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INFLUENCE OF CORRELATION LENGTH IN OPTIMAL SENSOR PLACEMENT

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Abstract. *Modern technologies for structural safety, such as active or semi-active protection systems towards seismic events and damage detection identification procedures, require control systems to monitor the structural behaviour during the whole operating life. Given a number of sensors, reliability of those sistems and the quality of the obtained information strongly depend on the sensor placement.*

In this paper, a comparison of results obtained by optimal sensor placement from different definitions of the correlation length is presented. The sensor placement procedure is based on the Information Entropy that depends on the modal matrix and on the prediction error of the signal acquired by two sensors in hypothetical locations. The solution of the optimisation problem is obtained maximizing the determinant of the so called Fisher Information Matrix (FIM). Unfortunately, the Fisher matrix determinant is zero if the number of sensor is smaller than the number of modes to identify. For this reason and for numerical conveniences, in this work the Fisher matrix is decomposed by the Cholesky factorization and the Singular Value Decomposition (SVD) is applied to obtain a stable estimation of the Fisher matrix determinant. The correlation length is introduced in the optimization procedure to control the relative distance between sensors and to avoid concentration of sensors in limited parts of the structure. The methodology is presented and applied to benchmark case studies in the field of the modal identification: a simply supported beam and a cantilever beam. The comparison between results obtained considering or neglecting the correlation length is also described. Moreover, an extension to spatial structures is also defined and results on a 3D-frame non-symmetrical in plan are reported.

1 INTRODUCTION

Modern technologies for structural safety, such as active or semi-active protection systems towards seismic events and damage detection identification procedures, require control systems to monitor the structural behaviour during the whole operating life. These technologies are based on the development of new numerical techniques of structural identification and on the adoption of increasingly reliable sensors. However the quality of the obtained information also depends on the number of sensors and on their placement. Usually, several alternative positions can be selected, although economic constraints and spatial restrictions tend to limit the set-up layout. Thus, it is necessary to optimize the position of a limited number of sensors, in order to obtain the greater amount of information from the measured data and to assure a reliable evaluation of the parameters of interest. In the past, the optimal sensor placement problem was solved through practical considerations, i.e. placing sensors as near as possible to points where maximum displacements were expected [1]. In [2] a priori analysis was performed; the proposed optimal sensor placement approach was based on the difference between modal information acquired with or without the contribution of sensor (for instance, the mode shape component in modal identification is estimated by means of a linear interpolation). An optimisation procedure is then performed to minimize the distance between the measured and reconstructed data for a given sensor number. However, the same sensor configuration have to identify more than one mode shape so that the solution is reached solving a combinatorial optimisation problem. The Effective Independence (EFI) method [3, 4], was developed for spatial structures on orbit. It determines the position of each candidate maximizing the Fisher Information Matrix (FIM) determinant. In [5] it is shown that maximize the FIM determinant means to maximize the mode shape signal strength and spatial independence. Different methods and variants based on the EFI method were proposed; for instance, the EFI-DPR (Driving Point Residue) method was developed to identify the damage in mechanical elements and the solution is reached using a combined Genetic Algorithm with Simulated Annealing [3, 6]. The Kinetic Energy Method (KEM) is also similar to EFI, but it maximize a kinetic energy measure of the structure; however, it is shown that it should be brought back to the Fisher matrix determinant maximization [3]. Finally, other energy-driven methods are the Eigenvalue Vector Product (EVP) [7] and the Non-Optimal Driving Point (NODP) [8, 9].

The Information Entropy is introduced in [10] and it is used as a performance measure of the sensor configuration. The method generalizes several other sensor placement procedures and it is shown efficient and robust. For this reason, the Information Entropy is briefly illustrated in the next paragraphs and it is used in analyses described in the following. The optimal sensor placement is formulated as an optimisation problem solved using, for example, Genetic Algorithm or Sequential Sensor Placement procedures. If the goal of the monitoring process is the modal identification, the Information Entropy only depends on modal matrix and on the prediction error of a signal acquired by two sensors in hypothetical locations. A correlation length of the prediction error can be also introduced to control the relative distance between sensors and to avoid concentration of sensors in limited parts of the structure. Results of optimisation process considerably depend on the definition of these parameters and on their evaluation. In [10] an exponential correlation function is assumed. However, the evaluation of the signal correlation and of the procedure applied in 3D structures need to be improved. So, this work proposes an alternative method to evaluate the “correlation length” and it is applied to different benchmark cases in the field of the modal identification: a simply supported beam and a cantilever beam. Moreover, results of the procedure applied in a 3D-frame non-symmetrical in plan are reported.

