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SUPPRESSING CHAOS IN A DOUBLE-WELL OSCILLATOR WITH LIMITED POWER SUPPLY USING ELECTROMECHANICAL DAMPED DEVICE

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Abstract. *In this paper, we analyzed chaotic dynamics of an electromechanical damped Duffing oscillator coupled to a rotor. The electromechanical damped device or electromechanical vibration absorber consists of an electrical system coupled magnetically to a mechanical structure (represented by the Duffing oscillator), and that works by transferring the vibration energy of the mechanical system to the electrical system. A Duffing oscillator with double-well potential is considered. Numerical simulations results are presented to demonstrate the effectiveness of the electromechanical vibration absorber. Lyapunov exponents are numerically calculated to prove the occurrence of a chaotic vibration in the non-ideal system and the suppressing of chaotic vibration in the system using the electromechanical damped device.*

1 INTRODUCTION

For the symmetric Duffing oscillator, the potential function can be expressed as follows [1]:

$$V(x) = \frac{a}{2}x^2 + \frac{b}{4}x^4 \tag{1}$$

where a and b are constants. Here we consider only the bounded case with positive b $(b > 0)$. Then, depending on the sign of a, the potential function becomes one of two different types: double-well potential for a $(a < 0)$ or single-well potential for a $(a > 0)$ For the Duffing oscillator with a double-well potential there are two stable equilibrium point at $x = \pm\sqrt{-a/b}$ and one unstable equilibrium point at $x = 0$. On the contrary, the Duffing oscillator with a single well has only a stable equilibrium point at $x = 0$.

The Duffing equation with a double-well potential (with a negative linear stiffness) is an important model. One physical realization of such a Duffing oscillator model is a mass particle moving in a symmetric double well potential. This form of the equation also appears in the transverse vibrations of a beam when the transverse and longitudal deflections are coupled [2]. The Duffing equation with negative linear stiffness also describes the dynamics of a buckled beam as well as a plasma oscillator [3].

The damped and forced double-well Duffing equation has been a subject of intensive study over the last few decades as a landmark chaotic system. Venkatesan and Lakshmanan [4] showed numerically and analytically the existence of bifurcations and chaos in a double-well Duffing oscillator. The stability and bifurcation of a van der Pol-Duffing oscillator with the delay feedback are investigated by Suqi et al. [5]. New methods have been used to suppress chaos by various authors, which considered the double-well Duffing equation [6, 7, 8]. These methods have been applied for systems whose energy sources are described by a harmonic function. However, in several mechanical experiments the oscillator cannot be driven by systems whose amplitude and frequency are arbitrarily chosen, since the forcing source has a limited available energy supply. Such energy sources have been called non-ideal, and the corresponding system a non-ideal oscillator. For this kind of oscillator, the driven system cannot be considered as given a priori, but it must be taken as a consequence of the dynamics of the whole system (oscillator and motor). Souza et al. [9] proposed a simple feedback control method to suppress chaotic behavior in oscillators with limited power supply.

In this work, we study the dynamics behavior of the double-well oscillator with limited power supply coupled to an electromechanical damped device. A single-well oscillator with limited power supply coupled to an electromechanical damping vibration absorber was known in current literature.

The purpose of this paper is to consider the dynamics of an electromechanical damping device, and that works by transferring the vibration energy of the mechanical system to the electrical system, consists of an electrical part coupled magnetically to a mechanical structure (modeled by a double-well Duffing oscillator). A linear electrical system is applied for suppressing the chaotic vibrations which limit the performance of the motion in the mechanical structure.

