NUMERICAL INVESTIGATION OF PIEZOELECTRIC ELEMENT COUPLING DEGRADATION IN ACTIVE BEAM SYSTEMS

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Abstract. *The objective of the present study is to develop modelling of the piezoelectric element edge delamination process and analysis of effects of the progressive adhesive interlayer damage on active beam dynamic behaviour. In the considered system piezoelectric patches are mounted to both opposite sides of the beam and operate as a collocated sensor/actuator pair with velocity feedback. Herein, the actuator coupling damage is described as a large reduction of the adhesive interlayer shear stiffness. It is assumed that the damaged zone of the decreased stiffness extends uniformly across the actuator from its edges to the centre. In the analysis, the beam is divided into sections due to its geometry and the supposed delaminated regions. The calculations are performed using the FE analysis. In the first step, the cantilever beam with piezoelectric elements bonded to the beam surface by a thin glue layer of uniform properties is analysed. The beam is excited by a time varying force. In order to validate the arranged FE model, the analytical and FE calculations of open and closed loop dynamic responses are performed. The compared results show quite good agreement and prove that the applied FE model of the considered system is correct and can be accepted for further numerical analysis. In the next step, FE simulations of the beam forced vibration suppressed by piezoelectric control system with velocity feedback are accomplished. The influence of the length of damaged zone on the dynamic characteristics and the control effectiveness is investigated. As expected, the size, type and location of the coupling damage influence on the dynamic characteristics of the examined beam and cause reduction of the system control efficiency.*

1 INTRODUCTION

Piezoelectric elements find a wide range of applications in many electromechanical systems for active vibration control, acoustic and pressure sensing, shape control and health monitoring of structures. For this purpose, piezoelectric patches are frequently attached to a controlled structure and are used as actuators or sensors. Acceptable operational effectiveness of vibration control of the structure requires forces, which are produced by deformation of piezoelectric transducers. This connection is mainly made with an adhesive layer. However, the connection may be weakened because of imperfections (flaws) arising during manufacturing process. Additionally, significant alternating in time interfacial shear stresses, created by the control action, may initiate and increase in degradation of coupling between piezoelectric element patches and the main structure, which is introduced as a delamination process. Such delamination causes degradation of the overall stiffness and strength of a structure. Hence, existence of any delamination could change dynamic characteristics of the damaged system. The size and location of the delamination could have significant influence on these changes [1]. Therefore, the design of a control system that contains piezoelectric elements requires an effective modelling of the electrical, mechanical and coupling properties of the main structure and piezoelectric elements. These problems have been extensively studied using analytical methods (e.g. [1, 2, 3, 4, 5, 6]). A brief review of works considering these problems could be find in the paper by Huang et all [2] and by Tylikowski [3].

Tylikowski [3], based on an analytical approach, studied the bending-extensional dynamic model of a simply supported beam with piezoelectric actuators bonded to lower and upper surfaces. He showed that an increase in the edge delamination decreases the magnitude of the transfer function relating the beam deflection to the applied actuator voltage, and also shear stresses in the bonding layer.

Using an analytical approach, Pietrzakowski [4] studied the active damping of structural vibration of a simply supported beam by using a collocated sensor/actuator pair. He investigated influence of the bonding layer parameters on the dynamic response of the controlled beam. He assumed that the sensor is perfectly connected to the main structure but the actuator is bonded with a viscoelastic adhesive layer. He showed that the stiffness of bonding layer affects significantly the active damping efficiency. Modelling of piezoactuator edge delamination described as a significant stiffness reduction of the bonding layer was presented in the paper by Pietrzakowski [5].

Analytical methods are limited for solving practical problems of complex geometry, for which only numerical methods can be used, especially the finite element method (FEM).

Kim et all [7] using the FEM studied the behaviour of a cantilever plate controlled with a piezoelectric sensor and actuator. They assumed that piezoelectric transducers were perfectly bonded to the main structure. In that paper results obtained by numerical computation were experimentally verified and good agreement was stated.

Trindade et all [8, 9] studied active–passive damped multilayer sandwich beams, consisting of a viscoelastic core sandwiched located between layered piezoelectric faces. The frequency-dependence of the viscoelastic material was handled through the inelastic displacement field model. They applied a modal reduction to the resulting augmented system. They made the validation of the used approach through modal analysis comparisons with numerical and experimental results found in the literature.

