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MODELING VIBRATIONS OF NONUNIFORMLY POLARIZED PIEZOCERAMIC BODIES

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Abstract. *In this paper we attempt to describe the effect of the internal structure of nonuniformly polarized polycrystalline ferroelectric materials on the physical characteristics of the modal and harmonic analysis of work items piezoelectric sensors and actuators made on their basis. Modeling work of uniformly polarized piezoceramic transducers were devoted a lot of works, what is impossible to tell about the models with nonhomogeneous polarization. Perhaps this is due to the fact that calculates the field of the pre-polarization of the polycrystalline ferroelectric sample is not so simple. This task, like the problems of the theory of plasticity, is quite complex in the mathematical sense. It is known that the piezoelectric properties of ceramic materials are created by pre-polarization in the strong electric fields. Thus the pre-polarization in the ceramic element can be carried out by a certain law, which will inevitably lead to a change of the internal structure of the material, and as a consequence, a change of the dynamical and physical characteristics in the external electrical and mechanical fields. Such materials can be attributed to the class of locally transversely isotropic materials in which the crystallographic axes can change its direction from point to point. The latter fact is due to the nonhomogeneity of piezoelectric, elastic and dielectric properties. To modeling nonuniformly polarized piezoelectric ceramic elements, first of all, we constructed the mathematical models to determine the nature of the heterogeneity of pre-polarization, and secondly, the models of the modal and harmonic analysis in the one-dimensional and axisymmetric cases with use of the finite element method. On an example of one-dimensional problem of longitudinal oscillations transversely polarized non-uniform manner band-pass ceramic transducer was shown that in such transducer can be excited only one mode of oscillation. In the case of axially symmetric nonhomogeneous polarized cylindrical transducer the heterogeneity of pre-polarization affects on the form of oscillation, this also may be used in manufacture of transducers.*

1 INTRODUCTION

Polycrystalline ferroelectric materials are widely used as electromechanical transducers for ultrasonic diagnostics in medicine, aviation and rail transportation, energy, oil and gas industry, in ultrasonic welding, cleaning surfaces, coating, drilling, etc. Products made on the basis of piezoelectric ceramics, are divided into groups: generators, sensors (sensors), actuators (piezodrive), converters and combined systems. In the future, we will be discussed the piezoelectric transducers which operate on the principle of inverse piezoelectric effect, and designed to convert electrical energy into mechanical energy. Piezoelectric effect was discovered in the XIX century, but its extensive use was made possible only in the XX century with the advent of piezoceramic materials. Remarkable piezoelectric, dielectric and mechanical properties of these materials allowed considering them as the most promising active materials of the XXI century [1]. However, not all of the wonderful properties inherent to piezoelectric ceramics were studied in full, and as a result, are not claimed by science, engineering and technology. As a result, new types of piezoelectric ceramics and composite elements are created, well-known piezoelectric elements and components are improving, technology inhomogeneous polarization is developing, etc.

Basically piezoelectric transducers are designed as thin circular disks or rectangular plates or in the form of a round of square rods with the electrodes, totally or partially covering the face surfaces of these elements. Polarization of the piezoceramic elements it is a long time excerpt the samples in a strong electric field that generated, as a rule, by the potential difference across the same electrodes, that will be used for further operation of these elements, but in small fields. After the pre-polarization the polycrystalline ferroelectric elements that elements are became the piezoelectric. When a variable electric field is directed along the thickness of disks and plates with the pre polarization directed across the thickness, in them are generated radial and planar oscillation modes. In the rod elements with longitudinal polarization are generated longitudinal vibrations by a longitudinal electric field. There may be cases of shear high-frequency vibrations plates by the transverse field. Homogeneity of the field preliminary polarization and the presence of certain geometrical symmetry leads to the known characteristics of vibration, such as resonance and antiresonance frequencies, vibration modes, the amplitude-frequency characteristic (AFC) of the resistance, etc. Results of the solution of such problems are well known [2, 3]. For example, in the rod transducer with transverse polarization, only odd harmonics are excited by a transverse electric field. The situation changes dramatically if the transducer is attached to a certain concentrated mass, or changed the boundary conditions. In this case, the change of the geometric conditions of symmetry results in a change of the symmetry of the solution. Other significant factor affecting the vibration modes is either the inhomogeneity of the electric field, or the field inhomogeneity preliminary polarization of the ceramics. In the first case, the symmetry of the loads is disturbed (the electric field distribution in the bulk), and in the second case the symmetry structure of material is broken. Each of these cases involves a violation of the symmetry of the solutions. Therefore, modeling of such problems is urgent and requires further mathematical research. Note, that the definition of non-uniform field of pre-polarization is an independent mechanical problem and complex in mathematical plan task, and requires special consideration. This is all the more relevant under the conditions miniaturization of automation elements where are possible violations of the thermal regime, or the unexpected application of small mechanical loads which, due to the small geometric dimensions can cause large mechanical stresses, leading to depolarization.

