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NONLINEAR ANALYSIS OF UNBALANCED MASS OF VERTICAL CONVEYER WITH NON-IDEAL EXCITERS

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Abstract. *In the area of mechanics and electronics, the behaviors of mechanical systems un-der periodic loadings have been examined by many researchers. Vertical conveyors are effec-tive examples observing various kinds of parameters of this problem. In this study, the nonlinear analysis of unbalanced mass of vertical conveyor with non-ideal DC motor. The results of numerical simulation are plotted and Lyapunov exponents are calculated.*

1 INTRODUCTION

The conveying machines are applied in all kind of industries [1,2,3]. The vibrating model of the system excited by the rotation of four equal unbalanced masses is shown in (Figure 1) [1,2,3]. We consider mathematical model with spring of cubic nonlinearity and linear damping and non-ideal exciters. In non-ideal system there is coupling between motion of the rotor and vibrating system. The excitation dependent of the system response [4].

A practical difficulty with unbalanced mass exciters, observed as early as 1904 by A Sommerfeld, is that local instabilities may occur in operating speed of such devices [4]. Presented an overview of the main properties of non-ideal vibrating systems by Balthazar, et al. [5,6,7].

Linear or non-Linear excited vibrations have been examined by many authors in the current literature, as an example, the physical model of the ideal vibrating system consists of linear spring and sinusoidal excitation (ideal source), the physical model of the vibrating system consists of linear spring and nonideal source [8,9,10] and the physical model of the vibrating system consists of a cubic non-linear spring and nonideal source [8].

Resonant-type vibrational conveyers are conveyers in which the vibration frequency is held close to the natural frequency. They are divided into two types as those working just below ore just above the resonance point. The most important characteristic of this type is that the required driving force is low. On the other hand, in conveyers of this type small change in the total mass of the system and the amortization caused by load's motion significantly.

A third type can be defined as conveyers having a vibration frequency 2-3 times greater than the natural frequency[8,11,12]. The exerted force in this sort of conveyers is greater in comparison to resonance conveyers; on the other hand changes in the system's operation conditions have less effect on the load's forward velocity.

In many cases, linear and ideal exciter analysis is insufficient to describe the behavior of the physical system adequately. One of the main reason for modeling a physical system as a nonlinear one is that totally unexpected phenomena sometimes occur in nonlinear systems-phenomena that are not predicted or even hinted at by linear theory[4]. Numerical simulations are performed and the simulation results are visualized by means of the phase portrait, Poincaré map, Liyapunov exponents, power spectrum and Phase portrait with mathematica [14,15].

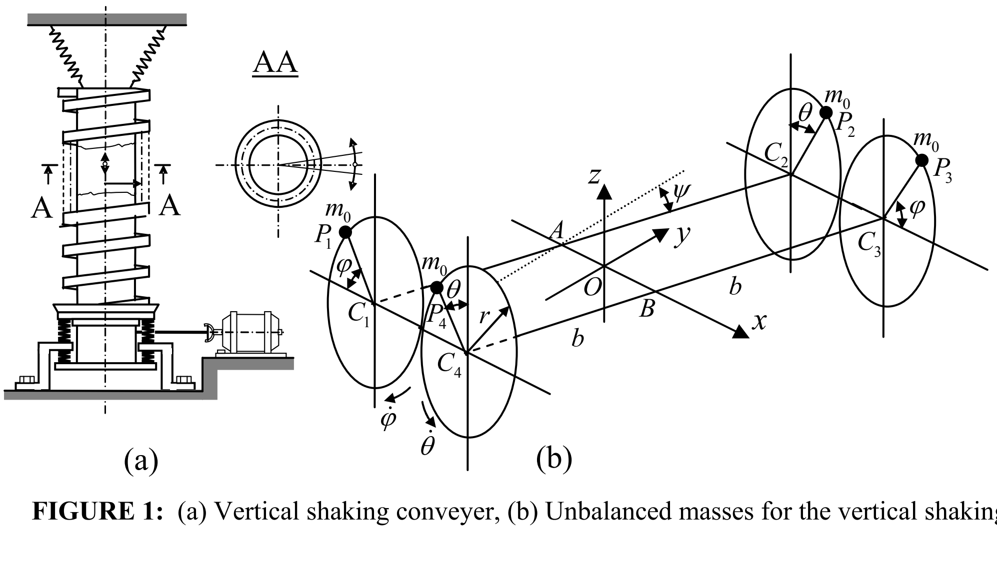


FIGURE 1: (a) Vertical shaking conveyer, (b) Unbalanced masses for the vertical shaking conveyer.

