11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

A METHODOLOGY FOR OPTIMAL SENSOR PLACEMENT IN STRUCTURAL IDENTIFICATION AND VERIFICATION BY FUZZY FINITE ELEMENT MODEL UPDATING

Yildirim S. Erdogan*[1], Necati F. Catbas[2], Pelin G. Bakir[1]

[1]Istanbul Technical University, Civil Engineering Faculty, Istanbul, Turkey
yserdogan@itu.edu.tr, gundesbakir@yahoo.com

[2]University of Central Florida, Civil, Environmental and Construction Engineering, Orlando, Florida
catbas@ucf.edu

Keywords: Structural Identification, Finite Element Model Updating, Fuzzy Numbers, Optimal Sensor Placement, Uncertainty Quantification.

Abstract. *This Optimum configuration of sensors in order to collect sufficient data in structural health monitoring applications is very crucial for the reliable identifications of civil engineering structures. In this study, a methodology is presented based on the sensitivities of model parameters which are to be updated using the data obtained from the optimal sensor locations. A crude finite element model is used to calculate the sensitivities of model parameters with respect to the structural response and a clustering approach is applied in order to find out the optimal sensor locations which is supposed to provide sufficient data for reliable St-Id. A performance measure indicates the capability of sensors in providing information about the model parameters is presented. The results are verified by the so-called fuzzy finite element model updating (FFEMU) approach which is proposed to quantify uncertainties in model and the response of the structure. The experimental data and the updating model parameters are represented by fuzzy numbers. For each sensor set, a model updating procedure is then applied and the amount of uncertainty in model parameters for different cases is compared in order to investigate the effect of the data amount on the updating results. In this study, the presented method is used to configure the displacement gages; however it is also applicable for any sensor type. A benchmark bridge structure developed for structural health monitoring purposes in University of Central Florida structural engineering laboratories is used to verify the proposed approach. It is demonstrated that the proposed methodology is very practical and a well-established approach in determining the optimal sensor set for St-Id applications.*

1 INTRODUCTION

Model-based structural identification (St-Id) techniques have gained considerable attention in last two decades especially for better evaluation of aged, critical or landmark structures using structural identification (St-Id) and structural health monitoring (SHM) data. A complete St-Id is crucial to understand the real behavior of existing structures and diagnose possible changes in structural parameters such as material properties, boundary conditions, member connections and mass properties for more informed decision-making for repair, retrofit or replacement of existing structures ([1],[2],[3]). Model calibration methods such as finite element model updating (FFEMU) can be efficiently used to developed numerical models, which best describe the real behavior of the structure by minimizing the difference between the model predictions and the measurements ([4], [5]).

The uncertainties both in modeling and measurements play a significant role on the quality of the FEMU results. However, uncertainty quantification is not a trivial task especially for the inverse problems like finite element model updating. This is mostly accomplished by probabilistic methods ([6], [7]) and recent years by non-probabilistic methods ([8], [9]). In this study, a stochastic finite element model updating technique is used in which the uncertain parameters are modeled by fuzzy numbers. This method is also used to verify the model quality updated using data sets obtained from different sensor configurations.

The success of St-Id largely depends on the quality and the amount of SHM data and the updating finite element model. The data should include the necessary information about the global and the local behavior of the structure. In order to identify the real-life structures such as bridges, high rise buildings, dams etc., an appropriate sensor configuration is required to provide data with high or, in most real life cases, with a “sufficient” spatial resolution. In SHM literature, various methods have been proposed for optimal sensor configuration. Some authors have used the Shannon’s Entropy Function in order to quantify the degree of separation between candidate configurations and some has used the orthogonality properties of the measured mode shape vectors ([10], [11]). A comparative study can be found in [12]. In the context of this study, an optimal sensor configuration methodology is proposed and verified by fuzzy finite element model updating method. The method is based on the sensitivities of the sensor locations to the updating model parameters and the clustering approaches. The method is applied to obtain the optimal displacement gage configuration however the mode shape components or strain values can also be used to configure the acceleration sensors and strain gages, respectively.