In section 1 we present a model that allows calculating the non-uniform field of polarization of the piezoceramic elements in 3-dimensional case [4] and showing the image of inhomogeneous polarization fields for some types of circular disk transducers.

In section 2, we consider the problem of longitudinal vibrations of a rod non-uniform transversely polarized transducer. Here we note the difference in solving a similar problem with a homogeneous polarization. Next, we will present the graphs of the amplitude frequency response of admittance and note the effect of internal damping on resonance and antiresonance frequencies and the corresponding vibration modes.

Modeling of high-frequency transducers with axially symmetric but with nonhomogeneous pre-polarization is devoted the section 3. It is noted that significant difference from the case of a uniform polarization are obtained for high frequencies [5].

The results of the study and the application are discussed in section 4.

2 THE FIELD OF PRELIMINARY POLARIZATION BY SOME TRANSDUCERS

For definiteness we consider two types of converters are commonly used in engineering. This is the rod or disc transducer with a transverse polarization, with electrodes on the front surface as shown in Figure 1. The number of electrodes for disk converters may vary.

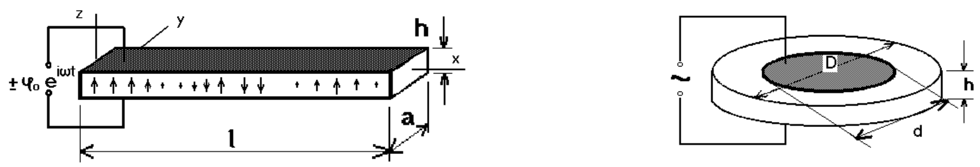

Figure 1: Two types of transducer.

For the rod transducer it is considered the changes in the residual polarization by the following law

$$\frac{\mathbf{P}_0}{p_{sat}} = \sum_{k=1}^{N} \mathrm{Sin}\,(\pi k x / l)$$

In other cases, a general theory of determination of residual polarization and strain in polycrystalline ferroelectric materials after the process of polarization is described in [4, 5]. Therefore, we shall confine ourselves only of the formulation the basic assumptions and the suggestions, required in the future. Specifically, we assume that the polarized ceramics satisfies locally transversely isotropic body. The fore, the physical characteristics of the material are the functional relationship of the form:

$$\mathbf{S} \equiv \mathbf{S}(\mathbf{P}_0^{(i)}) = \mathbf{R}^T \cdot \overset{0}{\mathbf{S}} \cdot \mathbf{R} = \mathbf{S}_0 + \frac{|\mathbf{P}_0^{(i)}|}{p_{sat}} \mathbf{R}^T \cdot \mathbf{S}_1 \cdot \mathbf{R},$$

$$\mathbf{d} \equiv \mathbf{d}(\mathbf{P}_0^{(i)}) = \mathbf{L}^T \cdot \overset{0}{\mathbf{d}} \cdot \mathbf{R} = \frac{|\mathbf{P}_0^{(i)}|}{p_{sat}} \mathbf{L}^T \cdot \mathbf{d}_1 \cdot \mathbf{R},$$

$$\text{э} \equiv \text{э}(\mathbf{P}_0^{(i)}) = \mathbf{L}^T \cdot \overset{0}{\text{э}} \cdot \mathbf{L} = \text{э}_0 + \frac{|\mathbf{P}_0^{(i)}|}{p_{sat}} \mathbf{L}^T \cdot \text{э}_1 \cdot \mathbf{L},$$

where $\mathbf{P}_0$ - the vector of the residual polarization, whose magnitude is a function of coordinates; $\mathbf{L}$, $\mathbf{R}$ - the transition matrix from the local to the global axes. All necessary features are determined by the rules laid down in the developed finite-element complex ACELAN, according to the algorithms described in the above-mentioned works.

