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A TWO-STAGE TIME-DELAY IDENTIFICATION WITH SUPERVISORY CONTROL OF DYNAMICAL SYSTEMS

Xiang Li[1], **Zhi-Chang Qin**[1], **Shun Zhong**[1] **and Jian-Qiao Sun***[2]

[1]Department of Mechanics, Tianjin University
Tianjin, 300072, China
[2]School of Engineering, University of California
Merced, CA 95343, USA
Corresponding author: J.Q. Sun (jqsun@ucmerced.edu)

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Abstract. *Time delay can cause instability of control systems and is often unknown. The unknown time delay makes the control design a difficult task as well. When the lower and upper bounds of an unknown time delay of dynamical systems are specified, one can design a supervisory control that switches among a set of controls designed for the sampled time delays in the given range so that the closed-loop system is stable and the control performance is maintained at an optimal level. While the supervisory control is designed to stabilize the system and targets at the closed-loop system performance, it also tracks the unknown time delay of the system. This provides the first stage estimate of the time delay. The accuracy of the first stage estimate depends on the behavior of the controllers designed for the sampled time delays as well as the true time delay relative to the sampled time delays.We propose that after the supervisory control converges, a system identification routine can be initiated to identify the unknown time delay nearby the associated time delay with the converged control. This is the second stage of estimation, which gives much more accurate prediction of the unknown time delay. Examples are shown in this work to demonstrate the two-stage identification process for linear time invariant system. The results are interesting and encouraging.*

1 Introduction

Whenever there is uncertainty, switching controls are used to maintain proper performance of dynamical systems. Uncertainty can be associated with the lack of the knowledge of the system leading to the model with uncertain parameters or functional structures. When there is time delay in the system, the delay is often uncertain as well. This paper presents a study of switching control for dynamical systems with uncertain control delay and an identification algorithm for estimating the time delay.

There are many studies of control systems with unknown and time-varying time delays. Chen *et al.* derived sufficient conditions for the existence of the guaranteed cost output-feedback controller in terms of matrix inequalities for uncertain dynamic systems with time delay [1]. A class of iterative learning controls with uncertain state delay and control delay has been studied [2]. The stability problem is treated in the integral quadratic constraint (IQC) framework. Kwon, Park and Lee [3] investigated delay-dependent robust stability for neutral systems with the help of the Lyapunov method. Robust stability of systems with random time-varying delay is studied in reference [4]. Sufficient conditions for the exponential mean square stability of the system are derived by using the Lyapunov functional method and the linear matrix inequality (LMI) technique.

The supervisory control can be applied to the systems with uncertain time delay when the uncertain time delay is bounded with known lower and upper bounds [5]. The supervisory control proposes to use several estimates of uncertain parameters for the system model [6, 7, 8, 9]. For each estimate of the parameter, a control is designed. A supervisor monitors the real-time response of the system, selects a plant model according to a switching criterion and implements the corresponding control. It turns out that the switching index of the supervisory control tracks very well the true time delay in the system, and thus provides a basis to identify the unknown time delay, particularly for unstable systems.

The rest of the paper is outlined as follows. First, we present the methods to formulate the control of linear dynamical systems with known time delay in Section 2. We then present the supervisory control and switching rule in Section 3 and an identification method for estimation of uncertain time delay in Section 4. Section 5 presents numerical examples of switching control and time delay identification of linear dynamical systems with unknown time delay. Section 6 concludes the paper.

2 Control Design with Known Delay

Consider a linear system with a delayed control,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t-\tau), \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ and $\mathbf{u} \in \mathbf{R}^m$. $\mathbf{A} \in \mathbf{R}^{n\times n}$, and $\mathbf{B} \in \mathbf{R}^{n\times m}$. Let us discretize the time delay τ into an integer N intervals of length $\Delta\tau = \tau/N$. From the general solution of the linear system

$$\mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{x}_0 + \int_{t_0}^{t} e^{\mathbf{A}(t-s)}\mathbf{u}(s-\tau)ds. \tag{2}$$