2 FUZZY FINITE ELEMENT MODEL UPDATING

In FFEMU, all model and the output parameters are modeled by using fuzzy numbers. The shape of the fuzzy numbers can be chosen depending on the problem by using expert knowledge and the experiences. In this study, triangular membership functions are used. The α-level representations is used in the formulation of the objective function. Hence, the fuzzy numbers are transformed into interval numbers for each α-level ([13]). The objective function is formulated based on the idea that the difference between the upper and the lower bounds of the interval experimental and the predicted responses are to be minimum. The equations for objective functions and related constraints that have to be strictly applied in order to make inverse problem to have a unique solution and capture all uncertainty in model responses are given in Eqs. (1-5).

$$\boldsymbol{min f(\theta^{int}) = \underline{r}(\theta^{int})^T W \underline{r}(\theta^{int}) + \overline{r}(\theta^{int})^T W \overline{r}(\theta^{int})} \tag{1}$$

$$\boldsymbol{\theta^{int} = [\underline{\theta}, \overline{\theta}]} \tag{2}$$

$$\underline{r}(\boldsymbol{\theta}^{int}) = \frac{\underline{\gamma}(\theta^{int}) - \underline{\gamma}^{e}}{\underline{\gamma}^{e}} \tag{3}$$

$$\overline{r}(\boldsymbol{\theta}^{int}) = \frac{\overline{\gamma}(\theta^{int}) - \overline{\gamma}^{e}}{\overline{\gamma}^{e}} \tag{4}$$

$$[\boldsymbol{\lambda}^{int}, \boldsymbol{\phi}^{int}, \boldsymbol{\varepsilon}^{int}] = f^{model}(\boldsymbol{\theta}^{int}) \tag{5}$$

where, $\boldsymbol{\theta}^{int}$ is the interval valued updating parameter vector, $\boldsymbol{W}$ is the weighting matrix, which might be determined intuitively considering the relative accuracy of the measurements, $\boldsymbol{\gamma} = [\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\varepsilon}]$ is the response vector, $f(\boldsymbol{\theta})$ is the objective function and $f^{model}(\boldsymbol{\theta})$ is the model function that governs the physical process. In Eq. (5), frequency, mode shape and strain vectors are denoted by $\boldsymbol{\lambda}$, $\boldsymbol{\phi}$ and $\boldsymbol{\varepsilon}$, respectively. The superscript *int* appeared in response and model parameters means that the given parameters are interval valued. Hence, the objective function given in Eq. (1) has to be minimized to obtain interval valued model parameters for each α-level. These interval model parameters are then combined to obtain final fuzzy parameters. Figure 1 depicts the α-level representation of the response and the model parameters. The bar above and below the response quantities denotes the upper and lower quantities.

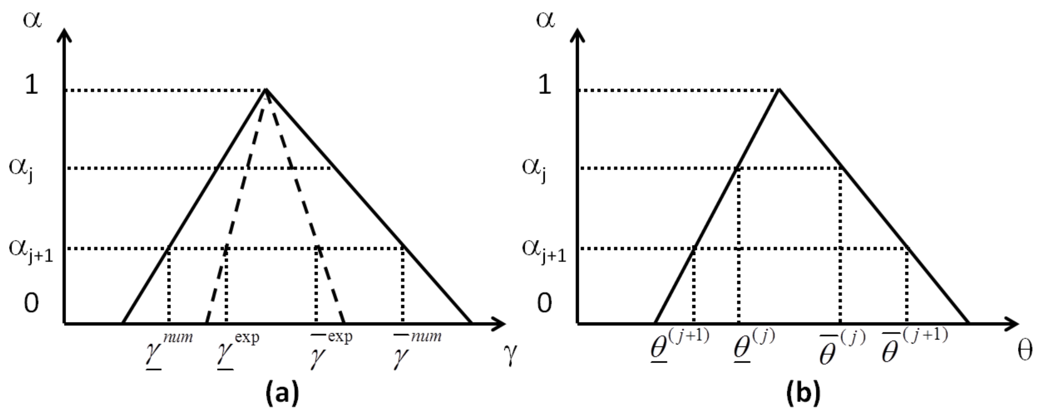

Figure 1: $\boldsymbol{\alpha}$-level representation of a) model response b) model parameters.