To demonstrate the field of the residual polarization, we consider two examples, two disks, on the face which are arranged circular and annular electrodes. The cross sections are shown in Figure 2, symbols “+ -“ indicates the signs of the electric potential at the electrodes.

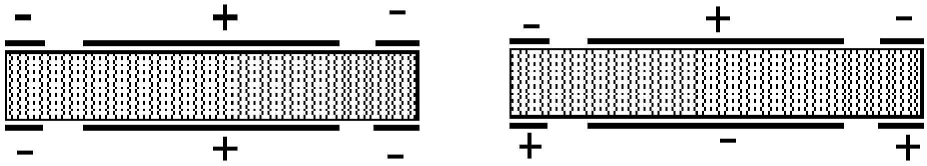

Figure 2: The cross sections of circular converters.

After completing the process polarization the field of residual polarization has the form shown in Figure 3. In the figures the arrows indicate the direction of the vector of the residual polarization, and the length of the arrow characterizes the value of polarization. By axial symmetry it has been shown only half section.

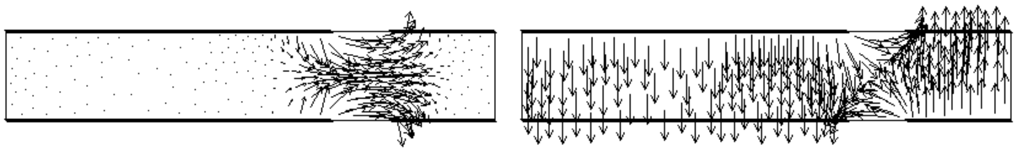

Figure 3: Half cross sections of circular converters after polarization.

From the figures it is seen as strongly influenced by changes in the polarity of the potential on change in the internal structure of the material.

3 THE OSCILLATIONS OF ROD TRANSDUCER WITH A NONUNIFORM POLARIZATION

In this case, the problem becomes one-dimensional, and is described by the follows equations: field equations of motion and electrostatics, the constitutive equations and geometric equations, relating strain and electric field with displacement and electric potential

$$\frac{\partial q_1}{\partial x} = -\rho\omega^2 u, \quad \frac{\partial D_3}{\partial z} = 0. \qquad \begin{aligned} \varepsilon_{11} &= (S_{11} - i\alpha)\sigma_{11} + (d_{31} - i\beta)E_3, & \varepsilon_{11} &= \partial u / \partial x, \\ D_3 &= (d_{31} - i\beta)\sigma_{11} + (\text{э}_{33} - i\gamma)E_3. & E_3 &= -\partial\varphi / \partial z. \end{aligned}$$

The solution of equations must be subordinated to the following boundary conditions

$$\varphi(x,t) = \pm\varphi_0 e^{i\omega t}, \quad (z = \pm\frac{h}{2}). \qquad q_1(x,t) = p e^{i\omega t}, \quad (x = 0, x = l).$$

Inhomogeneous polarization can be described by the following dependencies

$$S_{11}(p) = \overset{0}{S}_{11} + f_\gamma(p(x))\,\overset{1}{S}_{11}, \quad d_{31}(p) = f_d(p(x))\,\overset{1}{d}_{31}, \quad \text{э}_{33}(p) = \overset{0}{\text{э}}_{33} + f_{\text{э}}(p(x))\,\overset{1}{\text{э}}_{33}.$$