2 THEORETICAL MODEL

The model shown in Fig. 1 is a mechanical structure described by the Duffing oscillator with double-well potential coupled to an electromechanical vibration absorber. The structure consists of a mass m_1, a viscous damping coefficient b, a linear spring constant k_1 and a nonlinear spring constant k_2. The structure is excited to a source of limited power supply with mass unbalanced m_0 and eccentricity r. This system is called a main system. The vibration absorber consists by an electromechanical transducer and a RCL electrical circuit in series. The simplest transducer constant model is given by $S = 2\pi n l B$, where n is the number of turns in the coil, l is the radius of the coil and B is the uniform radial magnetic field strength in the annular gap. The transducer constant S also relates the electrical potential e, across to the terminals of the coil to the velocity of the coil, with respect to the permanent magnet. The electrical circuit consists by a linear inductor L, a linear capacitor C and a linear resistor R.

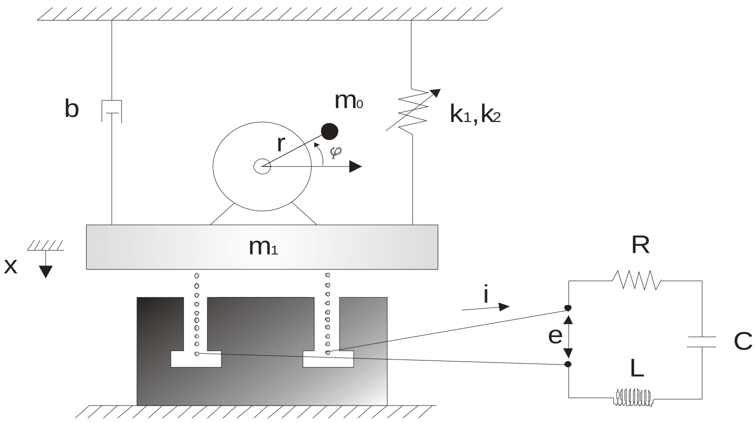

Figure 1: Schematic of a non-ideal structure coupled to nonlinear electromechanical vibration absorber device [10].

The kinetic energy and potential energy of the Duffing oscillator with double-well potential are described as:

$$K = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_0\left(\dot{x} - r\dot{\varphi}\cos\varphi\right)^2 + \frac{1}{2}m_0\left(r\dot{\varphi}sen\varphi\right)^2 + \frac{1}{2}I_0\dot{\varphi}^2 \tag{2}$$

$$V = -\frac{1}{2}k_1x^2 + \frac{1}{4}k_2x^4 \tag{3}$$

where x is the displacement of the main structure, φ is the angular displacement of the rotor, I_0 is the moment of inertia of the rotor.

The motion of the DC motor is governed by the following equations:

$$M(\phi') = Q(\phi') - H(\phi') \tag{4}$$

The motion of the system is governed by the following equations, modified from

$$\begin{aligned} &(m_0 + m_1)\ddot{x} + b\dot{x} + k_1x + k_2x^3 - S\dot{q} = m_0 r\left(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi\right) \\ &L\ddot{q} + R\dot{q} + \frac{1}{C}q + S\dot{x} = 0 \\ &(I_0 + m_0r^2)\ddot{\phi} - m_0 r\ddot{x}\cos\phi = u_1 - u_2\dot{\phi} \end{aligned} \tag{5}$$

It is convenient to rewrite Eq.(5) in terms of dimensionless variable:

$$\tau = \omega_m t \,,$$

$$x = r\chi_1 \,,\ \dot{x} = \omega_m r\chi_1' \,,\ \ddot{x} = \omega_m^2 r\chi_1'' \,,$$

$$q = q_0\chi_2 \,,\ \dot{q} = \omega_m q_0\chi_2' \,,\ \ddot{q} = \omega_m^2 q_0\chi'' \,,$$

$$\varphi = \phi \,,\ \dot{\varphi} = \omega_m\phi' \text{ e } \ddot{\varphi} = \omega_m^2\phi''$$