Consider the solution over a small time interval $t \in [t_k, t_{k+1}]$ where $t_k = k\Delta\tau$, $k = 0, 1, 2, \ldots$. Assume that $\mathbf{u}(t-\tau)$ is constant in the small time interval. We can obtain a mapping from Equation (2) as

$$\mathbf{x}(k+1) = \tilde{\mathbf{A}}\mathbf{x}(k) + \tilde{\mathbf{B}}\mathbf{u}(k-N), \tag{3}$$

where $\mathbf{x}(k) = \mathbf{x}(k\Delta\tau)$, $\mathbf{u}(t-\tau_j) = \mathbf{u}(k-j)$ where $\tau_j = j\Delta\tau$ and $j = 1, 2, \cdots, N$.

$$\tilde{\mathbf{A}} = e^{\mathbf{A}\Delta\tau}, \ \ \tilde{\mathbf{B}} = \int_0^{\Delta\tau} e^{\mathbf{A}(\Delta\tau - s)}\mathbf{B}ds. \tag{4}$$

Introduce the extended state $(n + Nm) \times 1$ vector

$$\mathbf{y}(k) = [\mathbf{x}(k), \mathbf{u}(k-N), \mathbf{u}(k-N+1), \cdots, \mathbf{u}(k-1)]^T, \tag{5}$$

The solution in Equation (2) can be written in terms of the following mapping,

$$\mathbf{y}(k+1) = \bar{\mathbf{A}}\mathbf{y}(k) + \bar{\mathbf{B}}\mathbf{u}(k), \tag{6}$$

where

$$\bar{\mathbf{A}} = \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \end{bmatrix}, \ \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix}. \tag{7}$$

Given the mapping in Equation (6), we can design a digital LQR optimal control to minimize a cost function [10],

$$J = \sum_{k=0}^{\infty} \frac{1}{2}\mathbf{y}^T(k)\mathbf{Q}\mathbf{y}(k) + \frac{1}{2}\mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k). \tag{8}$$

where $\mathbf{Q}$ is a $(n+Nm) \times (n+Nm)$ non-negative definite symmetric matrix and $\mathbf{R}$ is a $m \times m$ positive definite symmetric matrix. We obtain the optimal control $\mathbf{u}(k) = -\mathbf{K}_h\mathbf{y}(k)$ and the gain as

$$\mathbf{K}_h = [\mathbf{R} + \bar{\mathbf{B}}^T\mathbf{S}_\infty\bar{\mathbf{B}}]^{-\mathbf{1}}\bar{\mathbf{B}}^T\mathbf{S}_\infty\bar{\mathbf{A}}, \tag{9}$$

where $\mathbf{S}_\infty$ satisfies the algebraic Riccati equation [11],

$$\mathbf{S}_\infty = \bar{\mathbf{A}}^T[\mathbf{S}_\infty - \mathbf{S}_\infty\bar{\mathbf{B}}[\mathbf{R} + \bar{\mathbf{B}}^T\mathbf{S}_\infty\bar{\mathbf{B}}]^{-\mathbf{1}}\bar{\mathbf{B}}^T\mathbf{S}_\infty]\bar{\mathbf{A}} + \mathbf{Q}. \tag{10}$$

Since $\mathbf{K}_h \in \mathbf{R}^{m\times(n+Nm)}$, we refer to this control as the high order control.

Another way to design the feedback control for a given time delay is to make use of the mapping in Equation (3) after a lower dimensional feedback control $\mathbf{u}(k) = -\mathbf{K}_l\mathbf{x}(k)$ is substituted into it [12, 13, 14, 15, 16]. Note that $\mathbf{K}_l \in \mathbf{R}^{m\times n}$. We refer this control as the low order control. Define another extended state $n(1+N) \times 1$ vector

$$\mathbf{z}(k) = [\mathbf{x}(k), \mathbf{x}(k-1), \mathbf{x}(k-2), \cdots, \mathbf{x}(k-N)]^T. \tag{11}$$

We obtain a closed-loop mapping as a function of the feedback gain

$$\mathbf{z}(k+1) = \mathbf{\Psi}(\mathbf{K}_l)\mathbf{z}(k), \ \ k = 1, 2, \cdots \tag{12}$$