2 INFORMATION ENTROPY

For a n Degree-Of-Freedom (DOF) linear vibratory system, the measured response $\mathbf{y}(t)$, depending from the time t, is related to the model response $\mathbf{x}(t, \boldsymbol{\theta})$ by the observation equation

$$\mathbf{y}(t) = \mathbf{L}\mathbf{x}(t, \boldsymbol{\theta}) + \mathbf{e}(t, \boldsymbol{\theta}). \tag{1}$$

where the vector $\mathbf{x}(t, \boldsymbol{\theta})$ is the response displacement, $\mathbf{e}(t, \boldsymbol{\theta})$ is the prediction error due to the model error and the measurement noise, and $\mathbf{L}$ is the observation matrix, composed of zeros and ones, that specifies which DOFs of the system are measured. Moreover, $\boldsymbol{\theta}$ are parameters characterizing the dynamic structural behaviour [11]. According to the Bayesian theory [10], the Probability Density Function $p(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D)$ of parameters $\boldsymbol{\theta}$, obtained by the data D measured on the system, is

$$p(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D) = c \, \frac{1}{\sqrt{2\pi}^N \sqrt{\det \boldsymbol{\Sigma}_t}} \, \exp\left[\frac{NN_0}{2}\mathbf{J}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D)\right] \tag{2}$$

where $\pi(\boldsymbol{\theta})$ is a priori distribution of the parameter set $\boldsymbol{\theta}$, c is a normalizing constant chosen such that the PDF function in Eq.2 integrates to one, and $\boldsymbol{\Sigma}_t(\boldsymbol{\theta})$ is the covariance matrix of the prediction error $\mathbf{e}(t, \boldsymbol{\theta})$. $\mathbf{J}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D)$ is the Fisher Information Matrix (FIM) [12] and it represents the distance between the measured data and the model response

$$\mathbf{J}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D) = \frac{1}{NN_0} \sum_{k=1}^{N} [\mathbf{y}_k - \mathbf{L}\mathbf{x}_k(\boldsymbol{\theta})]^T \, \boldsymbol{\Sigma}_t^{-1} \, [\mathbf{y}_k - \mathbf{L}\mathbf{x}_k(\boldsymbol{\theta})]. \tag{3}$$

N is the number of sensors and N_0 is the time interval length. Since the PDF is a measure of the uncertainty in parameter values, the Information Entropy

$$\mathbf{h}(\mathbf{L}, \boldsymbol{\Sigma}_t, D) = E_\theta[-\ln(p(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D))] = -\int \ln(p(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D)) \, p(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D) \, d\boldsymbol{\theta} \tag{4}$$

provides a unique scalar measure of the uncertainty in the evaluation of structural parameters. In Eq. 4 E_θ denotes the expected value of $p(\boldsymbol{\theta}, \boldsymbol{\Sigma}_t \mid D)$ with respect to $\boldsymbol{\theta}$. An asymptotic approximation of $\mathbf{h}(\mathbf{L}, \boldsymbol{\Sigma}_t, D)$ is available for a large number of acquired data, i.e. for $N \cdot N_0 \rightarrow \infty$ [13]

$$\mathbf{h}(\mathbf{L}, \boldsymbol{\Sigma}_t, D) \approx \mathbf{h}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}) = \frac{1}{2} N_\theta \ln(2\pi) - \frac{1}{2} \ln[\det(\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}))]. \tag{5}$$

where $\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta})$ is the Fisher matrix and its asymptotic approximation is

$$\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}) \approx \sum_{k=1}^{N} [\mathbf{L}\nabla_\theta \mathbf{x}_k(\boldsymbol{\theta})]^T \left[\mathbf{L}\boldsymbol{\Sigma}_t\mathbf{L}^T\right]^{-1} [\mathbf{L}\nabla_\theta \mathbf{x}_k(\boldsymbol{\theta})] \tag{6}$$

in which $\nabla_\theta = \left[\frac{\partial}{\partial\theta_1}, \cdots, \frac{\partial}{\partial\theta_{N_\theta}}\right]$ is the gradient operator. The optimal sensor configuration is selected as the one that minimizes the Information Entropy since it gives a direct measure of uncertainty in the model parameter estimate. From Eq. 5, it is easily shown that minimize the Information Entropy corresponds to maximize the determinant of the Fisher Matrix

$$\mathbf{L}_{best} = \underset{\mathrm{L}}{\operatorname{argmin}}[\mathbf{h}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta})] = \underset{\mathrm{L}}{\operatorname{argmax}}[\det(\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}))]. \tag{7}$$

