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THE NONLINEAR ANALYSIS OF VIBRATIONAL CONVEYERS WITH NON-IDEAL CRANK-AND-ROD EXCITERS

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Abstract. *The crank-and-rod exciters vibrational conveyers have a trough supported on elastic stands which are rigidly fastened to the trough and a supporting frame. The trough is oscillated by a common crank drive. This vibration causes the load to move forward and upward. The moving loads jump periodically and move forward with relatively small vibration. The movement is strictly related to vibrational parameters. In this study is the transitional behavior across resonance, during the starting of a single degree of freedom vibratory system excited by non-ideal DC motor. The mechanical system depends on the motion of the DC motor. In this study, the working rang of vibrational conveyers with cubic nonlinear spring and non-ideal vibration exciter has been analyzed. Lyapunov exponents are numerically calculated to prove the occurrence of a chaotic vibration in the non-ideal system.*

1 INTRODUCTION

In this work, the vibrating system consists of a cubic non-linear spring and non-ideal exciter has been analyzed. Ideal system, if there is no coupling between motion of the rotor and vibrating system. In this case, the excitation is completely independent of the system response. Vibrating Systems with ideal and non-ideal excitations was investigated by number authors. Kononenko [1], presented the first detailed study on the non-ideal problem of passage through resonance. Some numerical studies on dynamic characteristics of a vertical pendulum whose base is actuated horizontally through a slider crank mechanism, where the crank is driven through a DC motor, were performed in [16, 17], investigations on the properties of the transient response of this nonlinear and non-ideal problem showed that near the fundamental resonance region. Presented an overview of the main properties of non-ideal vibrating systems by Balthazar, et al. [2, 3]. These authors analyzed, the physical model of the ideal vibrating system consists of non-linear spring and sinusoidal excitation (ideal source) [4], the physical model of the vibrating system consists of linear spring and non-ideal source [5] and the physical model of the vibrating system consists of a cubic non-linear spring and non-ideal source [6].

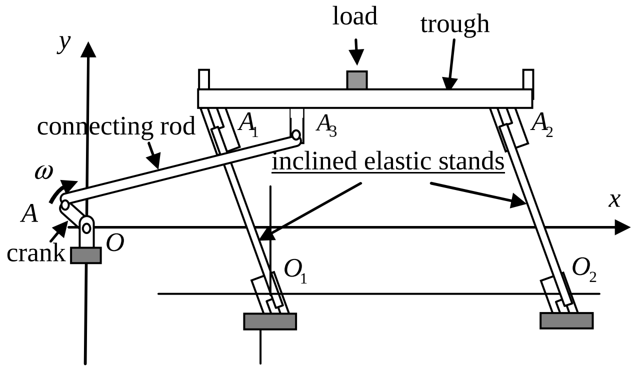


Figure 1: Single-mass, crank-and-rod driven vibrational conveyer.

In this study, vibrational conveyers are constituted by a trough and elastic stands of equal length connected to trough inclinedly. Forced vibration motion driven by the crank-and-rod mechanism is given to this system Figure 1. Crank-and-rod mechanism with elastic connecting rod can also be used, an elastic component added to the operation mechanism slowly increases the amplitude of the system from a low value to the maximum operation amplitude, it functions as elastic component at the initial motion of the conveyer and as rigid component at the continues operation state. During the vibration motion, the forward and upward motion of the load on the trough is provided. After each contact to the trough the velocity of forward movement of the load increases. The velocity of forward movement of the load increases until it reaches the maximum velocity of trough. After reaching the maximum velocity it continues its motion at this velocity [7].

In this study is the transitional behavior across resonance, during the starting of a single degree of freedom vibratory system excited by non-Ideal DC motor. The mechanical system depends on the motion of the DC motor. The vibrational conveyers have been analyzed numerically and analytically by the Method of Multiple Scales, for the primary resonance. The stability is analyzed by using an approximate analytical solution. The frequency-response curves show the behavior of the oscillator for the variation of the control parameters. Numerical simulations are performed and the simulation results are visualized by means of the phase portrait, Poincaré map, Liyapunov exponents, power spectrum and Phase portrait.