Consider a compact and bounded region $\mathbf{\Omega} \subset \mathbf{R}^{m\times n}$ where $\mathbf{K}_l \in \mathbf{\Omega}$. We can find the domains of stability and optimal control gains in the region to minimize the largest magnitude of the eigenvalues of $\mathbf{\Psi}$. This leads to the following optimization problem

$$\min_{\mathbf{K}_l \in \mathbf{\Omega}} [\max|\lambda(\mathbf{\Psi})|] \ \text{subject to } |\lambda|_{\max} < 1. \tag{13}$$

The control performance criterion is the decay rate of the mapping $\mathbf{\Psi}$ over one iteration. In the frequency domain, we have found that the optimal feedback gains maximize the damping of the dominant closed-loop poles of the system that are closest to the imaginary axis of the s-plane [17].

We should note that the optimization problem in Equation (13) is implicit. When the dimension of the gain $\mathbf{K}_l$ is large, the computation to search for the optimal gain can be quite prohibitive.

3 Switching Control with Uncertain Time Delay

Consider the system in Equation (1) again. Assume that the time delay τ is slowly time-varying, and lies in an interval $[\tau_{\min}, \tau_{\max}]$ where the minimum and maximum time delays are assumed to be known. We discretize $[\tau_{\min}, \tau_{\max}]$ into $M_\tau - 1$ intervals so that $\tau_{\min} = \tau^{(1)} < \tau^{(2)} \cdots < \tau^{(M_\tau)} = \tau_{\max}$. For each sampled time delay $\tau^{(i)}$ $(1 \leq i \leq M_\tau)$, we design a control using one of the two methods discussed earlier.

Assume that we have obtained a set of M_τ optimal gains $\mathbf{K}_i$ $(1 \leq i \leq M_\tau)$ such that $\mathbf{u}_i\left(t\right) = -\mathbf{K}_i \mathbf{y}_i\left(t - \tau^{(i)}\right)$ for the high order control or $\mathbf{u}_i\left(t\right) = -\mathbf{K}_i \mathbf{x}_i\left(t - \tau^{(i)}\right)$ for the low order control designed for the set of time delays $\tau^{(i)}$ sampled from the interval $[\tau_{\min}, \tau_{\max}]$. The control $\mathbf{u}_i\left(t\right)$ must be stable for all the time delay $\tau \in [\tau_{\min}, \tau_{\max}]$.

Following the concept of the supervisory control [6, 7, 8, 9], we define an estimation error as

$$\mathbf{e}_i = \mathbf{x}_i\left(t\right) - \mathbf{x}\left(t\right), \quad 1 \leq i \leq M_\tau, \tag{14}$$

where $\mathbf{x}\left(t\right)$ is the output of the system with unknown time delay. In the experiment, $\mathbf{x}\left(t\right)$ would be obtained from measurements. Consider a positive function of the estimation error $F_i(\mathbf{e}_i) > 0$. An example is $F_i(\mathbf{e}_i) = ||\mathbf{e}_i||^2$. Define a switching index $\pi_i(t)$ such that

$$\begin{aligned} \dot{\pi}_i(t) + \lambda_i \pi_i(t) = F_i(\mathbf{e}_i), \quad (\lambda_i > 0) \\ \pi_i(0) = 0, \end{aligned} \tag{15}$$

where the parameter λ_i defines the bandwidth of the low pass filter. The general solution for $\pi_i(t)$ can be obtained as

$$\pi_i(t) = e^{-\lambda_i t}\pi_i(0) + \int_0^t e^{-\lambda_i (t-\tau)} F_i(\mathbf{e}_i(\tau)) d\tau. \tag{16}$$

A switching algorithm to select a proper gain, known as the hysteretic switching rule, has been developed in [8, 9]. Assume that the system is sampled at time interval $\Delta\tau$. At the k^{th} time step, the system is under control with the gain $\mathbf{K}_j$ and the associated switching signal is $\pi_j(k)$. At the $(k+1)^{th}$ step, if there is an index i such that $\pi_i(k) < (1-\eta)\pi_j(k)$ where $\eta > 0$ is a small number, we switch to the gain $\mathbf{K}_i$. Otherwise, we continue with the gain $\mathbf{K}_j$. η is known as the hysteretic parameter and prevents the system from switching too frequently.