2 THE GOVERNING EQUATIONS OF THE MOTION

The equations of motion for the vertical shaking conveyer may be obtained by using Lagrange's Equation .

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)+\frac{\partial D}{\partial \dot{q}_i}-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=Q_i \tag{1}$$

Where q_i is the generalized coordinate, and T, the kinetic energy, is

$$T=\frac{1}{2}m z'^2+\frac{1}{2}I_z \psi'^2+\frac{1}{2}m_0\,(v_{P_1}^2+v_{P_2}^2+v_{P_3}^2+v_{P_4}^2) \tag{2}$$

Where prime indicate the time derivative, m is the mass of the trough of the conveyor, m_0 is the mass of the unbalanced materials , I_z the inertia moment of the conveyer about vertical axis, z is the vertical position of the trough, ψ is the torsional displacement of the trough, v_{p_i} is the velocity of the unbalanced masses

$$\vec{v}_{P_i}=[Rot_\psi]\frac{d\overrightarrow{OP_i}}{dt} \qquad (i=1,2,3,4) \tag{3}$$

Where $[Rot_\psi]$ is the rotation matrix for the angle ψ is

$$[Rot_\psi]=\begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix} \tag{4}$$

Unbalanced masses position can be computed as follows,

$$\begin{aligned}\overrightarrow{OP_1}&=\overrightarrow{OA}+\overrightarrow{AC_1}+\overrightarrow{C_1P_1}\\ \overrightarrow{OP_2}&=\overrightarrow{OA}+\overrightarrow{AC_2}+\overrightarrow{C_2P_2}\\ \overrightarrow{OP_3}&=\overrightarrow{OB}+\overrightarrow{BC_3}+\overrightarrow{C_3P_3}\\ \overrightarrow{OP_4}&=\overrightarrow{OB}+\overrightarrow{BC_4}+\overrightarrow{C_4P_4}\end{aligned} \tag{5}$$

$$\begin{aligned}OA&=\{-r,0,0\}\\ OB&=\{r,0,0\}\end{aligned} \tag{6}$$

Where r is the radius of the unbalanced masses.

$$\begin{aligned} AC_1 &= \{0,-b,0\} \\ AC_2 &= \{0,b,0\} \\ BC_3 &= \{0,b,0\} \\ BC_4 &= \{0,-b,0\} \end{aligned} \tag{7}$$

Where b is the half distance of the opposite disks .

$$\begin{aligned} C_1P_1 &= \{-rCos\phi, 0, rSin\phi + z\} \\ C_2P_2 &= \{rSin\theta, 0, rCos\theta + z\} \\ C_3P_3 &= \{rCos\phi, 0, rSin\phi + z\} \\ C_4P_4 &= \{-rSin\theta, 0, rCos\theta + z\} \end{aligned} \tag{8}$$

Where θ and ϕ is the angular position of the unbalanced masses.

Then kinetic energy rewritten as follows,

$$T = \frac{1}{2}((m + 4m_0)\, z'^2 + 4r^2 m_0\, \phi'^2 + (I_z + 4b^2 m_0 + 6r^2 m_0 + 2r^2 Cos2\phi m_0)\, \psi'^2)\,. \tag{9}$$

V, the potential energy, is

$$V = mgz + \frac{z^2 k_{z1}}{2} + \frac{z^4 k_{z2}}{4} + \frac{\psi^2 k_{\psi 1}}{2} + \frac{\psi^4 k_{\psi 2}}{4} + 4m_0 g Sin(\phi) \tag{10}$$