Balamurugan and Narayanan [10] developed the finite element model for the beam-like structure with partially covered, smart constrained layer damping (SCLD) treatment. The system consisted of a viscoelastic shear layer sandwiched between two layers of piezoelectric sensors and actuator. This composite SCLD bonded to a vibrating structure acted as a smart

treatment. The sensor piezoelectric layer measured the vibration response of the structure and a feedback controller regulated the axial deformation of the piezoelectric actuator.

Wu et all [11] using the commercial FE system Ansys studied the harmonic response, resonant frequency of the piezoelectric plate, and the equivalent piezoelectric circuit of the piezoelectric transducer. In their work numerical results were verified and confirmed by experimental researches.

The number of publications dedicated to the study of the influence of increased length of the delamination zone on the dynamic properties of the controlled system is limited. For this reason, the objective of the present study is to develop modelling of the piezoelectric element edge delamination and analysis of effects of the progressive damage process on the active beam dynamic behaviour.

2   ANALYSED PROBLEM

We consider a cantilever beam of the length L and width b excited by a time varying force $F(t)$. In the considered system piezoelectric patches are bonded to both opposite sides of the beam and operate as a collocated sensor/actuator pair with velocity feedback. The bonding layer of thickness t_g is assumed to be elastic. The schematic diagram of the analysed system is shown in Figure 1. Herein delamination is described as a large reduction of the adhesive interlayer shear stiffness. It is assumed that the damaged zone of the constant stiffness extends uniformly across the actuator from its ends to the centre. The influence of the length of delaminated section δ and the adhesive interlayer equivalent stiffness on the dynamic characteristics and the control effectiveness is investigated. In the first step, the cantilever beam with piezoelectric elements bonded to the beam surface by the thin adhesive layer of uniform stiffness is analysed. The beam is excited by a time varying force. In order to validate the arranged FE model, the analytical and the FE calculations of open and closed loop responses have been performed. In the next step, the FEM simulations focused on the influence of the delamination length on the dynamic characteristics and the control effectiveness of the system have been made.
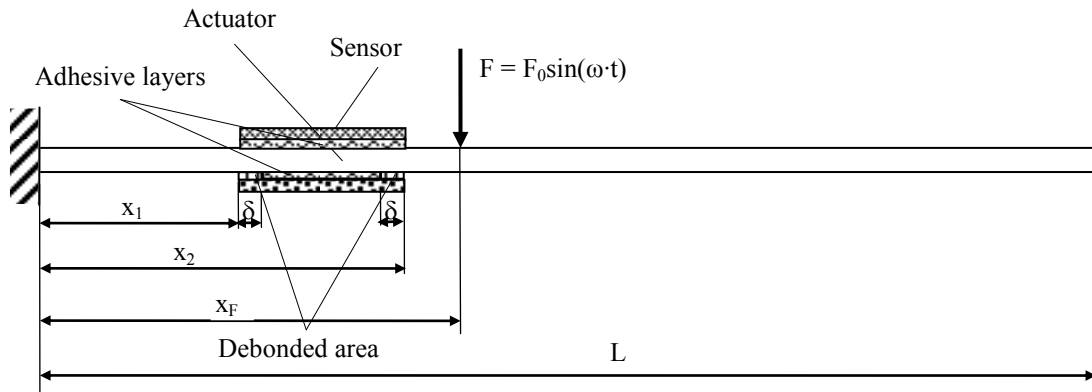

Figure 1. Schema of the analysed system.

3   ANALYTICAL MODELLING

The beam Bernoulli-Euler theory is used to obtain equations of motions of the considered system. It is assumed that only the actuator is bonded with an adhesive layer to the main structure. In the analysis, the beam is divided into three sections due to its geometry.

Taking into account the actuator extension and shear stresses transmitted by the bonding layer, the beam's activated section is governed by two coupled equations expressed here in terms of actuator strains ε_a and beam surface strains ε_b as follows

$$E_a t_a \frac{\partial^2 \varepsilon_a}{\partial x^2} - \frac{G_g}{t_g}(\varepsilon_a - \varepsilon_b) - \rho_a t_a \frac{\partial^2 \varepsilon_a}{\partial t^2} = 0$$

$$\frac{\widetilde{E} t_b^2}{12}\frac{\partial^4 \varepsilon_b}{\partial x^4} + \frac{t_b}{4}\frac{G_g}{t_g}\left(\frac{\partial^2 \varepsilon_a}{\partial x^2} - \frac{\partial^2 \varepsilon_b}{\partial x^2}\right) + \widetilde{\rho}\frac{\partial^2 \varepsilon_b}{\partial t^2} = 0 \qquad x \in (x_1, x_2) \qquad (1)$$

where: t_a, t_b and t_g - actuator, beam and bonding layer thickness, respectively, $\widetilde{\rho}$ - equivalent mass density of the activated beam section, ρ_a - actuator mass density, E_a - Young's modulus of the actuator material, $\widetilde{E}$ - equivalent Young's modulus of the activated beam section, G_g - Kirchhoff's modulus of the adhesive material.