2 THE GOVERNING EQUATIONS OF THE MOTION

The equations of motion for the vibrating model of system may be obtained by using Lagrange's Equation, *the conveyor empty;*

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)+\frac{\partial D}{\partial \dot{q}_i}-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=Q_i \tag{1}$$

where M_t is mass of the trough of the conveyor, T is the kinetic energy

$$T=\frac{1}{2}M_t\dot{z}^2+\frac{1}{2}I_m\dot{\phi}^2 \tag{2}$$

V, the potential energy, is

$$V=\frac{1}{2}k_1(z-z_A)^2+\frac{1}{4}k_2(z-z_A)^4+M_t g\sin\psi \tag{3}$$

D, the Rayleigh Dissipation Function, is

$$D=\frac{1}{2}c(\dot{z}-z_A)^2 \tag{4}$$

and q_i is the generalized coordinate. Applying Lagrange's Equation for the coordinate $q_1=z$, $q_2=\phi$ gives the differential equations of motion.
Here the measurement values are taken from the position, where the components in the same directions of the vertical force applied by the leaf spring on itself and the weight force balance each other. M_t is the trough of the conveyor mass, r is the length of the crank, one can write the generalized force Q_1. Mass of the connecting rod AA_3 is neglected and $\theta \cong 0$ is accepted. Because $\frac{r}{L}$ is small.

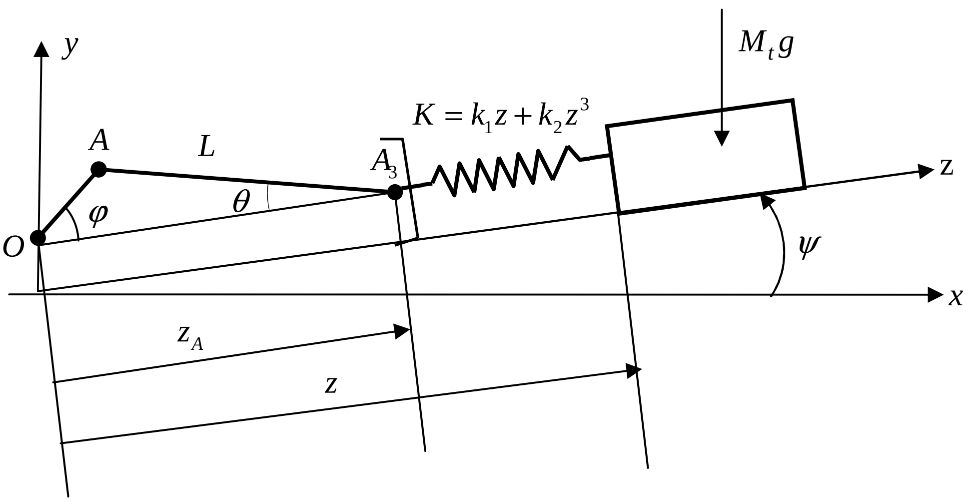


Figure 2: Vibrating model of system.

$$gM_t \sin\psi + k_1(-r\sin[\phi] + z + k_2(-r\sin[\phi]+z)^3 + c(\dot{z} - r\cos[\phi]\dot{\phi} = -M_t\ddot{z} \tag{5}$$

$$\begin{aligned} &-k_1 r\cos[\phi](-r\sin[\phi]+z - k_2 r\cos[\phi](-r\sin[\phi]+z)^3 - \\ &cr\cos[\phi](\dot{z} - r\cos[\phi]\dot{\phi}) + I_m\ddot{\phi} = M(\dot{\phi}) \end{aligned} \tag{6}$$

The motion of the DC motor is governed by the following equations:

$$M(\phi') = E(\phi') - H(\phi') \tag{7}$$

where the function $E(\phi')$ is the driving torque of the source of energy, the function $H(\phi')$ is the resistive torque that is applied to the motor. Note that, usually, the inductance taken to be as much smaller than the mechanical constant time of the vibrating system and, then in the stationary regime, I can take $E(\phi')$ as (linear) $E(\phi') = E_0 - E_1\sigma$, where E_0 is related to the voltage applied across to the armature of the motor and E_1 is a constant for each model of motor considered. I considered the resistive torque nulls.

