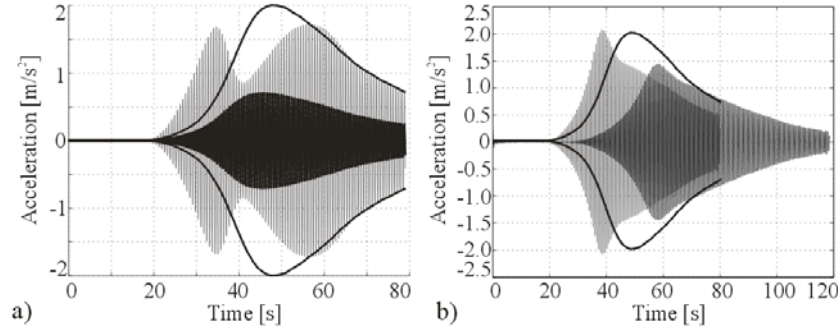


Figure 9: Acceleration response using (a) MOAM₁ (black) and MOAM₂ (grey) and (b) IPM₁ (black, 120s) and IPM₂ (grey, 80s). The envelopes represent the response to MHFM.

5 CONCLUSIONS

Main conclusions of this study can be listed as follows:

- Traditional MHFM cannot account for human-structure interaction. To estimate vibration response of slender lively structures reliably, models that account for human-structure interaction should be developed.
- Models based on both human body dynamics (MOAM) and human stepping behaviour (IPM) have potential for accounting for pedestrian-structure interaction.
- IPM replicates the kinematics of BCoM movement more closely than MOAM.
- Interaction in the IPM leads to the change in the pacing frequency, which is believed to be one of ways in which a pedestrian interact with structural vibration.
- The responses using both IPM and MOAM are sensitive to a number of input parameters. It is necessary to calibrate these models against experimental data before widely accepted interaction models can be developed.

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