Although it is not always the case, the response quantities are the monotonic functions of the model parameters (boundary spring stiffness) in the variation range of model parameters. The monocity of the response variables can be checked out by preliminary FE analysis. Thus, transformation method ([13]) can be applied to obtain the response variables as done in this study. This means that 2^m model calculations, which is not feasible for complex models, have to be performed for each input vector. However, instead of evaluating the complex model for fuzzy calculations, very simple equation provided by Gaussian Process model can be calculated. In addition to the objective function provided by Eqs. (1-5), some additional constraints have to be introduced. It should also be noted that the optimization problem given Eqs. (1-5) has to be solved for some specified numbers of α-level in order to capture the nonlinear relationship between inputs and outputs.

$$\underline{\theta}_i^{(j+1)} \leq \underline{\theta}_i^{(j)} \tag{6}$$

$$\overline{\theta}_i^{(j+1)} \geq \overline{\theta}_i^{(j)} \tag{7}$$

$$\underline{\gamma}_k^{num} \leq \underline{\gamma}_k^{exp} \ \forall j \in [0,1] \ k = 1, \dots .nm \tag{8}$$

$$\bar{\gamma}_k^{num} \geq \bar{\gamma}_k^{exp} \; \forall \, j \in [0,1] k = 1, \ldots . nm \tag{9}$$

In Eqs. (6-9) j is the α-level, k is the output (measurement) number, nm is the number of measurements. These equations guarantee that the fuzzy sets of the updated parameters are convex. The convexity is a requirement for a unique fuzzy set solution. Other constraints given in Eqs. (8) and (9) ensure that all uncertainty in the output parameters is captured. By this way, all output parameters (measurements) will be a member of the fuzzy set obtained from fuzzy FEM analysis using updated fuzzy model parameters. Actually, optimization problem turns out to be a constraint optimization problem with inequalities by introducing these limitations. The genetic algorithms provide a practical way to the solution of these type of problems. As seen in Eq. (10), constraints given in Eqs.(8) and (9) can effectively be satisfied by assigning some penalties to the infeasible regions in the output space domain. In Eq. (10), the term H is a number in which its value is very high compared to the objective function value. By this way, infeasible regions can be disregarded.

$$f^{infeasible}(\theta) = f(\theta) + H \tag{10}$$

3 OPTIMAL SENSOR CONFIGURATION

The In order to achieve a successful St-Id with measurement data obtained from a limited number of locations, optimum locations for sensor should be determined. Those locations can be identified according to the information that is needed to update the model parameters. Hence, the minimum sensor number and the location set should include the sensor locations, which are most sensitive to the structural model parameters or to the updating parameters. For the purpose of finding optimum sensor configuration, an initial sensor configuration, which might be very dense can be arranged based on engineering judgment. Then, a clustering method is employed to group the sensors according to their sensitivities on the specified updating parameters. From each sensor cluster, one sensor can be selected according to the relative sensitivity value and coefficient of variation of the sensor values. A parameter, which is the sum of absolute sensitivity and the coefficient of variation of the sensor values (displacements) is proposed to choose the sensor locations. The proposed approach consists of the following steps:

1) Determine the model parameters to be updated (boundary conditions, connection rigidities, mass etc.)

2) Determine the possible dimensions of the updating parameter space intuitively or by preliminary analysis (variation range of the updating parameters)

3) Select a certain number of points, say nm, in the parameter space randomly depending on the number of updating parameters. The number *nm* largely depends on the number of updating parameters, the size of the parameter domain and the input output relation. The nm can be estimated by considering an error threshold as done in Monte Carlo Simulation.

4) Calculate the absolute sensitivities $s_{ij}^k = \left|\frac{\partial d_i^k}{\partial a_j}\right| \; k = 1, \ldots nm$ of each sensor location (displacement values) to the updating parameters and the displacement at the selected sampling points in the parameter space. Superscript k corresponds to any selected point in the parameter space, s_{ij}^k is the absolute sensitivity of the ith sensor value to the jth updating parameter at the kth point, d_i^k is the displacement at the ith sensor location and a_j is the jth updating parameter.

5) Find the total absolute sensitivity matrix $\rho_{ij} = \sum_{k=1}^{nm} s_{ij}^k$. In this equation, ρ_{ij}corresponds to total absolute sensitivity of the ith sensor value to the jth updating parameter.

6) Choose the appropriate sensor set as described below:

I. Determine the initial sensor locations randomly or by engineering judgment

II. Cluster the sensor locations using absolute sensitivity values of each sensor locations. Linkage tree has been observed to be the most efficient way to differentiate the most appropriate clusters. Clustering can be carried out by creating a linkage tree using any measure of distance such as Euclidean, Mahalanobis, Hamming etc. between the sensitivity values of measurement locations . These linkage values then can be used to cluster sensor locations.