Rewriting Eqs. (5) and (6) in terms variables, it will obtained, where c is damping coefficient, k_1 is the linear spring coefficient, and k_2 is the cubic spring coefficient, ω_n is the natural frequency, $\omega = \sqrt{\dfrac{k_1}{M_t}}$, $\varepsilon = \dfrac{1}{M_t}$, $G = gM_t$, $\gamma_1 = \dfrac{k_1 r M_t}{I_m}$, $\gamma_2 = \dfrac{k_1 r M_t}{I_m}$, $\gamma_3 = \dfrac{crM_t}{I_m}$;

$$z'' + \omega_n^2 z = \varepsilon\left(k_1(r\sin[\phi])\right) - k_2(-r\sin[\phi]+z)^3 - c(z' - r\cos[\phi]\phi' - G\sin[\psi] \tag{8}$$

$$\begin{aligned} \phi'' = \varepsilon(&\gamma_1\cos[\phi](-r\sin[\phi]+z) + \gamma_2\cos[\phi](-r\sin[\phi]+z)^3 + \\ &\gamma^3\cos[\phi](z' - r\cos[\phi]\phi' + E(\phi') \end{aligned} \tag{9}$$

3 APPROXIMATE ANALYTICAL SOLUTION

The method of multiple scales is used to obtain approximate analytical solution of Eqs. (7) and (8) [9, 10, 11, 12]. I seek a second-order expansion in the form.

$$\begin{aligned} z(t,\varepsilon) &\approx z_0(T_0, T_1) + \varepsilon z_1(T_0, T_1) + \dots, \\ \phi(t,\varepsilon) &\approx \phi_0(T_0, T_1) + \varepsilon\phi_1(T_0, T_1) + \dots, \end{aligned} \tag{10}$$

where the fast scale $T_0 = t$ and the slow scale $T_1 = \varepsilon t$. The time derivatives transform according to

$$\begin{aligned} d/dt &= D_0 + \varepsilon D_1 + \dots, \\ d^2/dt^2 &= D_0^2 + 2\varepsilon D_0 D_1 + \dots, \end{aligned} \tag{11}$$

where $D_n = \partial/\partial T_n$. Then, substituting Eqs. (9, 10) into Eq. (7), (8) and equating coefficients of like powers ε, where Ω is excitation frequency, I obtain, order ε^0:

$$D_0{}^2 z_0 + \omega^2 z_0 = 0 \tag{12}$$

$$D_0{}^2 \phi_0 = 0 \tag{13}$$

equating coefficients of like powers ε, I obtain, order ε^1:

$$\begin{aligned} D_0{}^2 z_1 + \omega^2 z_1 = & -GSin[\psi] + k_1 rSin[\phi_0] + k_2 r^3 Sin[\phi_0]^3 - 2D_0 D_1 z_0 - \\ & cD_0 z_0 + crCos[\phi_0]D_0\phi_0 - 3k_2 r^2 Sin[\phi_0]^2 z_0 + 3k_2 rSin[\phi_0]x_0^2 - k_2 z_0^3 \end{aligned} \tag{14}$$

$$\begin{aligned} & D_0{}^2\phi_1 = E_0 - E_1\sigma[T1] - r\gamma_1 Cos[\phi_0]Sin[\phi_0] - r^3\gamma_2 Cos[\phi_0]Sin[\phi_0]^3 - \\ & 2D_0 D_1\phi_0 + \gamma_3 Cos[\phi_0]D_0 z_0 - r\gamma_3 Cos[\phi_0]^2 D_0\phi_0 + \gamma_1 Cos[\phi_0]z_0 + \\ & 3r^2\gamma 2Cos[\phi_0]Sin[\phi_0]^2 z_0 - 3r\gamma_2 Cos[\phi_0]Sin[\phi_0]z_0^2 + \gamma_2 Cos[\phi_0]z_0^3 \end{aligned} \tag{15}$$