In [5], the supervisory control has been studied for linear time-invariant as well as periodic systems with uncertain time delay where the lower order controls have been considered. It has been found that the switching algorithm can lead the controlled system to one with the best performance in terms of the switching index. In the meantime, the associated time delay of the converged control points to the unknown time delay. In the present work, we shall investigate the supervisory control in conjunction with the higher order control for linear time-invariant systems with uncertain time delay, and develop an algorithm to identify the uncertain time delay.

4 Identification of Time Delay

The switching control can stabilize the dynamical system with uncertain control time delay. The switching algorithm is designed to guarantee the index in Equation (16) to decrease and not to estimate the time delay directly. Nevertheless, the switching algorithm does track the time delay approximately. This approximation can be used as a first stage estimation of the time delay.

After the switching control converges or remains unchanged in a period of time, an identification procedure can be started to accurately estimate the unknown time delay in the system. This forms the second stage of the identification. In the second stage of identification, we shall continue to use the index in Equation (16). Note that a similar index to that in Equation (16) is used by Chen and Cai [18] and has provided an effective way to identify the time delay.

Let i_s be the index associated with the converged or steady control. It points to the true underlining time delay in the system. Therefore, we can subdivide the interval $[\tau^{(i_s-1)}, \tau^{(i_s+1)}]$ centered at $\tau^{(i_s)}$, sample a number M_{sub} of time delays in the interval, simulate the system with the same gain to compute the switching index denoted as $\pi_{ID}(t)$ and compare with the index associated with the converged control $\pi_{i_s}(t)$. Search for the sampled time delay such that

$$\min_{1\leq ID\leq M_{sub}} \pi_{ID}(t) \text{ subject to } \pi_{ID}(t) \leq \pi_{i_s}(t). \tag{17}$$

The one that leads to a smallest index is taken as the refined estimate of the unknown time delay. The accuracy of the estimate of the time delay is determined by the grid size of the refined sampling, and can be further improved by repeating the subdivision procedure.

5 Example of Linear Time-invariant System

Consider a linear time-invariant (LTI) system subject to a delayed control

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \dot{\mathbf{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-\tau) \tag{18}$$

Let $\omega_n = 2$, $\zeta = 0.05$. Assume that the delay τ is uncertain. Its nominal value is $\tau_{nom} = \pi/30$. The lower and upper bounds of the uncertain delay are known and are given by $\tau_{\min} = 0.1\tau_{nom}$ and $\tau_{\max} = 9\tau_{nom}$. We sample ten delays within the bounds: $[0.1, 1, 2, 3, 4, 5, 6, 7, 8, 9] \times \tau_{nom}$. We use the LQR method to design the optimal feedback gain. For the rest of the work, we implement the high order control with the gain in Equation (9). The discretization number of the delayed control is taken to be $N = 2^3$. The matrix $\mathbf{Q}$ for the LQR optimal control design is taken to be $Q_{ij} = 0$ for all i and j except for $Q_{11} = \omega_n^2$ and $Q_{22} = 1$, and $\mathbf{R}$, being a scalar in this case, is chosen to be 0.1. The optimal gains associated with the ten sampled delays are listed in Table 1.

Next, we implement the hysteretic switching algorithm for the following cases.

Varying τ

Assume that the time delay changes as a function of time given by

$$\tau(t) = \frac{\pi}{30}(9H(t) - 8.9H(t-1) + 5H(t-5)), \tag{19}$$

where $H(t)$ is the Heaviside function. The variation of the unknown time delay is within the lower $\tau_{\min}$ and upper $\tau_{\max}$ bounds. Let us assume that we randomly start with a control gain $\mathbf{K}_1$ designed for $\tau^{(1)}$. Figure 1 shows the switching signal $\pi(t)$ and the control index. Figure