Motion of other beam sections is described by the well-known equation

$$\frac{E_b t_b^2}{12}\frac{\partial^4 \varepsilon_b}{\partial x^4} + \rho_b \frac{\partial^2 \varepsilon_b}{\partial t^2} = 0 \qquad \begin{array}{l} x \in (0, x_1) \\ x \in (x_1, L) \end{array} \qquad (2)$$

where: E_b, ρ_b - Young's modulus and the mass density of the beam, respectively. Supposing viscoelastic material of the beam and the bonding layer, Young's moduli E_b, $\widetilde{E}$ and Kirchhoff's modulus G_g are complex.

The above dynamic equations (1) and (2) have to satisfy boundary conditions at the beam ends at $x = 0$ and $x = L$ for a cantilever beam, continuity of beam deflection, slope, curvature and transverse force at the borders of the sections at $x = x_1$ and x_2, and free edge conditions at the actuator ends.

The free edge conditions require zero normal stresses at the actuator ends and according to the stress equation $\sigma_a = E_a(\varepsilon_a - \lambda)$ become

$$\varepsilon_a(x_1^+) = \varepsilon_a(x_2^-) = \lambda \qquad (3)$$

where λ is the actuator longitudinal strain generated by the external electric field.

In the considered one-dimensional actuation the strain $\lambda(t)$ of the unconstrained actuator loaded by the external voltage $V(t)$ can be written as

$$\lambda(t) = d_{31}\frac{V(t)}{t_a} \qquad (4)$$

where d_{31} is the piezoelectric constant.

The actuator driving voltage is generated by the sensor and transformed according to the applied control function.

The details of the governing equations formulation and the steady-state response of the active beam-system one can find in [4].

4 THE FE MODEL OF THE ANALYSED SYSTEM

The beam is modelled using 3D-solid 20-nodes second order elements C3D20. The piezo-electric transducers are built using 3D-solid, second order, piezoelectric 20-nodes elements i.e. C3D20E elements from Abaqus element library [12]. These elements have additionally electric potential as the nodal quantity. Piezoelectric coupling is provided by introducing the piezoelectric and dielectric material coefficients. In Abaqus system, piezoelectric properties can be used in natural frequency extraction, transient dynamic analysis, linear and non-linear static stress analysis, and steady state dynamics analysis procedures. Adhesive layer between beam and piezoelectric transducers is modelled using 3D-solid 8-nodes cohesive elements COH3D8. These elements based on the assumption that the cohesive layer is subjected to only one direct component of strain, which is the through-thickness strain, and to two transverse shear strain components. The other two direct components of the strain (the direct membrane strains) and the in-plane (membrane) shear strain are assumed to be zero for the constitutive calculations. More specifically, the through-thickness and the transverse shear strains are computed from the element kinematics. However, the membrane strains are simply assumed to be zero for the constitutive calculations [12].

The displacement compatibility between particular elements of the structure is provided by "Tie" constrains [12]. These constrains make the translational and rotational motion as well as all other active degrees of freedom equal for a pair of nodes of constrained surfaces. These constrains are imposed for adjacent surfaces of connected elements.