It is analyzed this system the stability (a, γ) in the equilibrium point, using Eqs. (13), (14), where J, Jacobian matrix of Eq. (19), stability of the approximate solutions depends on the value of the eigenvalues of the Jacobian matrix J. The solutions are unstable if the real part of the eigenvalues is positives [9, 10]. Figure 3. shows the frequency-response curves for primary, subharmonic and superharmonic resonance of the vibratory conveyor. These curves show that the nonlinearity bends the frequency-response curves. The bending of the frequency-response curves leads to multivalued amplitudes and hence to jump phenomenon. In contrast linear systems, the mass-nonlinear spring system exhibits no resonance Figure 3.
Eliminating secular terms in equations, for analyzing the case of primary resonance, we chose the detuning parameter σ, where a and β are real,

$$\Omega = \omega_0 + \varepsilon\sigma \tag{16}$$

taking

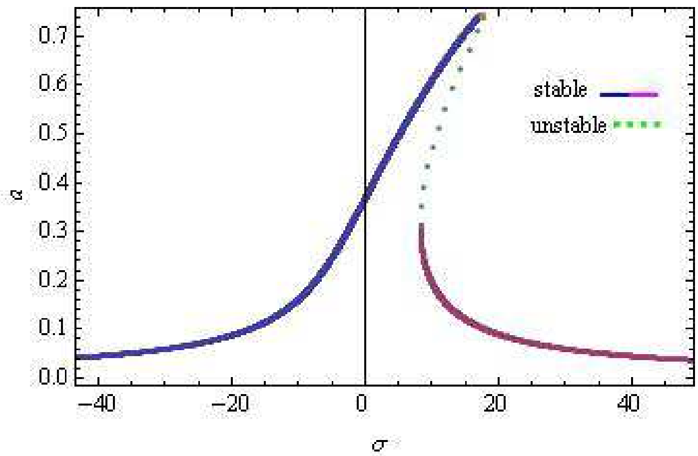
$$A[T_1] = a[T_1]e^{i\beta[T_1]} \tag{17}$$

and separating real and imaginary parts, the system of equations solved.

$$\begin{aligned} a' = & \frac{1}{16\omega}(-8c\omega a[T_1] - 4k_1 r\cos[\gamma] - 3k_2 r^3\cos[\gamma] - \\ & 12k_2 ra[T_1]^2\cos[\gamma] + 4cr\omega\sin[\gamma] + 6k_2 r^2 a[T_1]\sin[2\gamma] \end{aligned} \tag{18}$$

$$\begin{aligned} & \gamma' = \frac{1}{16\omega a[T_1]}(-12k_2 r^2 a[T_1] + 16\sigma\omega a[T_1] - 24k_2 a[T_1]^3 + 4cr\omega\cos[\gamma] \\ & \gamma = T_1\sigma[T_1] - \beta[T_1], \quad \beta'[T_1] = \sigma[T_1] - \gamma' \end{aligned} \tag{19}$$

$$f_1 = a',\ f_2 = \gamma', \qquad J = \begin{bmatrix} \dfrac{\partial f_1}{\partial a} & \dfrac{\partial f_1}{\partial \gamma} \\ \dfrac{\partial f_2}{\partial a} & \dfrac{\partial f_2}{\partial \gamma} \end{bmatrix} \tag{20}$$



Figre 3: Primary resonance, frequency-response curve with stability, – stable, ⋯ unstable, ($\omega = 14.1421$, $\varepsilon = 0.02$).