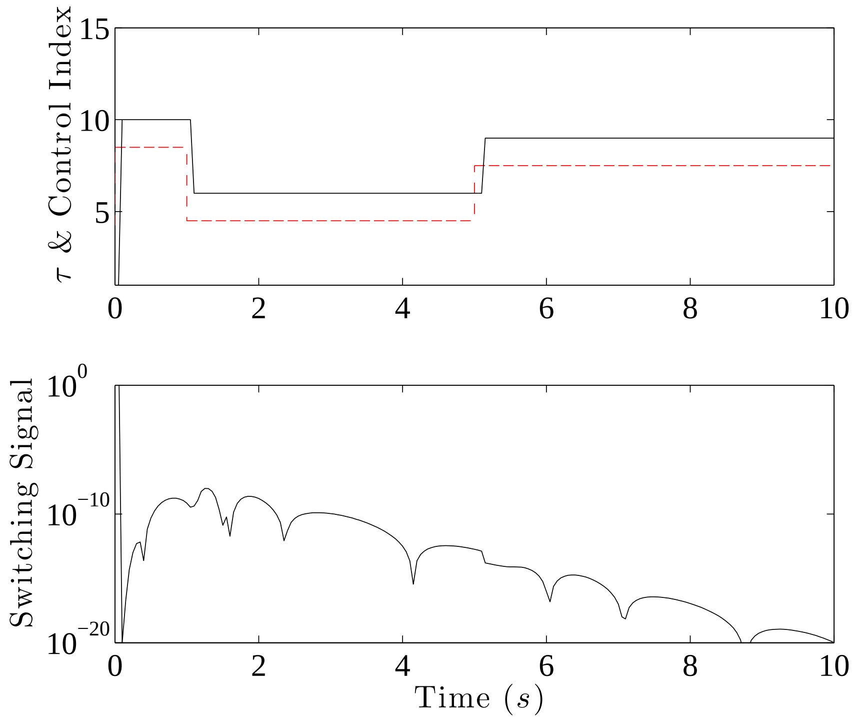

Figure 1: Switching index and switching signal of the LTI system with a time-varying time delay. Top: Solid line: Control index. Dashed line: Time-varying τ. Note that the control index tracks the time delay. Bottom: The switching signal of the high order control.

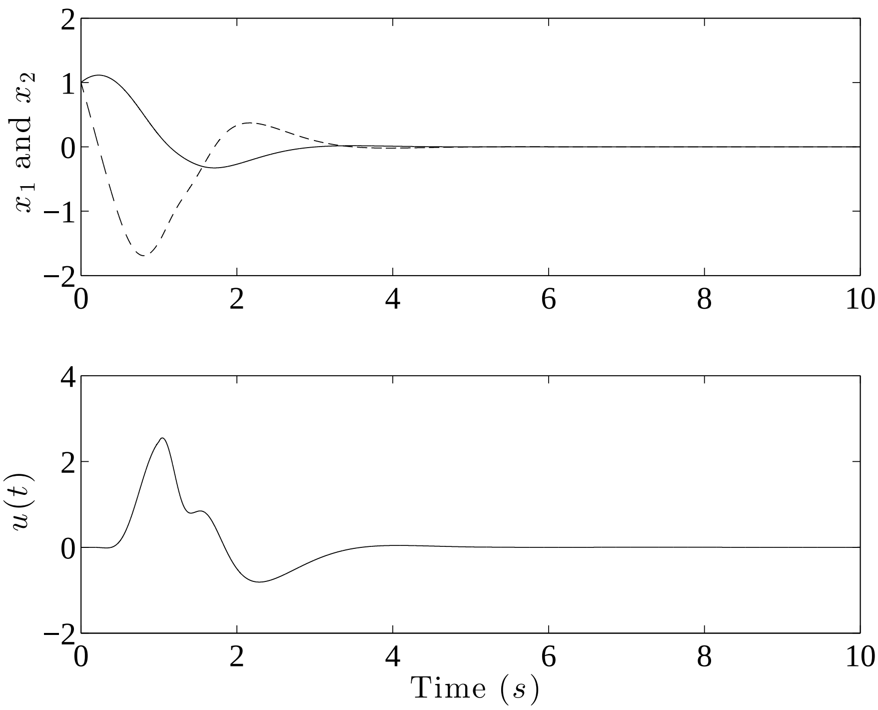

Figure 2: Top: The response x_1 (solid line) and x_2 (dashed line) of the system under the high order switching control. Bottom: The control signal.