In addition, we assume that the damping is not related to the nonhomogeneity of the polarization, whereas the coefficients α, β, γ are independent of the coordinate x. Consider a few common cases in the calculation of these converters. For all the following examples were chosen values ceramics PZT-19

$$\overset{0}{S}_{11} = 14.7059 \cdot 10^{-12}, \quad \overset{1}{S}_{11} = -2.4059 \cdot 10^{-12} (N/m^2); \quad \overset{1}{d}_{31} = -1.23 \cdot 10^{-10} (m/V);$$

$$\overset{0}{э}_{33} = 682.5\varepsilon_0, \quad \overset{1}{э}_{33} = -47.5\varepsilon_0, \quad \varepsilon_0 = 8.85 \cdot 10^{-12} (C^2/Nm^2).$$

3.1 Homogeneous polarization, there is no damping

This simple case is defined by a constant function of polarization $p(x) \equiv \dfrac{\mathbf{P}_0}{p_{sat}} = 1$, and is characterized by odd harmonics in the frequency response of the admittance, as shown in Figure 4.

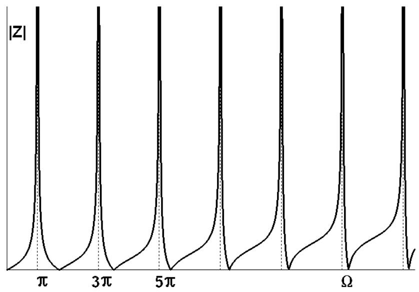

Figure 4: Odd harmonics of homogeneous polarized transducer.

3.2 Nonhomogeneous polarization, there is no damping

Some researchers in the calculation of the physical characteristics of nonuniformly polarized converters often neglect of the influence of heterogeneity on the elastic coefficients and dielectric permittivity. In this case, the functions describing the heterogeneity of the polarization, takes the form

$$S_{11}(p) = \overset{0}{S}_{11}, \quad d_{31}(p) = f_d(p(x))\,\overset{1}{d}_{31}, \quad э_{33}(p) = \overset{0}{э}_{33}.$$

Currently, when using finite-element approaches to solving complex problems, such assumptions do not lead to a simplification of solving the problem, but can lead to significant errors. Let us demonstrate this on the example of rod converter. Let the function describing the heterogeneity of polarization will include only one harmonic. Consider three cases. In the first case we have only the first harmonic $p(x) = \mathrm{Sin}(\pi x/l)$, in the second case, the second harmonic $p(x) = \mathrm{Sin}(2\pi x/l)$, and the third case, the third harmonic $p(x) = \mathrm{Sin}(3\pi x/l)$. The frequency response of the admittance for this case is shown in Figure 5.

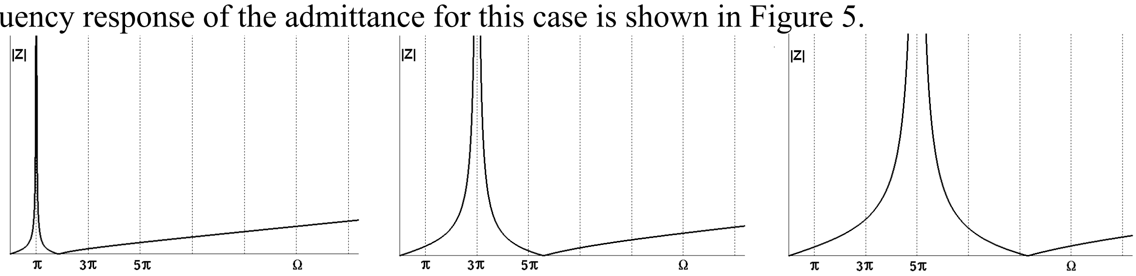

Figure 5: AFC of the cases

$p(x) = \mathrm{Sin}(\pi x/l)$, $p(x) = \mathrm{Sin}(2\pi x/l)$, $p(x) = \mathrm{Sin}(3\pi x/l)$ respectively.

Now consider the case when all the characteristics depend on the polarization. Let each of that functions change according to the same law, i.e.

$$S_{11}(p) = \overset{0}{S}_{11} + f_{\eta}(p(x))\overset{1}{S}_{11}, \quad d_{31}(p) = f_d(p(x))\overset{1}{d}_{31}, \quad э_{33}(p) = \overset{0}{э}_{33} + f_э(p(x))\overset{1}{э}_{33},$$

and $f_{\eta}(p(x) = f_d(p(x) = f_э(p(x)$.