4 NUMERICAL RESULTS

The numerical calculations of the vibrating system are performed with the help of the software Mathematica [13, 14]. Figure 4. shows the displacement-time response, the power spectrum, phase portrait and Poincaré map for the primary, subharmonic and superharmonic resonance. By Poincaré map I conclude that the motion of the oscillator is periodic with period-1.

I evaluate the Lyapunov exponents using the classical method described in Wolf et al. (1985). The main formula is

$$\lambda = \frac{1}{tN}\sum_{i=1}^{N} \ln \frac{d_i(t)}{d_i(0)} \tag{21}$$

where λ denotes the Lyapunov exponents, the index I corresponding initial positions, and d is the separation between two close trajectories .

Assume that λ_i $(i = 1, 2, 3, 4)$ are the Lyapunov exponents of system Eqs. (8) and (9), satisfying the condition $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$. The dynamical behaviours of system Eqs. (8), (9) can be classified as follows based on the Lyapunov exponents:

- The non-ideal system has a periodic attractor
  $\lambda_1 = 0$, $\lambda_2 < 0$, $\lambda_3 < 0$, $\lambda_4 < 0$

in all simulations. I consider that: λ_1, λ_2, λ_3 and λ_4.

The Lyapunov exponents of the solution to the non-ideal dynamical system, Eqs. (8), (9), are ploted in Figure 4. k_1 ranging form 1000 to 20000, k_2 ranging form 0 to 500, c ranging form 0 to 200. Different values can be found in the following situation:

- The non-ideal system has a chaotic attractor
  $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 < 0$, $\lambda_4 < 0$
- The non-ideal system is hyperchaotic
  $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 < 0$, $\lambda_4 < 0$

ε	I_m	c	ψ	E_1	E_0	k_1	k_2	g	M_t	r
0.01	100	5	$\frac{\pi}{180}20$	1.5	1.6	10^4	300	9.81	0.8	0.01

Table 1: Primary resonance: vibratory conveyer parameters in SI units.