Then, the governing equations of motion Eq.(5), itself reduce to the following non-dimensional equations:

$$\begin{aligned} &\chi_1'' - \omega_1^2\chi_1 = \varepsilon\left(\varphi''\cos\varphi - \varphi'^2 sen\varphi - \alpha_1\chi_1' - \beta_1\chi_1^3 + \gamma_2\chi_2'\right) \\ &\chi_2'' + \omega_2^2\chi_2 = \varepsilon\left(-\alpha_2\chi_2' - \gamma_1\chi_1'\right) \\ &\varphi'' = \varepsilon\left(\lambda\chi_1''\cos\varphi + \mu_1 - \mu_2\varphi'\right) \end{aligned} \tag{6}$$

where

$$\varepsilon\alpha_1 = \frac{b}{\sqrt{k_1(m_0+m_1)}} \,,\ \varepsilon\alpha_2 = \frac{R}{L\omega_m^2} \,,\ \ \varepsilon\beta_1 = \frac{k_2 r^2}{(m_0+m_1)\omega_m^2} \,,\ \varepsilon = \frac{m_0}{m_0+m_1} \,,\ \varepsilon\lambda = \frac{m_0 r^2}{\left(I_0 + m_0 r^2\right)} \,,$$

$$\varepsilon\gamma_1 = \frac{Sr}{L\omega_m q_0} \,,\ \varepsilon\gamma_2 = \frac{Sq_0}{(m_0+m_1)\omega_m r} \,,\ \varepsilon\mu_1 = \frac{u_1}{\left(I_0 + m_0 r^2\right)\omega_m^2} \,,\ \varepsilon\mu_2 = \frac{u_2}{\left(I_0 + m_0 r^2\right)\omega_m} \,,\ \omega_1 = 1 \,,$$

$$\omega_2 = \frac{\omega_e}{\omega_m} \,,\ \omega_m^2 = \frac{k_1}{m_0+m_1} \text{ e } \omega_e^2 = \frac{1}{LC}$$

3 NONLINEAR DYNAMICS AND CONTROL ANALYSIS OF THE SYSTEM

Eq. (6) will be solving by numerically integrated using the fourth order Runge-Kutta algorithm with variable step-time. In the considered numerical and numerical simulations, the values of system parameters are given in such a way that the local natural frequency of the main system ω_1 is equal to the local frequency of the absorber ω_2. The initial conditions are taken as being nulls. The responses are characterized by tracing the time evolutions, phase portrait and Lyapunov exponents.

The main aim of the electromechanical damping device, considered here expected, is the elimination or suppressing the mechanical chaotic vibrations or instability issue of the mechanical Duffing oscillator. We will also expect to have chaotic motion of the mechanical Duffing oscillator that need to be transformed to periodic motion. The main conclusion will be expected that that the electrical system eliminates the mechanical chaotic vibrations. Finally we would like to coupled Active and passive devices to help the keep the system into a periodic orbit, obtained through the method of numerically.

ε	ω_1	ω_2	γ_1	γ_2	μ_1	μ_2	α_1	α_2	λ	β_1
0.01	1	1	20	10	14	15	0.2	10	8	3

Table 1: Double-Well oscillator parameters in SI units.