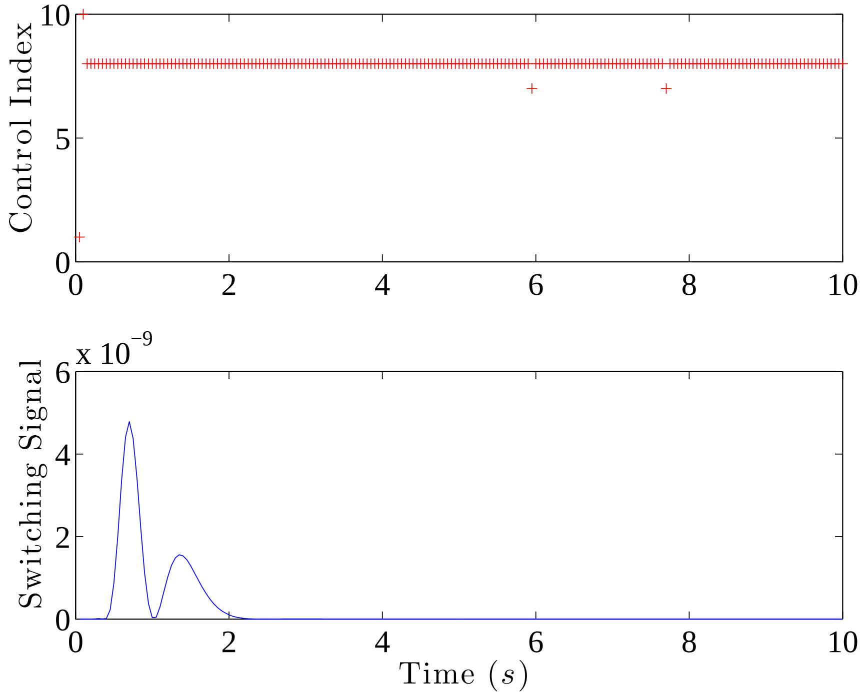

Figure 3: Switching index and switching signal of the LTI system with a constant time delay. Top: Control index. Bottom: Switching signal of the high order control.

2 shows the closed-loop response and control signal. The results indicate that the hysteretic algorithm is able to switch to achieve the performance and tracks the variation of time delay.

Let us now demonstrate the process of identification of uncertain time delay by making use of the tracking capability of the switching control. Assume that the real time delay of the system is constant $\tau_{\text{real}} = 0.6807$. Figure 3 shows the control index converges to 8 when the switching algorithm converges. This indicates that the system has a time delay near $\tau^{(8)}$. We then subdivide the interval $\left[\tau^{(7)}, \tau^{(9)}\right]$ centered at $\tau^{(8)}$ into 5 intervals. The algorithm in Equation (17) leads to the finding of the time delay as 0.6749. The difference between the real time delay and the estimated one is 0.85%. The accuracy is adequate. We have also conducted a study of the effect of subdivisions on the estimate of the time delay. The results are shown in Figure 4. It appears that the convergence of the estimate is reasonably fast.

Forced System

We are interested in the robustness of the switching control with respect to the external disturbances to the system. For this purpose, we assume that the system is forced by a harmonic function $f(t) = \sin(8t)$ as following,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \dot{\mathbf{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-\tau) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \sin(8t). \tag{20}$$

Similarly we start the control with the gain $\mathbf{K}_1$ while the system true delay is $\tau_{\text{real}} = 0.4817$. Figures 5 and 6 show that the switching algorithm can achieve the desired control performance even in the presence of external excitations. In particular, we point out that the switching index converges to 6 suggesting that the time delay is very close to $\tau^{(6)}$ and that this convergence is insensitive to external excitations. This implies that the subdivision identification algorithm for time delay is still applicable to forced systems.