If the aim of the structural monitoring is the modal identification, the objective of the optimal sensor procedure is to find the configuration that provides for the most information on modal coordinate vector $\boldsymbol{\xi}(t)$. The model parameter vector thus corresponds to the modal coordinate vector, i.e. $\boldsymbol{\theta} = \boldsymbol{\xi}(t)$. Using the classic coordinate transformations, the structural response vector is related to the modal coordinate vector by the mode shape matrix $\boldsymbol{\Phi}$

$$\mathbf{x} = \boldsymbol{\Phi}\boldsymbol{\xi}(t) = \boldsymbol{\Phi}\boldsymbol{\theta}. \tag{8}$$

Considering that $\nabla_\theta \mathbf{x} = \boldsymbol{\Phi}$ and substituting into Eq. 5, the Fisher Information matrix takes the form

$$\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}) = [\mathbf{L}\boldsymbol{\Phi}]^T \left[\mathbf{L}\boldsymbol{\Sigma}_t\mathbf{L}^T\right]^{-1} [\mathbf{L}\boldsymbol{\Phi}] \tag{9}$$

which is independent to the modal coordinates $\boldsymbol{\xi}(t)$ but it is related with the sensor positions by the observation matrix $\mathbf{L}$ and with the mode shape matrix $\boldsymbol{\Phi}$.

2.1 Numerical procedure to solve the optimisation procedure

The sensor location estimate exhibits indeterminate solution in the case of a limited number of sensors: the $\det(\mathbf{Q})$ is zero for all sensor configuration if the sensor number N_0 is smaller than the contributing mode number N_M. To avoid the aforementioned indetermination, some numerical procedure can be adopted. Since every matrix is similar to a diagonal matrix having the eigenvalues on the diagonal, and the determinant of a diagonal matrix is simply the product of diagonal elements, [13] proposes to solve the problem maximizing only the non-zero eigenvalue product of the FIM, instead of maximizing the product of all eigenvalues. However, for numerical approximations in the diagonalization procedure and in evaluation of the term $\left[\mathbf{L}\boldsymbol{\Sigma}_t\mathbf{L}^T\right]^{-1}$, some near-zero values are taken into account or neglected depending on the imposed threshold value, giving wide fluctuations to result. In this work, a Cholesky factorization of $\left[\mathbf{L}\boldsymbol{\Sigma}_t\mathbf{L}^T\right]^{-1}$, is first performed

$$\left[\mathbf{L}\boldsymbol{\Sigma}_t\mathbf{L}^T\right]^{-1} = \mathbf{C}\mathbf{C}^T \tag{10}$$

so obtaining

$$\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}) = [\mathbf{L}\boldsymbol{\Phi}]^T\mathbf{C}\mathbf{C}^T[\mathbf{L}\boldsymbol{\Phi}]. \tag{11}$$

Then, a Singular Value Decomposition of $\mathbf{A} = \mathbf{C}^T[\mathbf{L}\boldsymbol{\Phi}]$ is calculated since the square of singular values $\boldsymbol{\sigma}(\mathbf{A})_k$ are equals to non-zero eigenvalues of $\mathbf{Q}$ matrix. The proposed procedure has a lower computational effort and gives more stable results instead of considering non-zero eigenvalues product. Now, it is possible to determine the best configuration as indicated in Eq. 7 as

$$\mathbf{L}_{best} = \underset{\mathrm{L}}{\mathrm{argmax}}[\det(\mathbf{Q}(\mathbf{L}, \boldsymbol{\Sigma}_t, \boldsymbol{\theta}))] = \underset{\mathrm{L}}{\mathrm{argmax}}[\prod_k \sigma(\mathbf{A})_k]. \tag{12}$$

Different algorithms are available to find the configuration that solves the maximization problem defined in Eq. 12, as Genetic Algorithm [14] and Sequential Sensor Positioning procedures (SSP) [10]. An SSP algorithm subtype is the FSSP (Forward Sequential Sensor Positioning) procedure that puts on the structure one sensor at time evaluating which position, for each subsequent sensor, satisfies Eq.12. For details about the algorithm see [13]. The computational effort involved in the FSSP procedure is much more smaller than the ones involved for a direct search method, especially when a large amount of sensors is considered. The sensor configuration computed by the FSSP algorithm cannot be guaranteed to be the optimal, but numerical applications show that results coincide with, or are very close to the exact solution [10]. For this reason, in benchmark cases described in the following, the latter procedure is preferred.