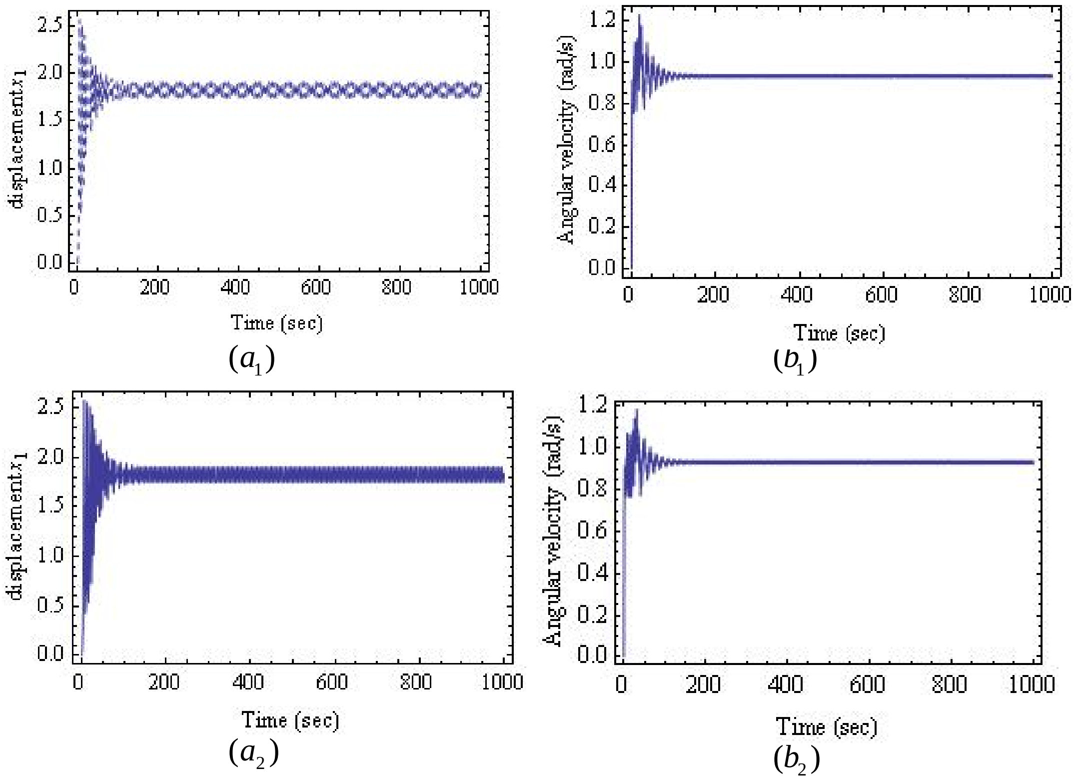

Figure 2: Primary resonance, displacement-time response: (a_1) without absorber, (a_2) with absorber. Angular velocity-time response: (b_1) without absorber , (b_2) with absorber.