III. Calculate the total differential $td = \sum_{j=1}^{n} \rho_{ij}$ of sensitivity values (n denotes the number of variables) and coefficient of variation (*cv*) of the displacement values for each sensor location which belongs to the same cluster. Larger coefficient of variation for any sensor value means that this sensor can separate more sets of models ([10]).

IV. Normalize the *td* and cv to unity in order to give equal weight to each parameter. Calculate the $\beta = td + \omega \times cv$ for each sensor location at same cluster. Choose the sensor location which has max β value. In this formulation, a parameter ω is used to involve the cv in the equation in a weighted manner. This parameter is explained in the next paragraph.

V. Repeat the steps III-IV for each sensor cluster.

In step IV, the coefficient of variation value is included into the formula with a weighting parameter ω which takes values less than one. This parameter is introduced because of the fact that some displacement values could be very small and insensitive to the parameters. However, they could have high cv values. Hence, cv values can be included into the formula in a weighted manner. The value of ω can be chosen intuitively considering values of response quantities.

4 NUMERICAL VERIFICATION

4.1 Optimal sensor configuration

In this section, optimum sensor locations are determined using the proposed method explained previously with a demonstration on a numerical model (Figure 2), which has been developed as a part of an international benchmark study ([14]). The updating parameters are chosen as the moment of inertia of the beam segments located at the connections between the cross and the main girders together with the mass of all connections. As a result, a total of 11 model parameters are selected to be updated. Total of 20 sensor locations (Figure 2b) are selected for the initial sensor configuration. The sensitivities of each sensor value are calculated for 10 different loading combinations and the average values of the total sensitivities are used to cluster the displacement sensors. The initial domain of the updating parameters is given in Table 1. The bounds of the initial parameter domain could be determined according to the literature search, material tests, geometric measurements and/or engineering judgment. The selected domain of parameters should be large enough such that it should include all possible parameter values and also should be small enough to sample the whole parameter space reliably. Updating parameters are normalized to unity and given as $\alpha = \theta / upper\ bound$. Hence, for moment of inertia α varies between 0.3 and 1, for mass and spring stiffness α varies between 0 and 1, respectively.

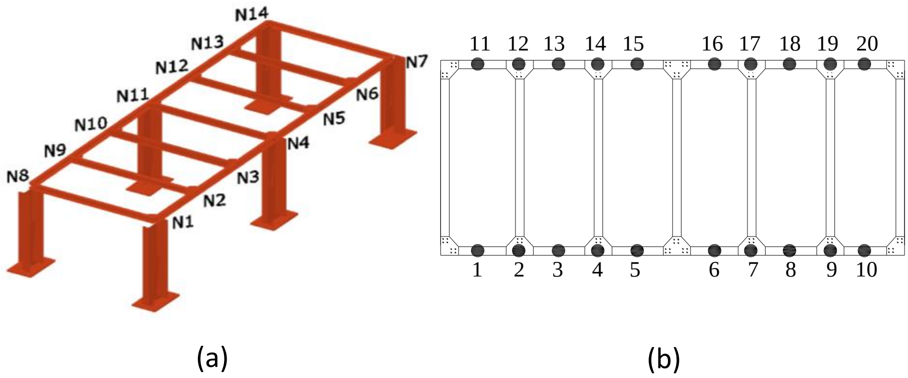

Figure 2 : Benchmark structure a) 3D model b) Initial sensor configuration

Updating parameters	Initial value	lower bound	upper bound
Moment of Inertia	$1.049 \times 10^{-6} m^4$	$0.9 \times 10^{-6} m^4$	$3 \times 10^{-6} m^4$
Mass of all connections	$1.5\, kg$	$1.2\, kg$	$4\, kg$

Table 1: Updating parameters and domain of definition

The clustering results for the sensor locations using the total sensitivities of the response quantities to each updating parameter are given in Figure 3. Four different cluster numbers (2, 4, 8, 12 and full) can be selected according to the linkage values. In Figure 3, the linkage values represent the distance between the different clusters. For example, the distance between the two groups marked by rectangles for the Cluster 1 is very large (approximately 1.6) relative to the others. This means that these two clusters provide different information for the model parameters. The selected sensor locations for different cluster numbers by using the proposed method are given in Table 2. As apparent in Table 2, the linkage values for the full sensor set (FSS), sensor set 1 (SS1) and sensor set 2 (SS2) does not differ significantly. There is slight difference between those sensor sets and SS3. Finally, the difference is larger than the first three sets for the SS4. As mentioned previously, this would mean that the data obtained from the first three sensor sets will provide similar information about the model parameters. Hence, it can be deducted that the optimum sensor set might be the SS2 in which its linkage values do not significantly differ from the full sensor set and have the lowest number of sensors.