2.2 Covariance of prediction error and correlation length

The definition of the covariance $\mathbf{\Sigma}_t(\boldsymbol{\theta})$ of the prediction error $\mathbf{e}(t, \boldsymbol{\theta})$ is necessary for the optimal sensor configuration settlement. The prediction error depends on a term $\mathbf{e}_{meas}$ counting the measurement error, usually sensor position independent, and on a term $\mathbf{e}_{model}$ that considers the model error. Assuming the independence between these two terms, the covariance $\mathbf{\Sigma}_t(\boldsymbol{\theta})$ is given by

$$\mathbf{\Sigma}_t = \overline{\mathbf{\Sigma}} + \mathbf{\Sigma} \tag{13}$$

where $\overline{\mathbf{\Sigma}}$ and $\mathbf{\Sigma}$ are the measurement error and model error covariance matrix, respectively. Due to the measurement error independence from the sensor position, only the term $\mathbf{\Sigma}$ can be taken into account. The model error covariance matrix generally depends on the distance between sensors; indeed, the prediction error of the structural response has a correlation between the measure acquired in neighboring points. In [13] an exponential correlation function for DOFs i and j is assumed as

$$\Sigma_{ij} = \sqrt{\Sigma_{ii}\Sigma_{jj}}\ \exp[-\delta_{ij}/\lambda] \tag{14}$$

where δ_{ij} is the spatial distance between the DOFs i and j, and λ is a measure of the spatial correlation length. The auto-correlation terms $\Sigma_{ii}\,\Sigma_{jj}$ are assumed equal to one. Elements of the model error covariance matrix vary between one in the case of perfect spatial correlation and tend to zero when the distance between sensors increases. In [13] the role of correlation length is investigated, considering λ as constant values. In this paper, a new definition of the term $1/\lambda$ is proposed, considering the inverse of the correlation length between sensors i and j as the norm between modal vector computed in the i-th and j-th positions for each N_M contributing modes

$$\frac{1}{\lambda_{ij}} = |\boldsymbol{\varphi}_i - \boldsymbol{\varphi}_j|, \tag{15}$$

combining the distance between sensors with the information given by mode shapes.

3 BENCHMARK CASE-STUDIES

Benchmark structures have been considered and the Information Entropy approach has been applied with a FSSP procedure. Results obtained by both constant and mode-dependent correlation length are compared to those obtained by neglecting the influence of the correlation length, i.e. assuming $\lambda = 0$ and, thus, considering Σ as an identity matrix. First, the procedure applied to a beam with free-free boundary conditions is described to better understand the role of the correlation length. Then, the FSSP algorithm is used to find the optimal sensor placement for a simply supported beam and for a cantilever beam.

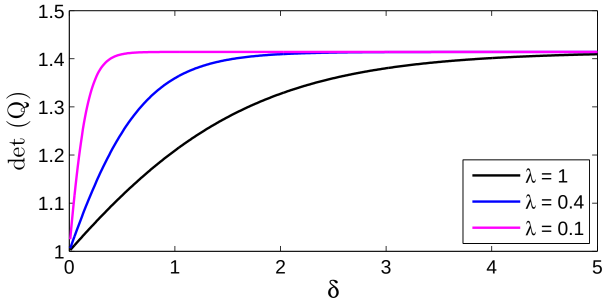


Figure 1: Influence of λ and δ on det$\mathbf{Q}$.

	Case A	Case B	Case C
cross correlation Σ_{ij}	0	$\sqrt{\Sigma_{ii}\Sigma_{jj}}\ \exp[-\delta_{ij}/\lambda_{ij}]$	$\sqrt{\Sigma_{ii}\Sigma_{jj}}\ \exp[-\delta_{ij}/\lambda_{ij}]$
auto correlation Σ_{ii}	1	1	1
correlation length λ_{ij}	–	0.4	$1/\lvert\boldsymbol{\varphi}_i - \boldsymbol{\varphi}_j\rvert$

Table 1: Correlation length components for method A, B and C.