Where $k_{z1}, k_{z2}, k_{\psi_1}, k_{\psi_2}$ is the linear, and nonlinear spring constant of the vertical and torsional spring respectively, D, the Rayleigh Dissipation Function, is

$$D = \frac{1}{2} c_z \dot{z}^2 + \frac{1}{2} c_\psi \dot{\psi}^2 \tag{11}$$

where c_z and c_ψ is the dumping constant of the vertical and angular spring respectively. Applying Lagrange's Equation for the three coordinates $q_1 = z$, $q_2 = \psi$ and $q_3 = \phi$ gives the differential equations of motion as

$$(m + 4m_0) z'' + k_{z1} z = -mg - z^3 k_{z2} - c_z z' + 4rm_0\, \phi'^2 Sin\,\phi \tag{12}$$

$$(\mathrm{I}_z + 4b^2 + 6r^2 + 2r^2Cos2\phi)m_0\psi'' + k_{\psi1}\psi = -\psi^3k_{\psi2} - c_\psi\psi' + 4rm_0\phi'(b\phi'Cos(\phi) + 2r\psi'Sin(\phi)) \quad (13)$$

$$4m_0r^2\phi'' = L(\phi') - 2m_0gCos(\phi) + rm_0(6z'\phi'Sin(\phi) + 6b\phi'\psi'Cos(\phi) + r(2Sin(\phi) + Sin(2\phi))\psi'^2) \quad (14)$$

Where $L(\phi')$ is the active torque generated by the electric circuit of the DC motor

These four disks should be rotate synchronously in order to make same angle between the unbalanced masses on the crossing disks in Figure 1b. This situation is provided by the gears.
The angle θ is equal to φ if the angle of unbalanced masses which are same side of axes of disks are taken equal, and the opposite side 90^0.

$$z'' + \frac{k_{z1}}{(m+4m_0)}z = \frac{1}{m+4m_0}(-mg - k_{z2}z^3 - c_zz' + 4rm_0\phi'^2Sin\phi) \quad (15)$$

$$(\mathrm{I}_z + 4b^2 + 6r^2 + 2r^2Cos2\phi)m_0\psi'' + k_{\psi1}\psi = -\psi^3k_{\psi2} - c_\psi\psi' + 4rm_0\phi'[b\phi'Cos(\phi) + 2r\psi'Sin(\phi)] \quad (16)$$

$$\phi'' = \frac{L(\phi')}{4m_0r^2} - \frac{g}{2r^2}Cos(\phi) + \frac{1}{4r}\{6z'\phi'Sin(\phi) + 6b\phi'\psi'Cos(\phi) + r[2Sin(\phi) + Sin(2\phi)]\psi'^2\} \quad (17)$$

Eqs(15-17) can be written nondimensional form as,

$$z'' + \omega_z^2z = \grave{o}(-G - k_{z2}z^3 - c_zz' + 4rm_0\phi'^2Sin\phi), \quad (18)$$

$$\psi'' + \omega_\psi^2\psi = \grave{o}\left\{-q_1\psi^3 - q_2\psi' + q_3\phi'\left[b\phi'Cos(\phi) + 2rSin(\phi)\psi'\right]\right\}, \quad (19)$$

$$\phi'' = \grave{o}\left\{E(\phi') - 2m_0gCos(\phi) + q_4Sin(\phi)z'\phi' + q_5Cos(\phi)\phi'\psi' + q_6\left[2Sin(\phi) + Sin(2\phi)\right]\psi'^2\right\}, \quad (20)$$

where

$$G = mg\ ,\ \grave{o} = \frac{1}{(m+4m_0)}\ ,\ \omega_z = \sqrt{\frac{k_{z1}}{(m+4m_0)}}\ ,\ \omega_\psi = \sqrt{\frac{k_{\psi1}}{I_z + 2m_0(2b^2+3r^2)}}\ ,$$

$$\sigma = \frac{\phi' - \omega_z}{\grave{o}}\ ,\ q_1 = \frac{k_{\psi2}}{\grave{o}\left[I_z + 2m_0(2b^2+3r^2)\right]}\ ,\ q_2 = \frac{c_\psi}{\grave{o}\left[I_z + 2m_0(2b^2+3r^2)\right]}\ ,$$

$$q_3 = \frac{4m_0r}{\grave{o}\left[I_z + 2m_0(2b^2+3r^2)\right]}, q_4 = \frac{2g}{4\grave{o}r^2}\ ,\ q_5 = \frac{6}{4\grave{o}r},\ q_6 = \frac{6b}{4\grave{o}r},\ E(\phi') = \frac{L(\phi')}{\grave{o}4m_0r^2}$$