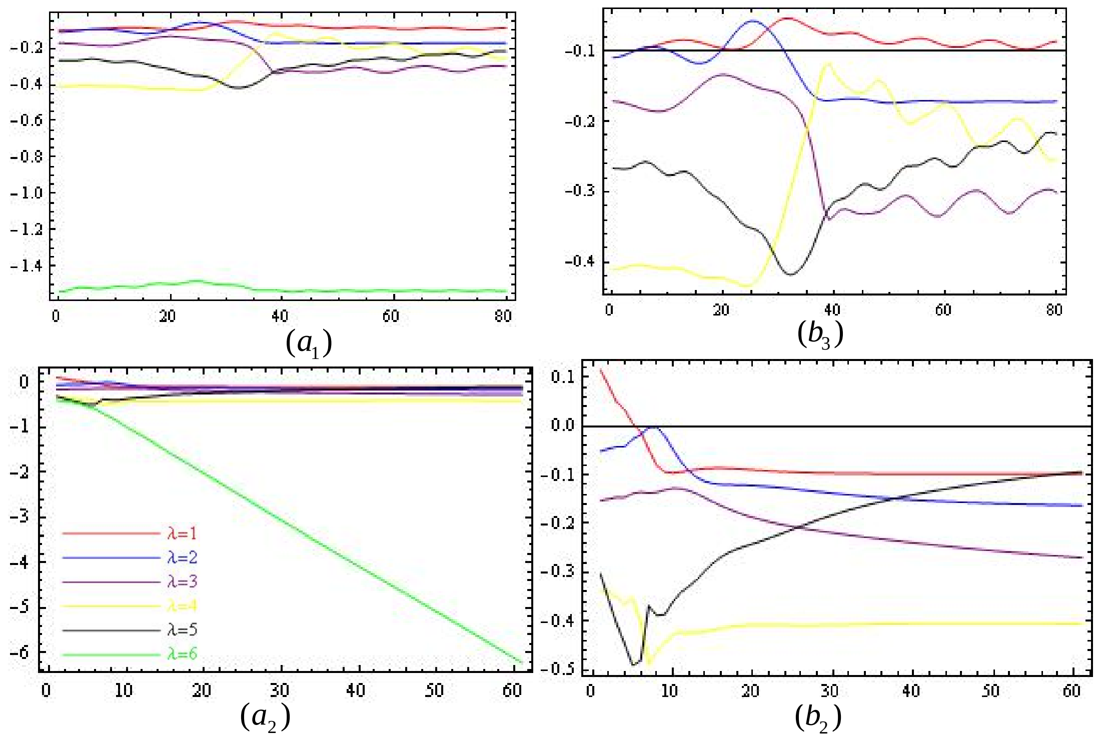

Figure 3: Primary resonance, Lyapunov exponents of the system, without absorber: (a_1) for μ_1 (a_2) for μ_1, (b_1) for μ_2, (b_2) for μ_2.

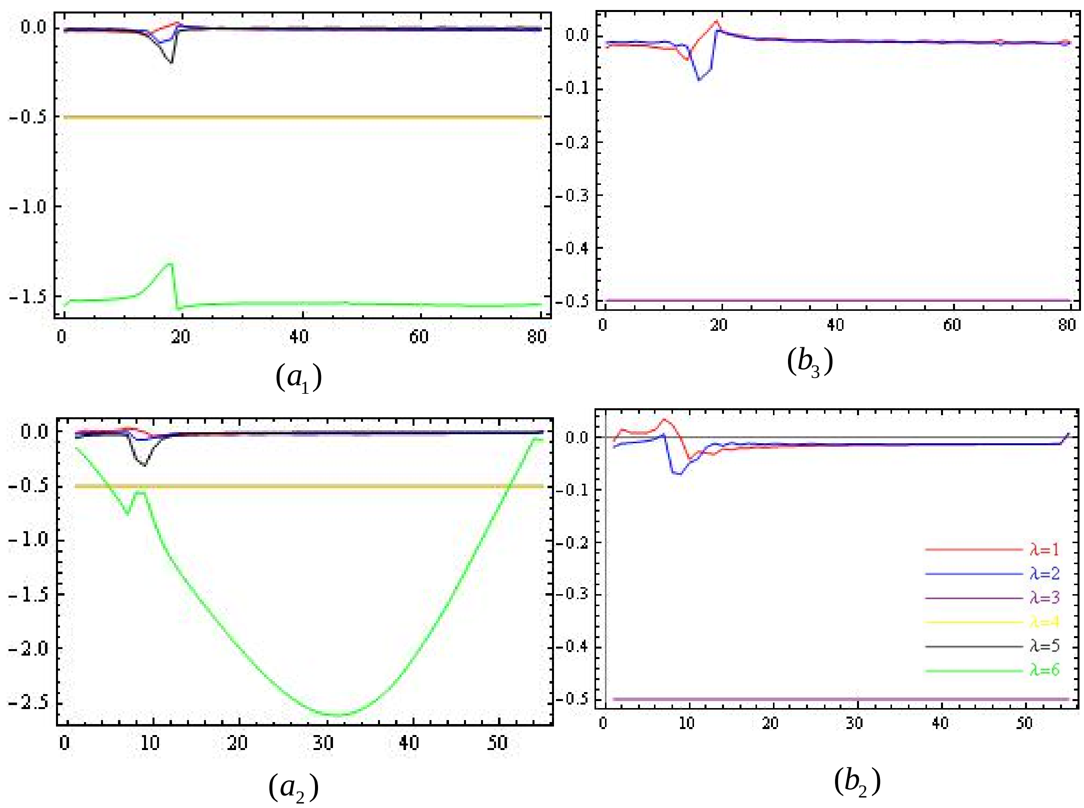

Figure 4: Primary resonance, Lyapunov exponents of the system, with absorber: (a_1) for μ_1 (a_2) for μ_1, (b_1) for μ_2, (b_2) for μ_2.