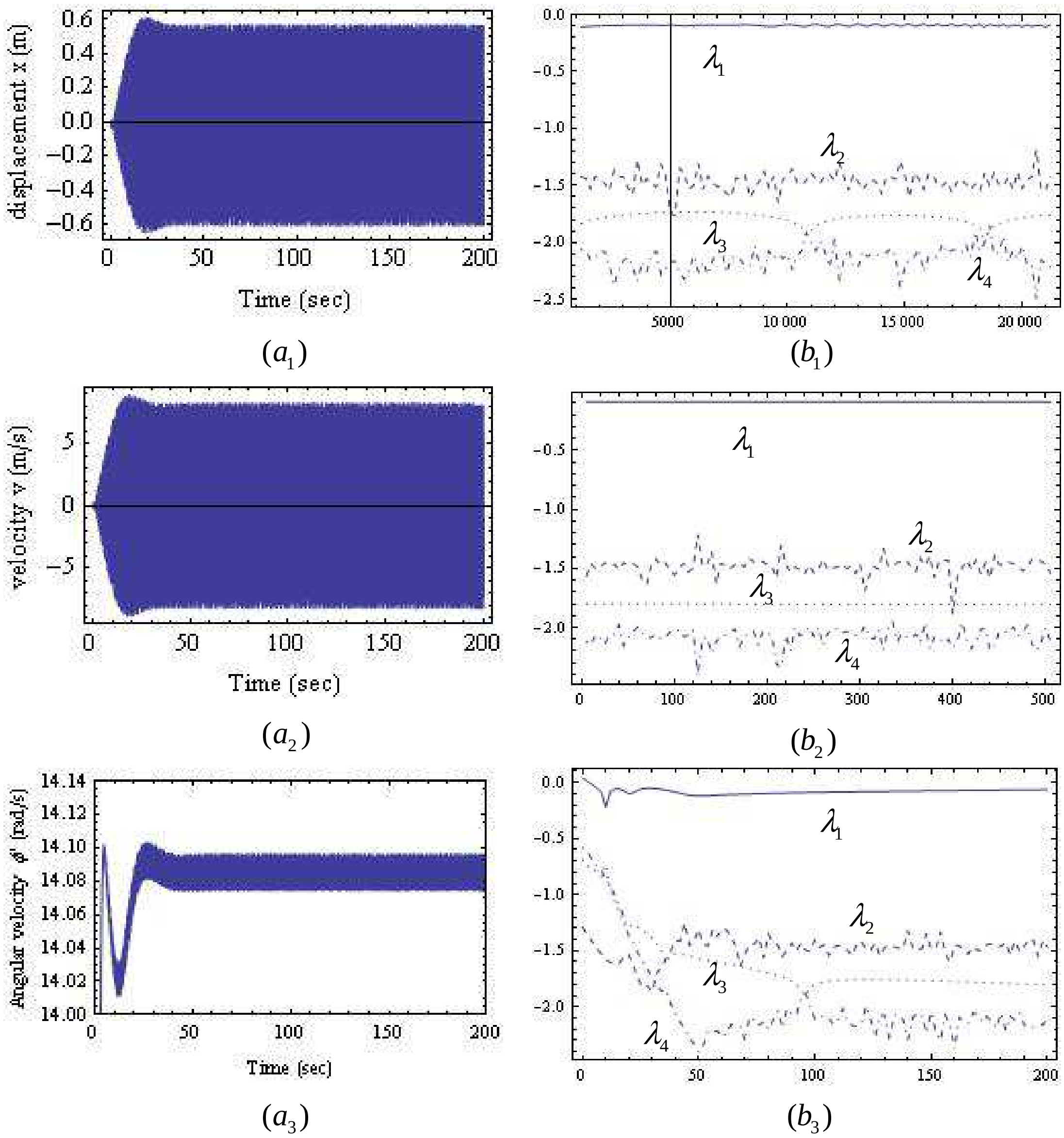


Figure 4: Primary resonance, (a_1) displacement-time response (a_2) velocity-time response, (a_3) angular velocity-time response. Lyapunov exponents of the system: (b_1) for k_1, (b_2) for k_2, (b_3) for c.