3 NUMERICAL RESULTS

The numerical calculations of the vibrating system are performed with the help of the software Mathematica [14]. Figure 2 shows the vertical and torsional displacement and velocity-time history of the conveyer trough. Figure 3. shows the phase portrait and Poincaré map for the vertical movement. By Poincaré map I conclude that the motion of the oscillator is periodic with period-1. Figure 4. Shows the Lyapunov exponent for the variable damper and spring coefficients. All the curves obtained numerically by solving the eqs.(18-20) are plotted. Periodic solution in the case of the angular velocity taken above the resonance (after stabilization) is illustrated in Figure 3. The main characteristic values used in this study are given in the Table 1.
It can be evaluated the Lyapunov exponents using the classical method described in Wolf et al. [15,16]. The main formula is

$$\lambda = \frac{1}{tN}\sum_{i=1}^{N} \ln \frac{d_i(t)}{d_i(0)} \tag{21}$$

where λ denotes the Lyapunov exponents, the index i corresponding initial positions, and d is the separation between two close trajectories .
Assume that λ_i $(i = 1,2,3,4,5,6)$ are the Lyapunov exponents of the system , satisfying the condition $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq \lambda_5 \geq \lambda_6$. The dynamical behaviours of this system can be classified as follows based on the Lyapunov exponents:

- The non-ideal system has a periodic attractor
  $\lambda_1 = 0$, $\lambda_2 < 0$, $\lambda_3 < 0$, $\lambda_4 < 0, \lambda_5 < 0, \lambda_6 < 0$
- The non-ideal system has a chaotic attractor
  $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 < 0$, $\lambda_4 < 0, \lambda_5 < 0, \lambda_6 < 0$
- The non-ideal system is hyperchaotic
  $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 < 0$, $\lambda_4 < 0, \lambda_5 < 0, \lambda_6 < 0$

in all simulations.
The Lyapunov exponents of the solution to the non-ideal dynamical system, Eqs. (18-20) are ploted in Figure 4. k_1 ranging form 2000 to 50000, c ranging form 0 to 250. Different values can be found in the following situation:

Figure 4. shows this system only has a chaotic attractor for $\boldsymbol{c} < 130$ Ns./m and $\boldsymbol{k}_1 = (9500 - 10500)$

$\boldsymbol{k}_1$(N/m^2)	$\boldsymbol{k}_2$(N/m^2)	$\boldsymbol{c}$(N·s/m)	$m\,(kg)$	$m_0\,(kg)$	$r\,(m)$	$\omega(rad/s)$	$I_z\,(kg.m^2)$	ε
20000	200	150	100	1	0.15	13.9	100	0.0096

Table 1: Vertical conveyer parameters in SI units.