In the piezoelectric transducers area, the mesh density is selected so that the stress distribution in the adhesive layer is adequately smooth but the number of elements is minimum. This requirement causes the mesh is much more dense in the transducers area. The used FE model is shown in Figure 2.
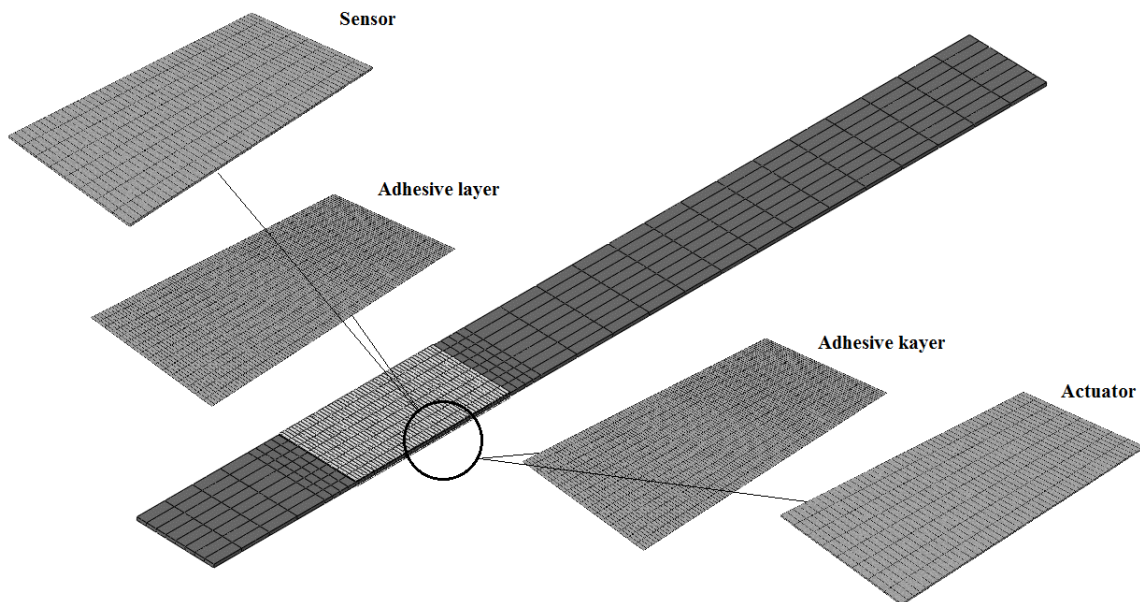

Figure 2: FE mesh of the examined beam.

The finite element mesh of the beam consists of 608 elements and 4679 nodes, every piezoelectric transducer is composed of 400 elements and 3093 nodes, and every adhesive layer

consists of 1600 elements and 3434 nodes. Thus the entire FE mesh consists of 4608 elements and 17733 nodes.

5   CALCULATIONS RESULTS

5.1   Physical properties of the analysed system

Numerical simulations are performed for the beam of length $L = 270$ mm, width 25 mm and thickness 1 mm. The beam is loaded by the harmonic force $F(t)$ of amplitude 0.1 N and acting at $x_F = 106$ mm. The sensor/actuator pair is located between $x_1 = 46$ mm and $x_2 = 96$ mm. The thickness of the actuator is 0.381 mm and the thickness of the sensor is 0.254 mm. The thickness of the adhesive layers is 0.01 mm. The material properties of the beam, the piezoelectric transducers and adhesive layers are listed in Table 1.

Material parameter	Beam	Actuator	Sensor	Adhesive
Mass density ρ [kg/m^3]	7800	5780	5900	1600
Young's modulus E [MPa]	$2.19 \cdot 10^5$	$3.3 \cdot 10^4$	$2.5 \cdot 10^4$	$2.5 \cdot 10^3$
Piezoelectric constant d_{31} [m/V]	-	$2.3 \cdot 10^{-10}$	$2.5 \cdot 10^{-10}$	-
Dielectric constant ϵ_{33} [F/m]	-	$1.416 \cdot 10^{-8}$	$1.416 \cdot 10^{-8}$	-

Table 1 Material properties.

5.2   Comparative calculations

In order to determine the correctness of the arranged FE model comparative calculations of the analysed beam are performed. The calculations are carried out using analytical and FE approach. Amplitude-frequency displacement characteristics of a point located at the beam tip are calculated for open loop feedback. These characteristics are computed for the case without degradation of the adhesive layers (Figure 3). Next using both methods, calculations of the beam forced vibration suppressed by a piezoelectric control system with velocity feedback are accomplished. The maximum amplitudes at resonance frequency for various feedback factors k_v are computed. The relative difference between the amplitudes achieved using the analytical and FE methods is less than 3 percent (Table 2). The obtained results show quite good agreement and prove that the applied FE model of the considered system is correct and can be accepted for further numerical analysis.
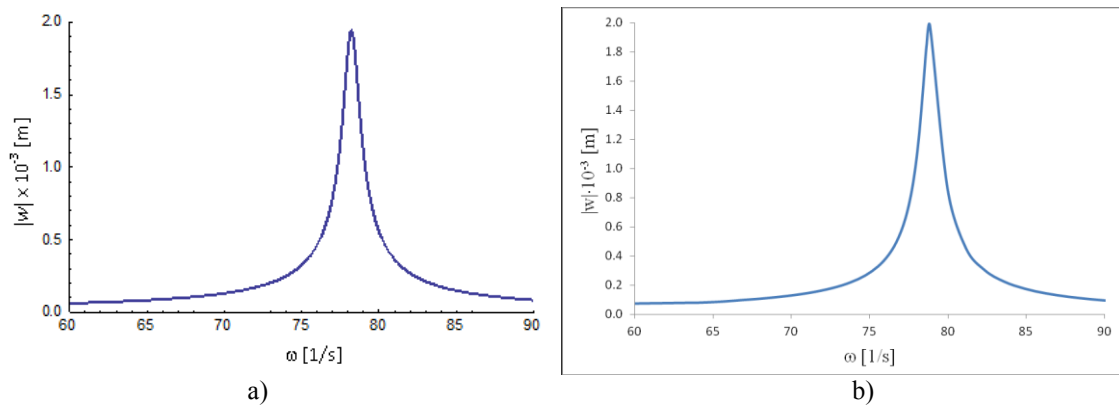