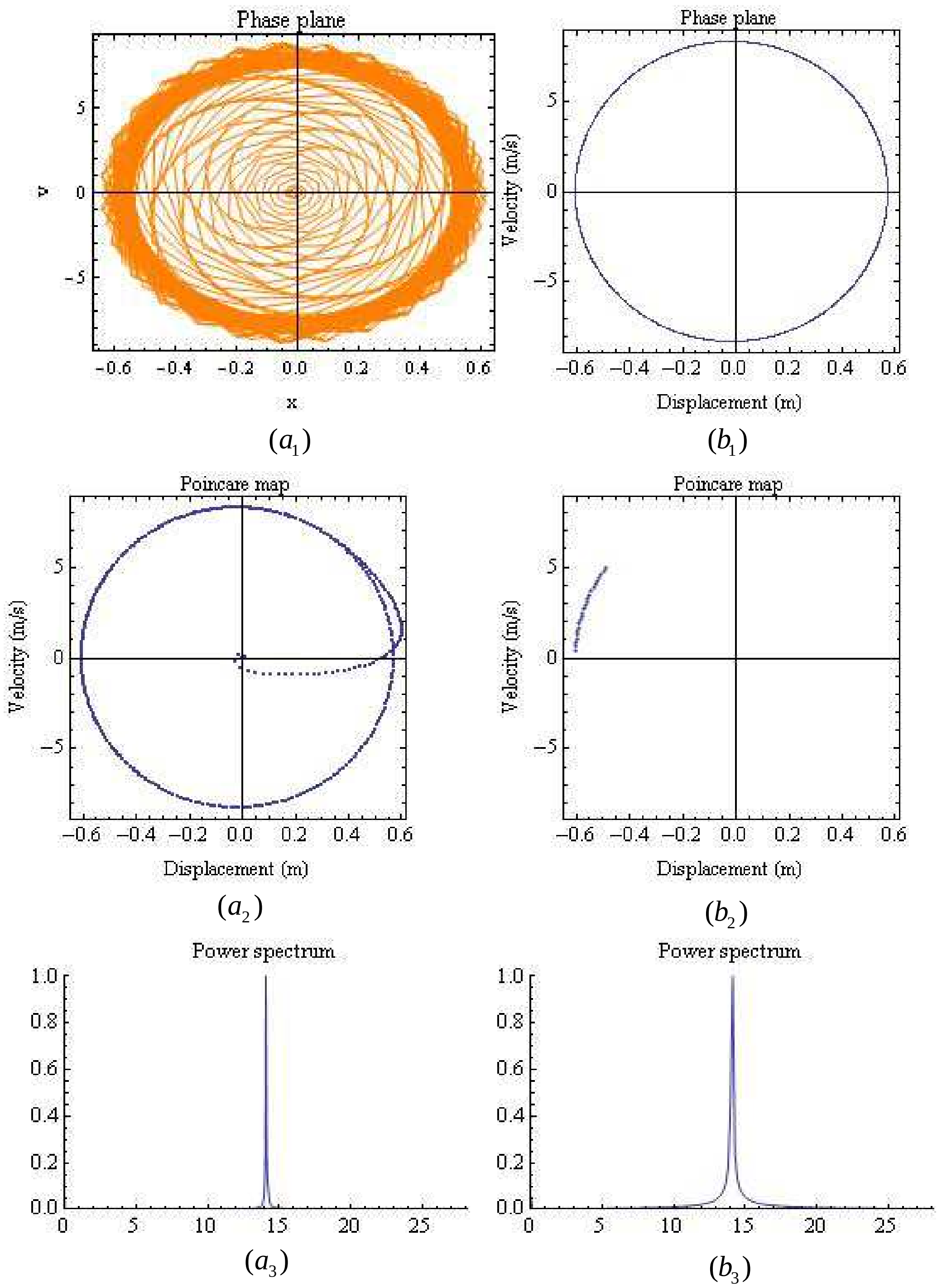


Figure 5: Primary resonance, before stabilization: (a_1) phase portrait, (a_2) Poincaré map, (a_3) power spectrum. After stabilization: (b_1) phase portrait, (b_2) Poincaré map, (b_3) power spectrum.

In Figure 4. the curves obtained numerically and analytically by solving the Equations (8) and (9) are plotted. The main characteristics values used in this study are given in the Table 1.

5 CONCLUSIONS

In this study, the transition over resonance of a nonlinear vibratory system, excited by crank-and-rod, is important in terms of the maximum vibrational amplitude produced on the drive for the cross-over. The maximum amplitude of vibration is then of interest in determining the structural safety of the vibrating members. The shaded region in the amplitude-frequency plot is unstable; the extend of unstableness depends on a number of factors such as the amount of damping present, nonlinearity of spring, and the rate of chance of the exciting frequency Figure 3. Results of the numerical simulations, obtained from the analytical equations, showed that the important dynamic characteristics of the system such as damping, nonlinearity and the amplitude excitations effects, still presented a periodic behavior for these situations. The bending of the response is due to the nonlinearity and is responsible for a jump phenomenon. In the motion of the system near resonance the jump phenomenon occurs. A periodic solution in the case of the angular velocity taken above the resonance (after stabilization) is illustrated in Figure 4**.**

Lyapunov exponents Figure 4., Poincaré sections and phase portraits Figure 5. have been used to examine the system dynamics.

Comparing the results obtained by applying the approximate analytic method with those obtained numerically it is concluded that the difference is negligible, proving the correctness of the analytic procedure used.

In this study is investigated the mechanical system with the non-ideal source, non-ideal vibrating systems are those for which the power supply is limited. Non-ideal problems are more realistic, the model should take into account also influence of the dynamics of the oscillating mechanical elements on electrical properties of the DC motor [3].

In the future, it is possible to investigate this system for subharmonic and superharmonic resonances.

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