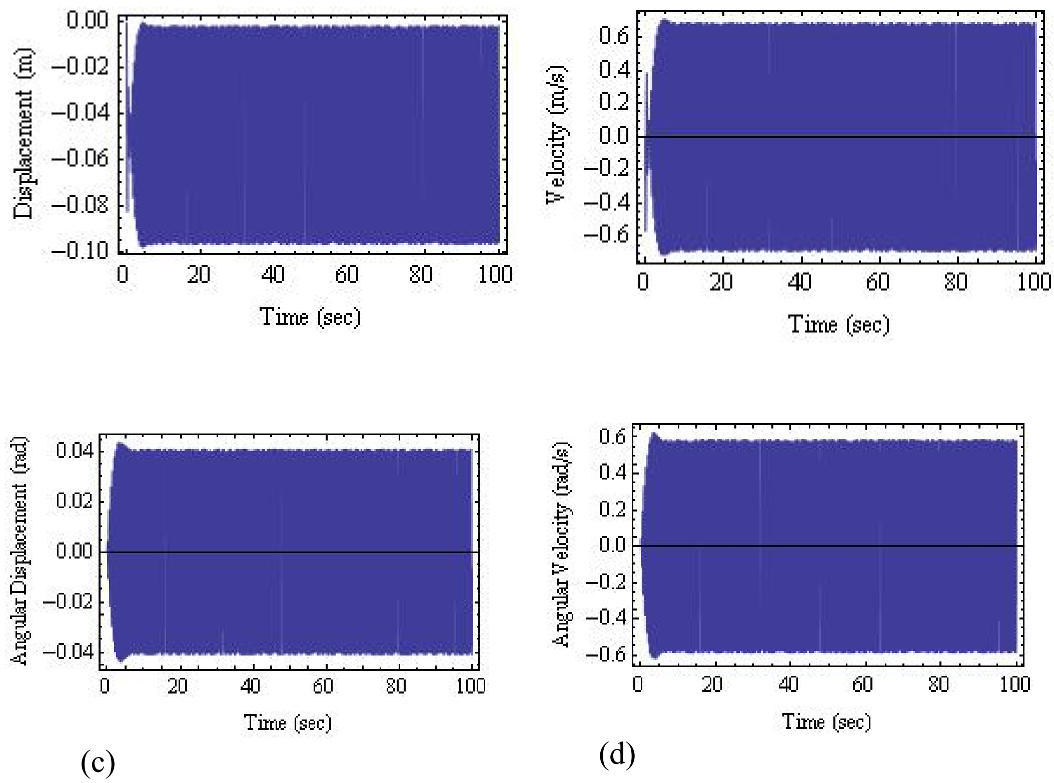


FIGURE 2: (a) Vertical displacement-time response, (b) vertical velocity-time response, (c) Angular displacement-time response, (d) angular velocity-time response.