Figure 3: Dynamic characteristics obtained using: a) the analytical method, b) FEM

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k_v	Analytical |w| [mm]	FEM |w| [mm]	Relative differ. [%]
0.08	0.874	0.856	2.1
0.10	0.769	0.752	2.2
0.12	0.689	0.668	3.1
0.14	0.622	0.603	3.0

Table 2 Suppressed amplitudes achieved using the analytical method and FEM.

5.3 Investigation of the system with the degraded adhesive layer

The influence of the delamination length δ on the dynamic characteristics and the control effectiveness of the system described above is investigated. FE simulations of the beam forced vibration damped by piezoelectric control system with velocity feedback are accomplished. As mentioned above, the damaged zone is formed by a large reduction of the bonding layer stiffness between the actuator and the beam. The increase of the delamiantion zone is modelled by increase its length. Two types of delamination are considered: the first one beginning from the actuator edge located near the beam fixed end (Figure 1), the second one, beginning from both outer edges of the actuator and growing into its centre. The numerical calculations are performed for four values of the length of delaminated zones δ = 0, 5, 10 and 15 mm (Figure 1) and four values of the feedback coefficient k_v.
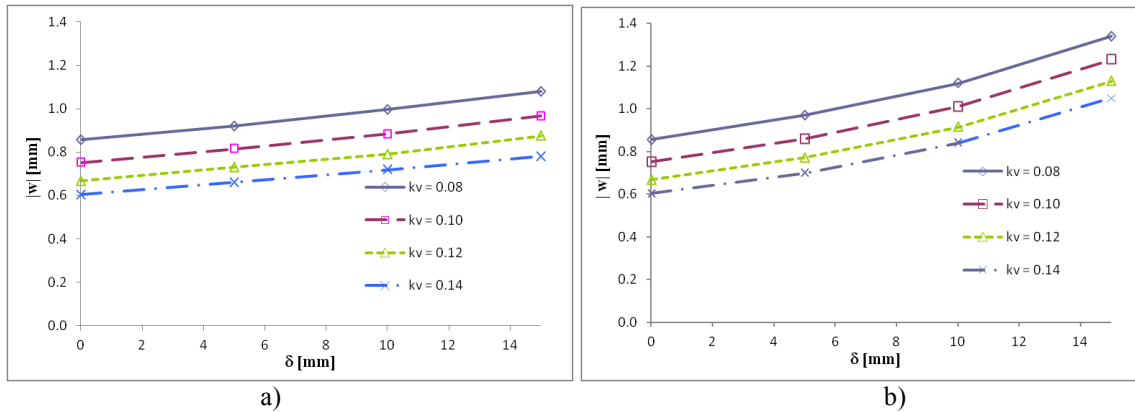
Figure 4: Influence of the delamination length δ on the maximum resonant amplitude; a) one-sided delamianation, b) two-sided delamination.

δ	Feedback coefficient			
	k_v =0.08	k_v = 0.10	k_v = 0.12	k_v = 0.14
5	7.48	8.38	9.28	9.62
10	16.47	17.55	18.41	19.24
15	26.17	16.36	19.01	19.57

Table 3 Decrease in damping efficiency caused by one-sided delamination.

Firstly, the influence of the delamination length δ on the maximum resonant amplitude of the beam excited at resonance frequency is investigated. The relationship curves obtained for constant feedback coefficients are shown in Figure 4. In the case of one-sided delamination the dependence is nearly linear (Figures 4a and 5a) but in the case of two-sided delamination it is weakly non-linear (Figures 4b and 5a). The distance between the particular curves is nearly constant (Figure 4).