3.1 Beam with free-free boundary conditions

First, consider a beam with free-free boundary conditions. Two sensors are positioned and only the first mode shape is taken into account. Giving the position of the first sensor, for example at the left end, the goal is to assign the position of the second sensor. The first mode shape is a rigid-body motion of the beam and the modal matrix $\mathbf{\Phi}$ becomes vector $\boldsymbol{\varphi}_1$ of constant values (set to one for the sake of simpleness). The second sensor best position will be determined maximizing the Fisher matrix determinant in Eq. 9. In this simple case, the value of $\det(\mathbf{Q})$ can be easily calculated

$$\det(\mathbf{Q}) = 2\frac{1+\exp(-\delta/\lambda)}{1-\exp(-2\delta/\lambda)}. \tag{16}$$

If the distance between sensors δ increases, the value of $\det(\mathbf{Q})$ in Eq. 16 increases. As expected, the second sensor tends to be placed as far as possible to the first one. Moreover, if λ increases, $\det(\mathbf{Q})$ becomes more sensitive with respect to the distance, as shown in Fig. 1.

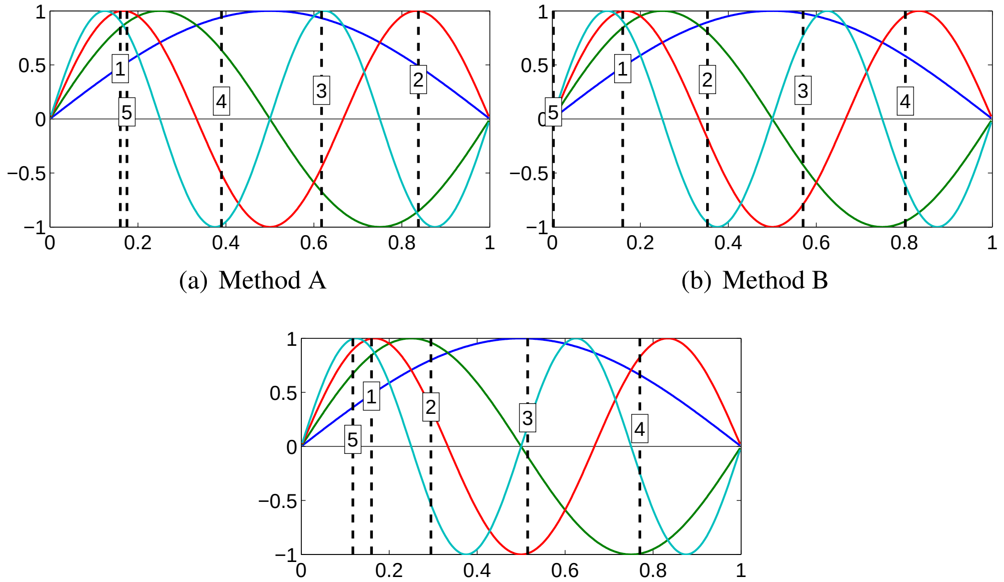
3.2 Sensor placement in a simply-supported beam and in a cantilever beam

Optimal sensor placement problem is first applied to a simply-supported beam with length $h = 1$. Modal shape matrix components has the well known form

$$\boldsymbol{\varphi}_n = \sin\left(\frac{n\pi\mathbf{x}}{h}\right) \tag{17}$$

where n denotes the mode number. A total of N=5 contributing modes is selected. $\mathbf{x}$ is the vector containing all possible sensor positions.

The purpose is to compare results using the 2 correlation length definitions explained above with those obtained neglecting the correlation contribution. Differences among formulations are summarized in Tab. 1 where the capital letters A, B, C denote respectively the method



(a) Method A (b) Method B (c) Method C

Figure 2: Sensors best position in simply supported beam for 4 mode shapes, numbers indicate the sensor order.

	sensors order				
	1st	2nd	3rd	4th	5th
Method A	0.1600	0.8375	0.6175	0.3900	0.1750
Method B	0.1600	0.3525	0.5700	0.8025	0.0025
Method C	0.1600	0.2950	0.5150	0.7700	0.1175