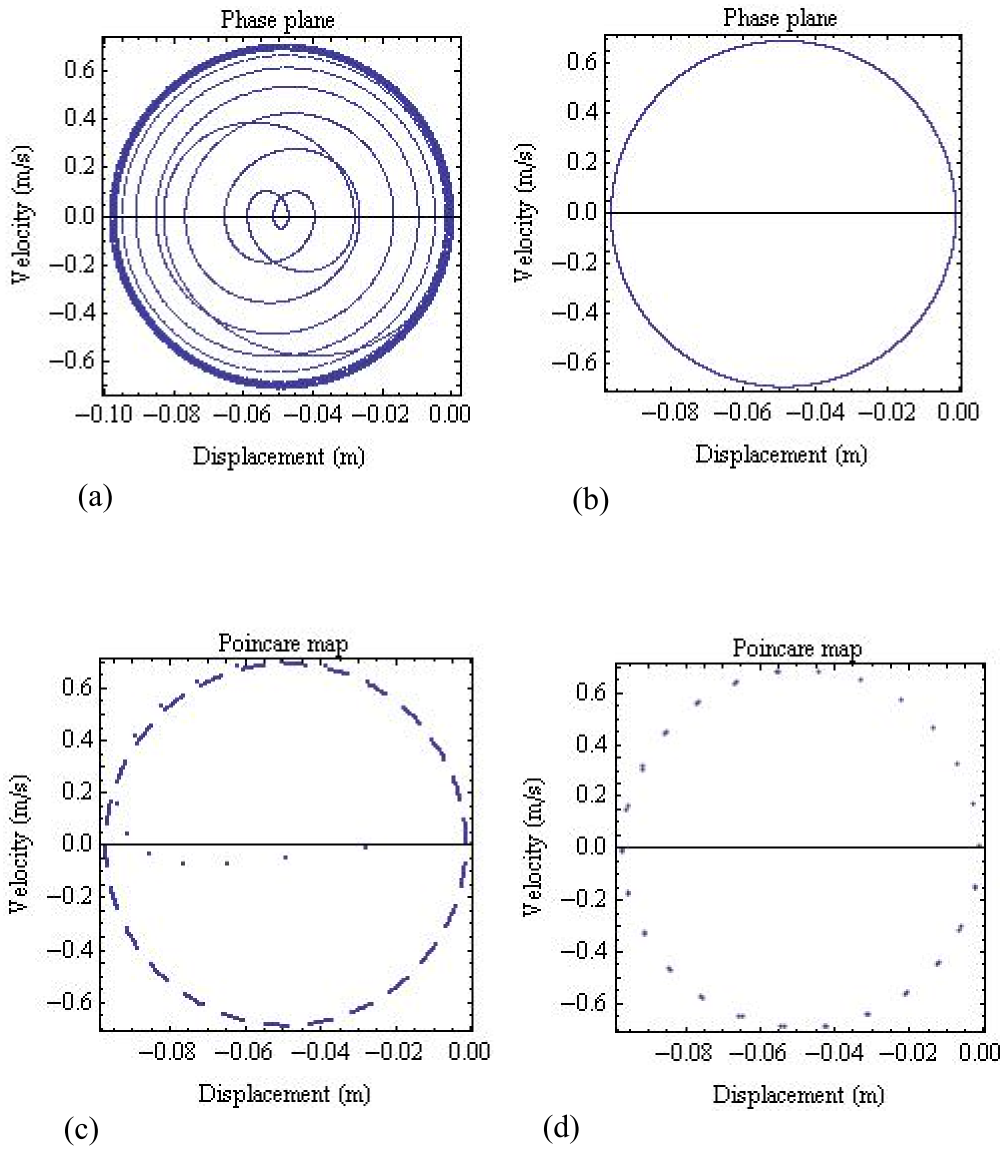
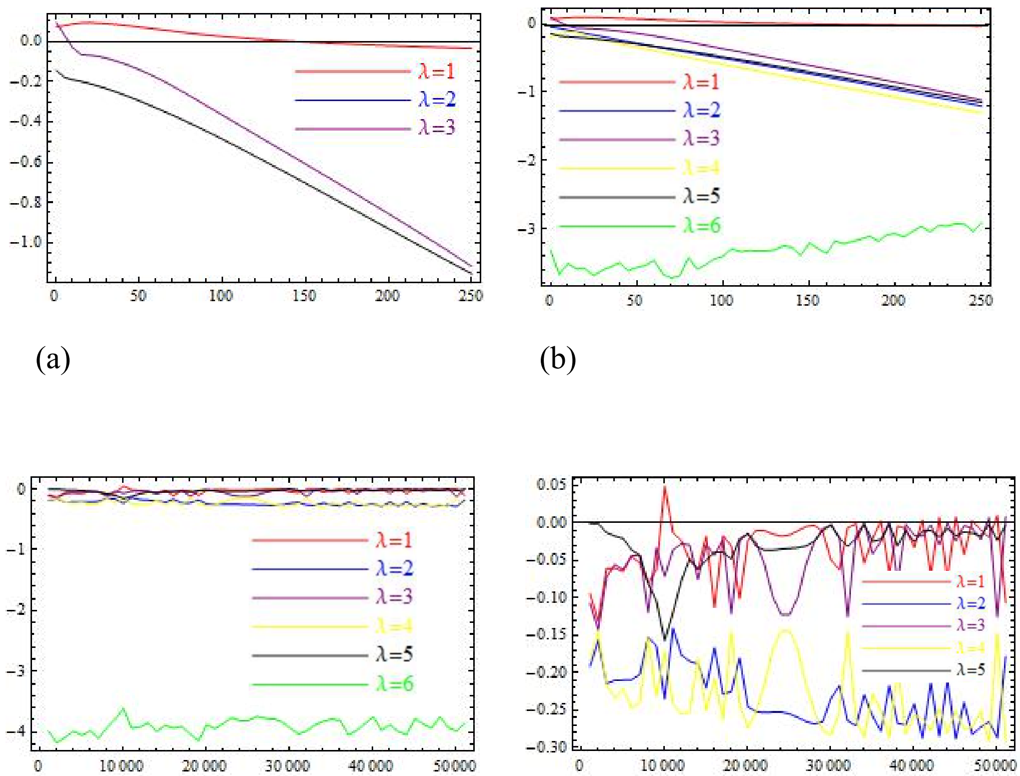


FIGURE 3: (a) phase portrait (before stabilization)., (b) phase portrait (after stabilization)., (c) Poincaré map (before stabilization) (d) Poincaré map (after stabilization).