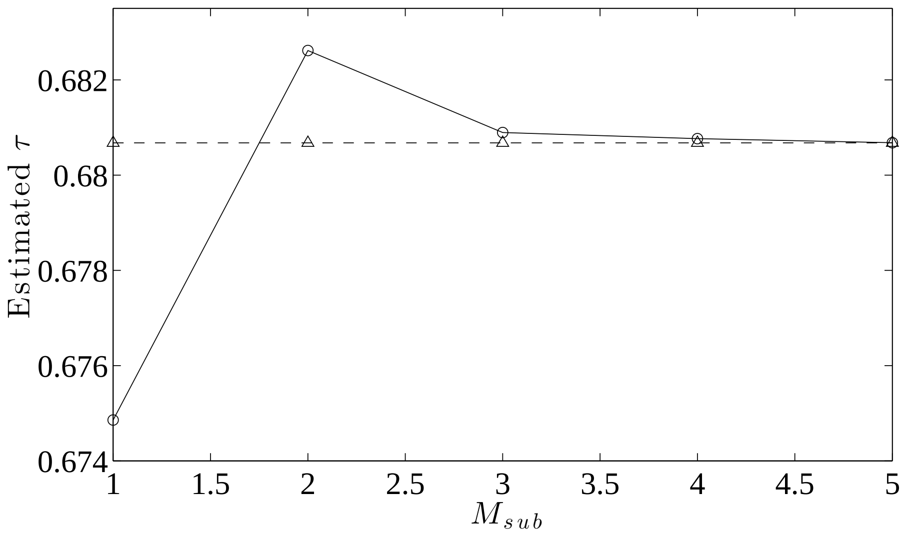

Figure 4: Variation of the estimate of time delay of the LTI system under higher order control with the subdivision level M_{sub}. $\circ$: Estimate of time delay; $\triangle$: The real time delay.

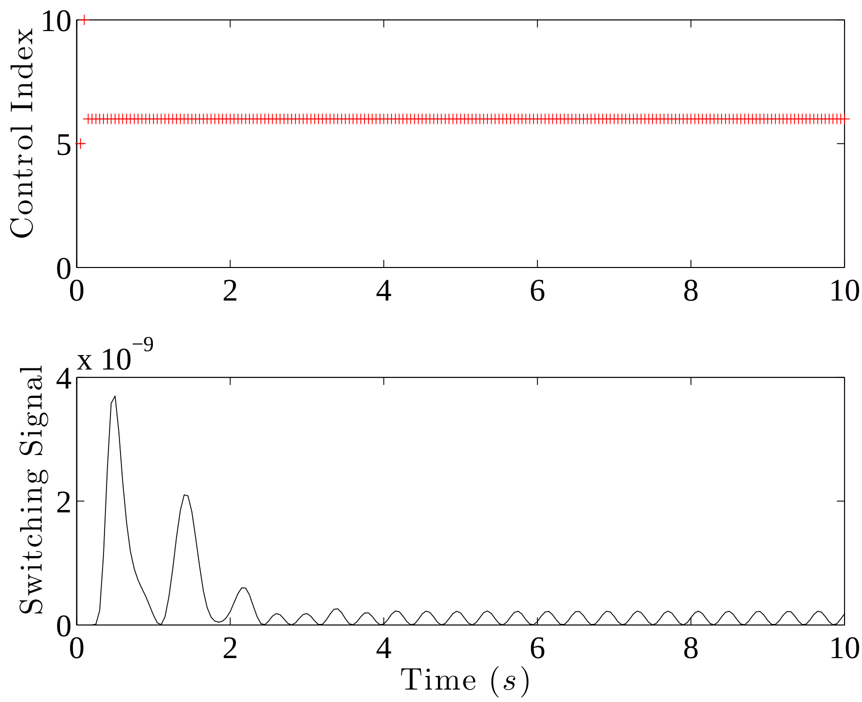

Figure 5: The switching index (top) and switching signal (bottom) of the LTI system under the switching high order control. The system true time delay is $\tau_{\text{real}} = 0.4817$.