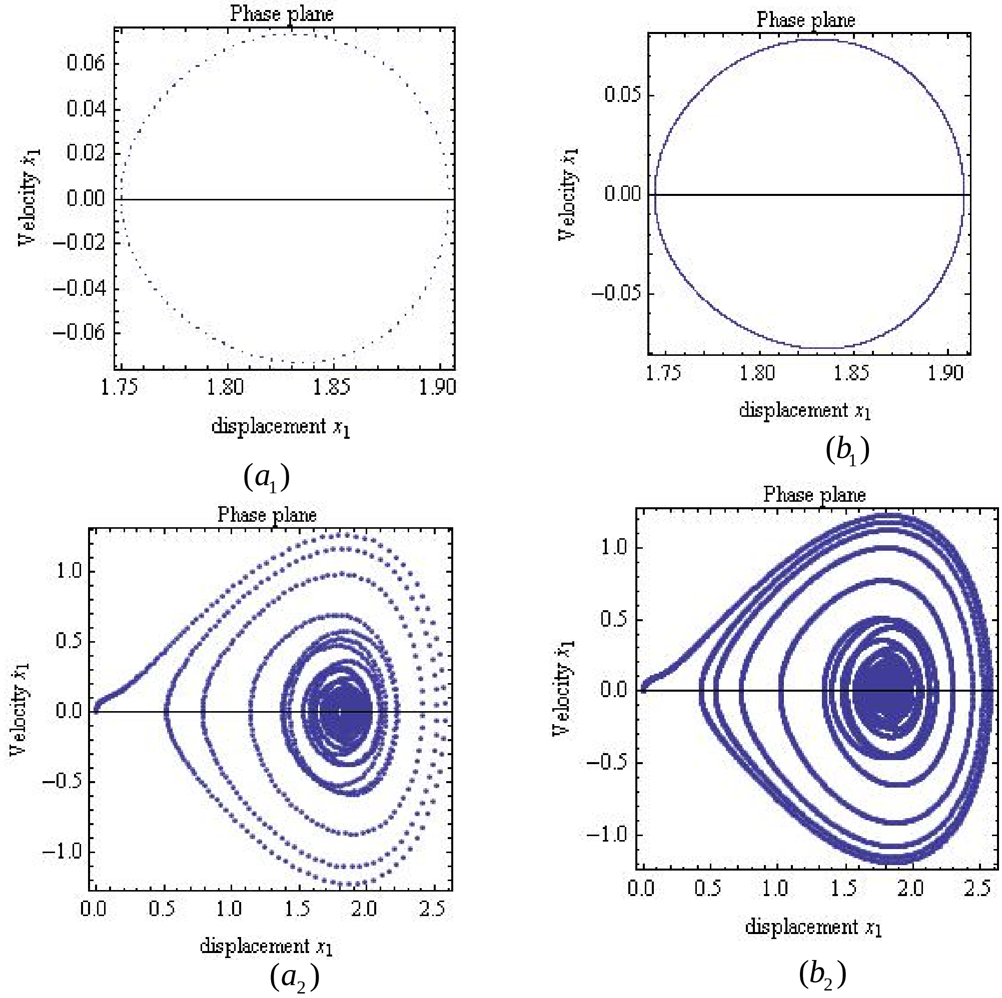

Figure 4**:** Primary resonance, phase portrait (a_1) without absorber, (a_2)without absorber, (b_1) wit absorber , (b_2) with absorber.

4 CONCLUSIONS

In this paper, we have considered the dynamics of a double-well Duffing oscillator coupled to a rotor (a source of limited power supply) and an electromechanical damped device (vibration absorber device). The main aim was analyze the behavior of an electromechanical absorber in a chaotic oscillator. The reduction of amplitude of vibration was considered in [10]. We have also found that the chaotic motion of the mechanical Duffing oscillator has been transformed to periodic motion, see as a quenching of chaotic vibrations. The main conclusion is that the electrical system eliminates the mechanical chaotic vibrations.

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