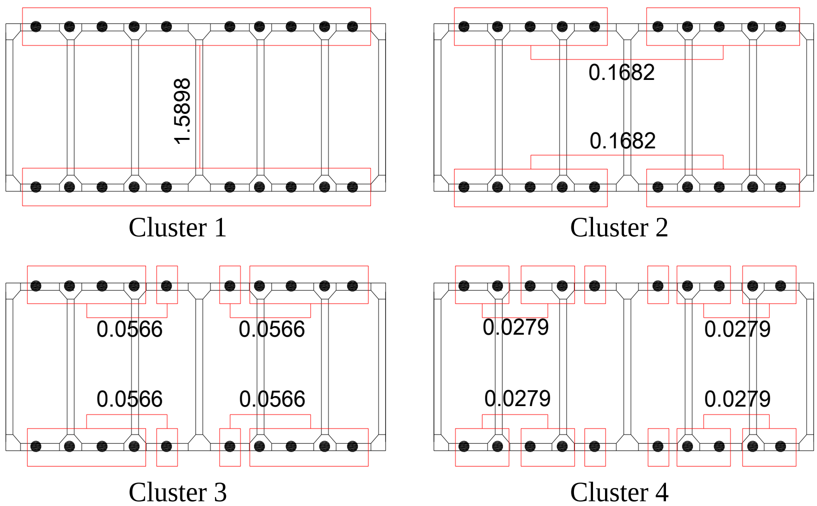

Figure 3 : Sensor Clusters a) cut off < 1.5898 b) cut off < 0.1682 c) cut off < 0.0566 d) cut off < 0.0279

Sensor Set	Cluster number	Linkage value	Sensor locations
FSS	20	0	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SS1	12	0.0279	2 3 5 6 8 9 12 13 15 16 18 19
SS2	8	0.0566	2 5 6 9 12 15 16 19
SS3	4	0.1682	2 9 12 19
SS4	2	1.5898	2 12

Table 2 : Sensor sets obtained by clustering

4.2 Verification by fuzzy finite element model updating

The optimum sensor set determined in the previous section are verified by fuzzy finite element model updating. The performance of each sensor set are assessed based on two criteria: 1) objective function values, 2) imprecision in the updating parameters. The amount of the relative imprecision in updating parameters is the area of the membership functions divided by the parameters value which corresponds to the α level 1. In all calculations, noisy measurements are used (%10 for accelerations and %5 for displacements). First five natural frequencies and displacements obtained from 3 different loading combinations are included into the objective function (error function). In fuzzy updating, uncertainty in response quantities is modeled by symmetric triangular membership functions in which the core value of the fuzzy response quantities are the measured values. Lower and upper values of the support of the fuzzy numbers are $\pm 2.5\%$ of the core value as seen in Figure 4. In order to simplify the calculations, fuzzy updating are carried out for two α-cut levels, one at $\alpha = 1$ and the other one $\alpha = 0$. In order to capture the nonlinearity between the inputs and the outputs, fuzzy updating procedure can be applied at additional α-levels.