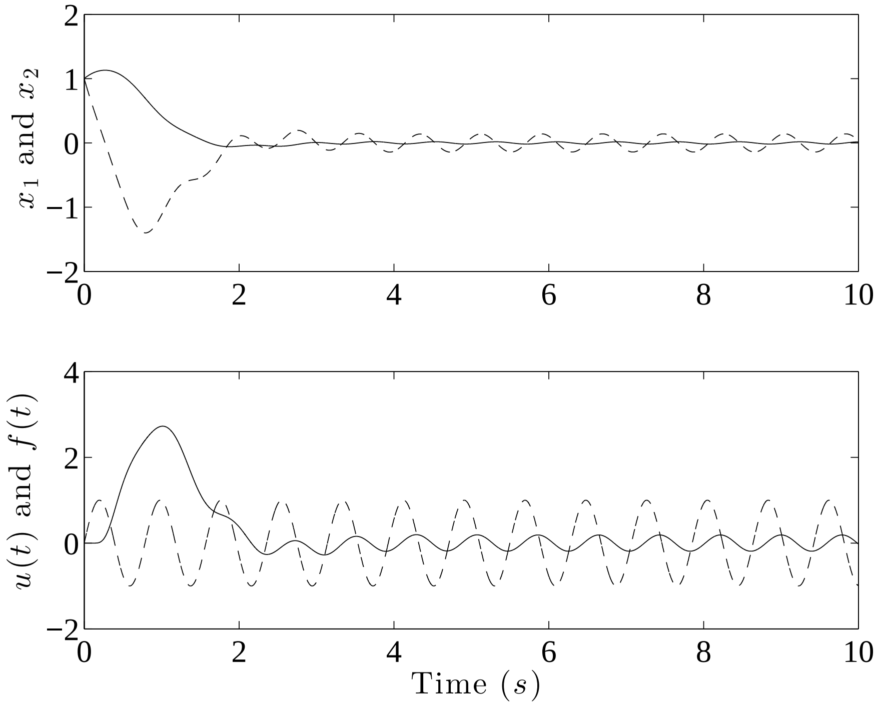

Figure 6: The responses, control signal and forcing term of the LTI system with a constant unknown time delay $\tau_{\rm real} = 0.4817$ under the switching high order control. Top: Solid line x_1; Dashed line x_2. Bottom: Solide line $u(t)$; Dashed line Excitation $f(t)$.

6 Concluding Remarks

We have demonstrated that the hysteretic switching algorithm can deal with feedback control with unknown time delay. When the upper and lower bounds of the unknown control delay are known, the set of feedback controls designed for the sampled time delays within the bounds can stabilize the system. The controls can be designed with the mapping method or the high-order design method. Furthermore, we have demonstrated that the switching index of the supervisory control tracks the uncertain time delay and can be used as a criterion to identify the uncertain time delay in the system. We then develop a subdivision technique to improve the accuracy of estimation of uncertain time delay. Numerical examples have been presented to demonstrate the application of subdivision technique for identification of time delay and the robustness of the switching control and identification algorithm with respect to external disturbance.

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Table 1: The high order control gains for the sampled time delays of the linear time invariant system.

Delay Index	High Order Control Gain **K**
1	$[3.3187, 3.9512, 0.0051, 0.0051, 0.0051, 0.0051, 0.0052, 0.0052, 0.0052, 0.0052]$
2	$[1.8271, 4.1138, 0.0488, 0.0500, 0.0510, 0.0518, 0.0525, 0.0530, 0.0534, 0.0536]$
3	$[0.1888, 4.1302, 0.0928, 0.0971, 0.1008, 0.1037, 0.1058, 0.1072, 0.1080, 0.1082]$
4	$[-1.3761, 3.9919, 0.1327, 0.1415, 0.1488, 0.1544, 0.1581, 0.1600, 0.1603, 0.1591]$
5	$[-2.8221, 3.7189, 0.1688, 0.1830, 0.1947, 0.2031, 0.2080, 0.2093, 0.2073, 0.2023]$
6	$[-4.1126, 3.3318, 0.2015, 0.2218, 0.2380, 0.2490, 0.2542, 0.2533, 0.2466, 0.2346]$
7	$[-5.2194, 2.8517, 0.2311, 0.2578, 0.2786, 0.2917, 0.2958, 0.2907, 0.2763, 0.2537]$
8	$[-6.1217, 2.2999, 0.2579, 0.2911, 0.3162, 0.3305, 0.3322, 0.3203, 0.2951, 0.2580]$
9	$[-6.8059, 1.6976, 0.2819, 0.3217, 0.3508, 0.3653, 0.3627, 0.3414, 0.3020, 0.2465]$
10	$[-7.2651, 1.0654, 0.3036, 0.3496, 0.3821, 0.3958, 0.3869, 0.3536, 0.2966, 0.2190]$