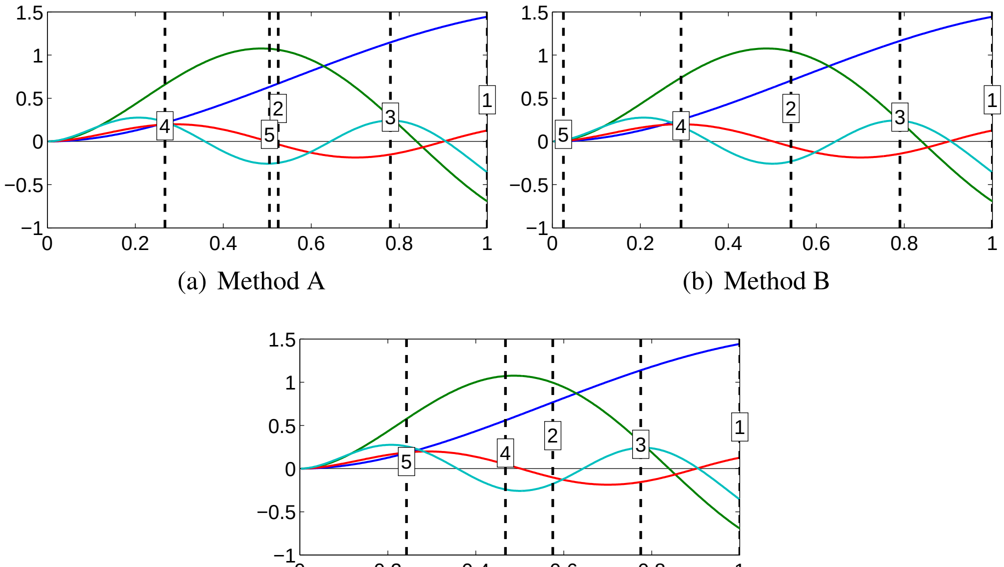
Table 2: Comparison among methods A, B and C in a simply supported beam.

considering Σ as identity matrix, the method where λ is a constant (i.e. $\lambda = 0.4$) and the proposed definition where λ is set as defined in Eq. 15.

Considering 5-sensor monitoring system to keep the first 4 modes of interest, the FSSP procedure has been applied to maximize the singular value product of the Fisher matrix. Results are shown in Fig. 2; sensors are placed in the obtained order as indicated in Tab. 2.

All methods tend to define the first 4 positions to permit the identifiability of all the 4 modes. With exception to some small differences, the obtained positions are almost coincident. The fifth sensor is placed very close to the first one in case A; this means that no more information are needed because of the mode shapes are perfectly defined by the 4 sensors so the new sensor is placed in a position almost close to the previous ones. Also next sensors (not given in figure) follow the same criteria obtaining a crowded concentration in only 4 positions. Guided by the aim to well separate the position of each sensor, method B assigns the fifth position close to the left end (x_5=0.0025). No useful information about mode shapes is substantially added. Method C defines the new position close to the maximum value for each mode of interest and the imposed correlation appears to improve the results obtained for method A, spacing better the sensors.

Similar results are obtained for a cantilever beam (see Fig. 3). Mode shape matrix for each



(a) Method A (b) Method B

(c) Method C

Figure 3: Sensors best position in cantilever beam for 4 mode shapes, numbers indicate the sensor order.

	sensors order				
	1st	2nd	3rd	4th	5th
Method A	1.0000	0.5250	0.7800	0.2675	0.5050
Method B	1.0000	0.5425	0.7900	0.2925	0.0250
Method C	1.0000	0.5750	0.7750	0.4675	0.2425

Table 3: Comparison among methods A, B and C in a cantilever beam.

n contributing mode is given by the relation

$$\boldsymbol{\varphi}_n = \sin\left(b_n \frac{\mathbf{x}}{h}\right) - \sinh\left(b_n \frac{\mathbf{x}}{h}\right) \alpha_n \left[\cos\left(b_n \frac{\mathbf{x}}{h}\right) - \cosh\left(b_n \frac{\mathbf{x}}{h}\right)\right] \tag{18}$$

where b_n is a constant depending on the mode shape, and

$$\alpha_n = (\sin b_n + \sinh b_n)/(\cos b_n + \cosh b_n). \tag{19}$$

To define a hierarchy of importance, mode shape vectors are normalized with modal participating factor. The three methods previously described are then compared, considering four modes of interest and five sensors. The placement order is reported in Tab. 3.