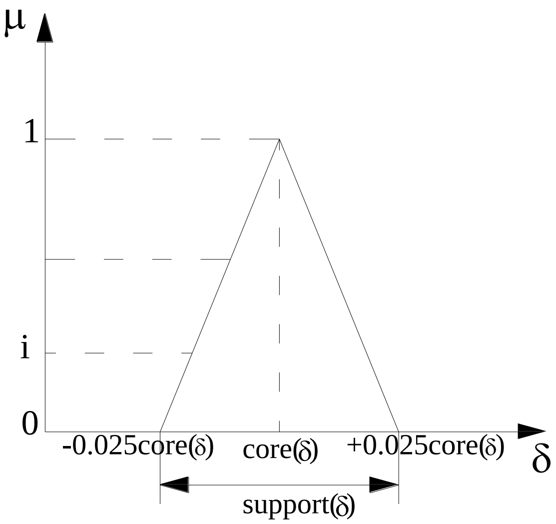

Figure 4 : Fuzzy measured response quantities

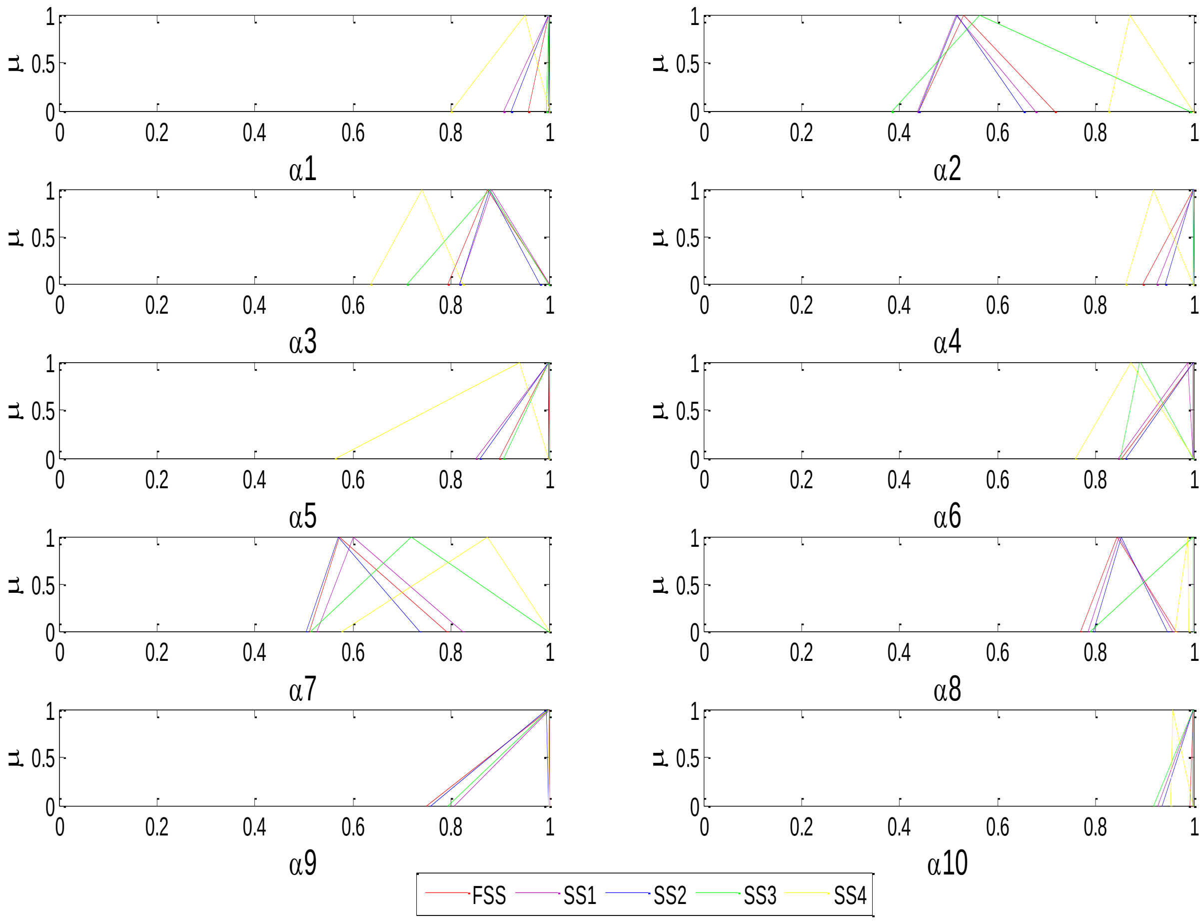

Figure 5 : Updated fuzzy parameters for connection damaged case

The updated fuzzy model parameters and the amount of imprecision are given in Figure 5 and Table 3. As apparent from Figure 5, second and the seventh updating parameter values are reduced roughly half of the initial parameter value. This indicates that the changes in these connections are identified correctly. In addition to the change in the second and seventh parameters, slight changes in some other parameters are also detected. However, these changes arise from both modeling and measurement noise. As apparent from both Table 4 and Fig. 7, the amount of imprecision in SS3 and the SS4 are larger than the other sensor sets. Nevertheless, the minimum amount of uncertainty and RMSE (Root Mean Square Error) are minimum for the SS2, which means that this sensor set provides sufficient information about the model parameters while it makes the uncertainty level minimum.