(a) (b)

(c) (d)

FIGURE 4: (a) Liyapunov exponent for variable damping coefficient for $\lambda = 1,2,3$ (b) Liyapunov exponent for variable damping coefficient for $\lambda = 1,2,3,4,5,6$ (c) Liyapunov exponent for variable linear spring coefficient for $\lambda = 1,2,3$, (d) Liyapunov exponent for variable linear spring coefficient for $\lambda = 1,2,3,4,5,6$

4 CONCLUSIONS

In this study, the transition over resonance of a nonlinear vibratory system, excited by unbalanced mass with nonideal DC motor. is important in terms of the maximum vibrational amplitude produced on the drive for the cross-over. The maximum amplitude of vibration is then of interest in determining the structural safety of the vibrating members.It is showed that the important dynamic characteristics of the system such as damping, nonlinearity and the amplitude excitations effects, still presented a periodic behavior for these situations.

Figure 4. shows this system have chaotic attractor about $\boldsymbol{k}_1 = 10000\boldsymbol{N}/\boldsymbol{m}$ and $\boldsymbol{c} < 130$ Ns./m .

In the future, it is possible to investigate this system for subharmonic and superharmonic resonances.

REFERENCES

[1] A. O. Spivakovasky and V. K., Dyachkov, *Conveying Machines*, Volume (I, II), Mir Publichers Moscow, (1985).

[2] M.A. Parameswaran, S. Ganapathy, Vibratory Conveying-Analysis and Design: A Review. *Mechanism and Machine Theory* Vol. 14, pp. 89-97, (1979).

[3] Lawrence W. Hallanger, *The daynamic stability of an unbalanced mass exciter*, Thesis, California Institute of Technology Pasadena, California, (1967).

[4] V. O. Konokenko, *Vibrating Problems With a Limited Power Supply*, Ilife, London, (1969).

[5] J. M. Balthazar, D. T. Mook, H. I. Weber, R. M. L. R. F. Brasıl, A. Fenili, D. Belato, J. L. P. Felix, An Overview on Non-Ideal Vibrations, *Mechanica*, 38: 613-621, (2003).

[6] J.M. Balthazar, R. M. L. R. F. Brasıl, H. I. Weber, A. Fenili, D. Belato, J. L. P. Felix, and F.J Garzelli, *A Review of New Vibration Issues due to Non-Ideal Energy Sources*, CRC Press, LLC, (2004).

[7] M. R. Bolla, J.M. Balthazar, J. L.P. Felix, D.T. Mook, On an approximate analytical solution to a nonlinear vibrating problem, excited by a nonideal motor. *Nonlinear Dyn*, 50:841-847, (2007).

[8] A. H. Nayfeh, and D. T., Mook, *Nonlinear Oscillations*. Wiley, New York, (1979).

[9] G. Schmidt, and A. Tondl, *Non-Linear Vibrations*. Cambridge University Press, 1986.

[10] A.H. Nayfeh, B. Balachandran, *Applied Nonlinear Dynamics: Analitical, Computation and Experimental Methods.* Wiley, 2004.

[11] H. Bayıroğlu, Nonlinear analysis of unbalanced mass of vertical conveyor:primary, subharmonic, and superharmonic response. *Nonlinear Dyn* , 71:93-107, (2013).

[12] S. Ganapathy, and M.A. Parameswaran, Transition over resonance and power requirements of an unbalanced mass driven vibratory system. *Mech. Mach. Theory*, 21, 73-85 (1986).

[13] J. Awrejcewicz and C.H. Lamarque, *Bifurcation and chaos in nonsmooth mechanical systems*. [electronic resource]. River Edge, NJ : World Scientific, 2003.

[14] S. Lynch, "Dynamical Systems with Applications using Mathematica", Boston.Basel. Berlin, 2007.

[15] V. Piccirillo, J.M. Balthazar, B. R. Pontes JR., J. L.P. Felix, On a nonlinear and chaotic non-ideal vibrating system with shape memory alloy (SMA). Journal of theoretical and Applied Mechanics, 46, 3, pp. 597-620, Warsaw 2008.

[16] A.Wolf, J.B.Swift., H.L. Swinney, J.A. Vastano, 1985, Lyapunov exponenets from a time series, *Physica* D,16, 285-317