Decrease of percentage damping efficiency obtained in the case for one-sided delamination is presented in Table 3, whereas for two-sided delamination in Table 4.

δ	Feedback coefficient			
	$k_v = 0.08$	$k_v = 0.10$	$k_v = 0.12$	$k_v = 0.14$
5	13.43	14.36	15.57	16.09
10	30.84	34.31	36.98	39.30
15	56.54	63.56	69.16	74.13

Table 4 Decrease in damping efficiency caused by two-sided delamination.

The total length of debonding zone in the case of two-sided delamination is two times longer than in the case of one side delamiantion, namely $\delta = 10$ mm in the case of one-sided delamination corresponds $\delta = 5$ mm for two-sided delamiantion. Comparing results presented in second row of Table 3 and in the first row in Table 4 can be seen that in the first case the influence of debonding zone length is more significant than in the second case. This is due to the fact that the second edge of the piezoactuator is bonded to the beam area with a minor curvature.
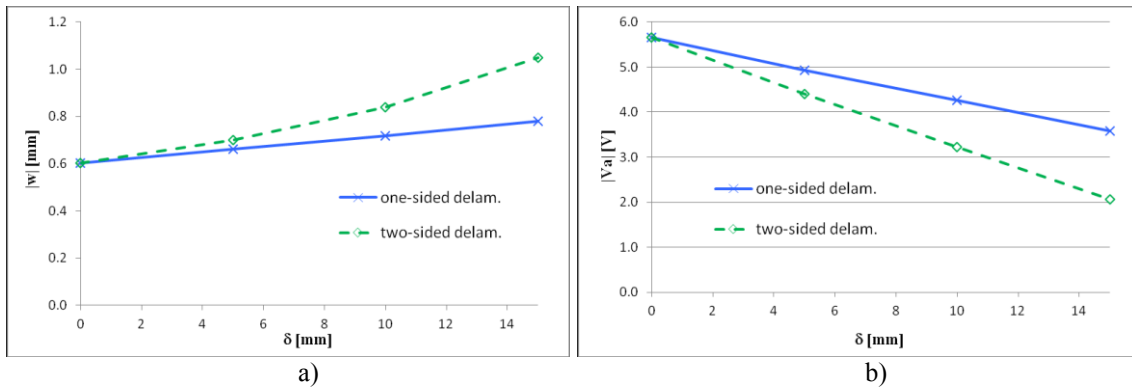

a)

b)

Figure 5: Influence of the delamination length δ on: a) maximum displacement amplitude at the resonant frequency and feedback factor $k_v = 0.14$, b) actuator voltage in case of the open feedback loop.

Mode	Delamination length δ			
	0	5	10	15
1	78.80	78.18	77.58	77.03
2	455.89	454.95	454.19	453.62
3	1284.20	1276.70	1267.70	1257.60
4	1630.20	1617.50	1605.30	1593.40
5	1855.00	1851.80	1848.70	1845.80
6	2565.80	2554.00	2537.80	2516.40
7	4265.80	4234.50	4197.00	4113.30
8	4444.40	4430.10	4412.90	4380.30
9	6471.60	6428.10	6348.20	5609.20
10	7648.30	7614.30	7566.90	5841.60

Table 5: Influence of the delamination length δ on natural resonance frequencies of the system.

When the system operates in an open loop, the existence of delamination can be detected by measuring the voltage i.e. the voltage measured on the actuator electrode significantly decreases when the length of delamiantion zone becomes greater (Figure 5b)

The natural frequencies of the analysed system are computed as well. It is stated that the influence of length of the delamination zone on natural frequencies is nearly negligible (Table 5).

6   CONCLUSIONS

- The analytical and FE models of a beam with bonded piezoelectric sensor and actuator are created. The FE model considering damage of the adhesive layer bonding the actuator to the beam is proposed. The damage of the adhesive layer is modelled by a large reduction of the material stiffness of this layer.

- The influence of the damaged zone length of the adhesive layer which mounting the actuator to the beam on the dynamic behaviour of the analysed system is examined. The one and two-sided delamination are considered.

- As expected, the active damping efficiency decreases when the delamination length becomes greater. In the case of one-sided delamination, the damping efficiency diminishes almost linearly when the damaged zone expands, whereas for the two-sided delamination the dependency is weakly non-linear.

- The existence of delamination can be detected by measuring the actuator voltage for the open-loop control system. The generated voltage significantly decreases with increasing the length of delamination zone.

- The influence of the actuator edge delemination on the natural frequencies of the examined system is practically negligible.

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