As shown in Fig. 3, the first sensor is placed at the free end of the beam. Then, sensors are located in similar position and differences are given only for sensor no. 5. As for the simply supported beam, method C is a good compromise between the need of distributing sensors in the whole beam length and to have new information about the mode shapes.

4 CORRELATION LENGTH IN SPATIAL STRUCTURES

Usually the prediction error has been considered uncorrelated in two different directions. However the measurements in x and y directions are correlated because of coupled torsional-flexural mode shapes in non symmetrical spatial structures. In this work the correlation length

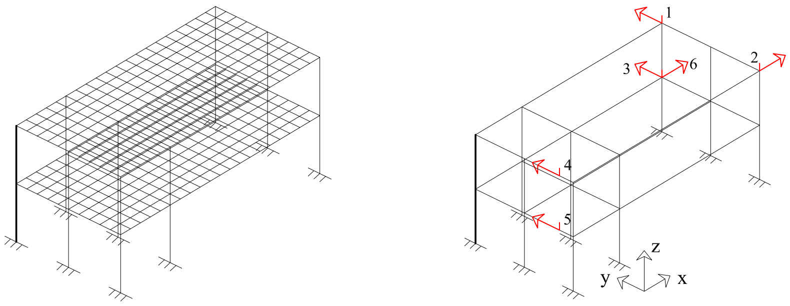


Figure 4: Sensors position in the 2-floor 3D frame.

in Eq. 15 is extended considering the following relation

$$\begin{aligned} \lambda_{i_x j_x} &= 1/|\boldsymbol{\varphi}_{ix} - \boldsymbol{\varphi}_{jx}| \quad \lambda_{i_y j_x} = 1/|\boldsymbol{\varphi}_{iy} - \boldsymbol{\varphi}_{jx}| \\ \lambda_{i_x j_y} &= 1/|\boldsymbol{\varphi}_{ix} - \boldsymbol{\varphi}_{jy}| \quad \lambda_{i_y j_y} = 1/|\boldsymbol{\varphi}_{iy} - \boldsymbol{\varphi}_{jy}| \end{aligned} \tag{20}$$

where subscripts i, j indicate hypothetical sensor positions and x, y indicate the measurement direction for all the N_M contributing modes.

4.1 Spatial frame non-symmetric in plan

To test the reliability of the proposed method, a 2 floor spatial frame is finally considered. The frame is non-symmetric in plan due to unsymmetrical distribution of stiffness in both x and y directions. Columns are $300 \times 300\text{mm}^2$ with exception to one that has a $600 \times 600\text{mm}^2$ cross section, see Fig. 4. Considering a rigid diaphragm at each floor, modal matrix is wrote considering displacements of centroids for all the six modes of interest. Mode shapes are normalized by an “importance factor”, defined as the norm of the modal participating factor in direction x, y and rotation. Floor decks are subdivided in 20×10 possible sensor positions, see Fig. 4, and the case C as described in section 3.2 is applied to determine the best 6 sensor positions. Results obtained are shown in Fig. 4 in which sensors are indicated as an increasingly number to define the obtained placement order.

It can be noted that sensors are placed in both x and y directions and in both levels. The distribution of sensors is able to get the rigid motion of each floor: 2 sensor are placed in y direction and one in the x direction. It is worth noting that only one sensor for level is placed in x direction due to the smaller displacements with respect to the y direction. Finally, to better estimate the floor rotation, all sensors are placed on edges, avoiding inner positions.

5 CONCLUDING REMARKS

The problem of optimal sensor placement has been investigated with reference to the Information Entropy approach. First of all, the problem of computing the Fisher Matrix determinant when sensors are limited to a number less than interest modes has been solved. A stable procedure is used, based on a Singular Value Decomposition and on a Cholesky factorization. Afterwards the correlation length has been considered, to connect model error carried out by sensors. Different procedures have been applied to benchmark structures. As a matter of facts, sensors are all placed in groups near maximal displacement points if the correlation contribution is not considered. However fixing a constant parameter in the correlation length definition, the sensors are placed throughout the structure, also in near-zero displacement points. A proposal concerning a new definition of the correlation length is described, taking into account the mode matrix contribution. Results show that in this way grouping sensors and near-zero point placement are avoided finding a good compromise between the need of distributing sensors in the whole beam and to have new information about the mode shapes. The modified method is also verified and applied to a 3D frame.

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