Sensor set	$RMSE$ (%)	Imprecision in updated parameters
FSS	0.2207	0.8076
SS1	0.2338	0.8168
SS2	0.2148	0.7435
SS3	0.5051	1.0688
SS4	1.0207	0.9381

Table 3 : Performance of the sensor sets

5 CONCLUSIONS

In this study, a methodology, which can be used to determine the optimum sensor configuration for achieving reliable St-Id using structural health monitoring data is presented. The validity of sensor configurations is investigated by considering the uncertainties propagated through the updated parameters. It is shown that the data obtained from sensors should contain enough information in order to keep the uncertainty existing in the model parameters at certain levels. In addition, data should be sufficient such that it can be used to simulate the complete behavior of the structure under different loadings and conditions. It is seen that the sensor sets SS3 and SS4 do not provide enough information about the damaged structure. The amount of uncertainty is also high for these sensor sets compared to the FSS, SS1 and SS2. In addition, it is shown that the linkage values, which indicate the distance between the different clusters could be used as a criterion to determine optimum number of required sensors. Linkage values provide a cut-off point in which the distance between the clusters becomes insignificant. The sensor set which is determined after that cut-off point could be selected as the optimum sensor set as there is no additional information that can be gained from additional sensors installed to the structure.

REFERENCES

[1] A. E. Aktan, F. N. Catbas, A. Turer, Z. Zhang, Structural Identification: Analytical Arts, *J. Struct. Eng.* ASCE, **124**(7) , 817- 829, 1998.

[2] J. M. W. Brownjohn, P. Xia, H. Hao, Y. Xia, Civil structure condition assessment by FE model updating: methodology and case studies, *Finite Elem. Anal. Des.*, **37**, 761-775, 2001.

[3] F. N. Catbas, S. K. Ciloglu, O. Hasancebi, K. A.Grimmelsman, A. E. Aktan, Limitations in Structural Identification of Large Constructed Structures, *J. Struct. Eng.* ASCE, **133**(8), 1051-1066, 2007.

[4] J. E. Mottershead, L. Michael, M. I. Friswell, The sensitivity method in finite element model updating: A tutorial, *Mech. Syst. Signal Process.*, **25**(7), 2275–2296, 2011.

[5] P.G. Bakir, E. Reynders, G. De Roeck, Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm, J. Sound Vib., **305**, 211–225, 2007.

[6] H. Khodaparast, J. Mottershead, M.I. Friswell, Perturbation methods for the estimation of parameter variability in stochastic model updating, *Mech. Syst. Signal Process.* **22** (8), 1751–1773, 2008.

[7] J.L. Beck, S.K. Au, Bayesian Updating of Structural Models and Reliability using Markov Chain Monte Carlo Simulation, *J. Eng. Mech.* **128**(4), 380-391, 2002.

[8] Y.S. Erdogan and P.G. Bakir Inverse propagation of uncertainties in finite element model updating problem through use of fuzzy arithmetic *Eng. Appl. Artif. Intel.*, **26**(1) 357-367, 2013.

[9] D. Degrauwe, G. De Roeck and G. Lombaert, Uncertainty quantification in the damage assessment of a cable-stayed bridge by means of fuzzy numbers, *Compt. Struct.* **87**, 1077–1084, 2009.

[10] Y.R. Nicoud, B. Raphael, I.F.C. Smith, Configuration of measurement systems using Shannon's entropy function, *Compt. Struct.* **83**, 599–612, 2005.

[11] T.H. Yi, H.N. Li, M. Gu, Optimal sensor placement for structural health monitoring based on multiple optimization strategies, *Struct. Design Tall Spec. Build.* **20**, 881–900, 2011.

[12] P.G. Bakir, Evaluation of optimal sensor placement techniques for parameter identification in buildings, *Math. Comput. Appl.* **16**(2), 456-466, 2011.

[13] M. Hanss, *Applied Fuzzy Arithmetic—An Introduction with Engineering Applications*. Springer, Berlin, 2005.

[14] F.N. Catbas, J.M. Caicedo, S.J. Dyke, Development of a Benchmark problem for bridge health monitoring, *Proceedings of the International Conference on Bridge Maintenance, Safety and Management*, IABMAS, Porto, Portugal, 2006.