Then these same cases give us the characteristics shown in Figure 6.

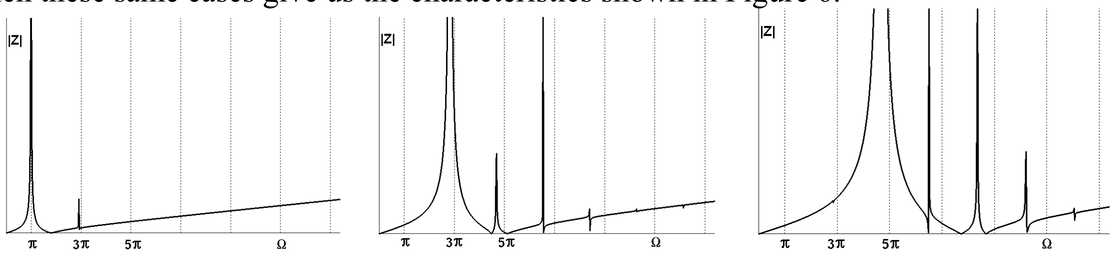

Figure 6: AFC of the cases $f_{\eta}(p(x) = \mathrm{Sin}(\pi x / l)$, $f_d(p(x) = \mathrm{Sin}(2\pi x / l)$, $f_э(p(x) = \mathrm{Sin}(3\pi x / l)$ respectively

3.3 Inhomogeneous polarization and a damping

However, all these cases were considered in the absence of damping. We introduce in a model of the damping then obtain the form of AFC, shown in Figure 7.

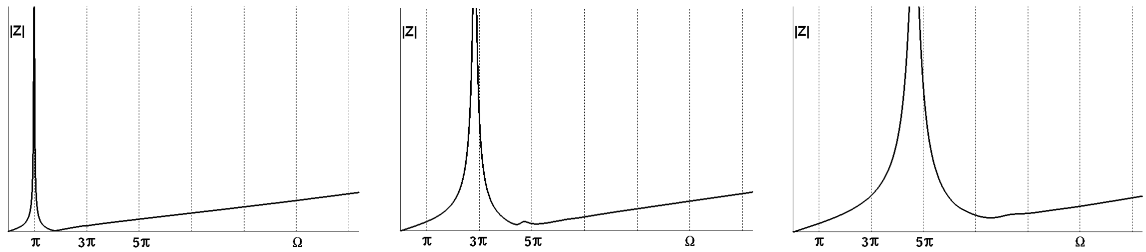

Figure 7: AFC of the cases

$$\alpha = 0.221 \cdot 10^{-17}(N/m^2), \quad \gamma = 0.124 \cdot 10^{-14}(C^2/Nm^2)(m/V), \quad \beta = 0.15 \cdot 10^{-16}(m/V).$$

The results of this study showed that by changing the nature of the pre-polarization, you can control the characteristics of the rod transmitter. In particular it is possible to excite only one mode of vibration which connected with a nature of the function of polarization. Allowance the damping shows that not arise here parasitic modes.

4 TIME-DEPENDENT PROBLEMS FOR ROD NONUNIFORMLY POLARIZED CONVERTER

In this case, we again consider a one-dimensional problem. Here, too, for the integration along the coordinates used technique of the finite-elements, and for the integration along the time a Newmark method is considered. The main objective of our study is to evaluate the influence of heterogeneity of the polarization on the physical characteristics of the problem. Let us take as the unknown function the displacement of the right end of the rod relative to the left. Limiting by only one result, when to the electrodes suddenly applied of a constant difference of electric potential, let us compare the displacement of the rod with a uniform and nonuniform polarization. The function of the inhomogeneity of the polarization we take as

$$p(x) = \sum_{k=1}^{N} \mathrm{Sin}\ (\pi k x / l), \quad N = 5.$$

The moving of the right end of the rod is shown in Figure 8.

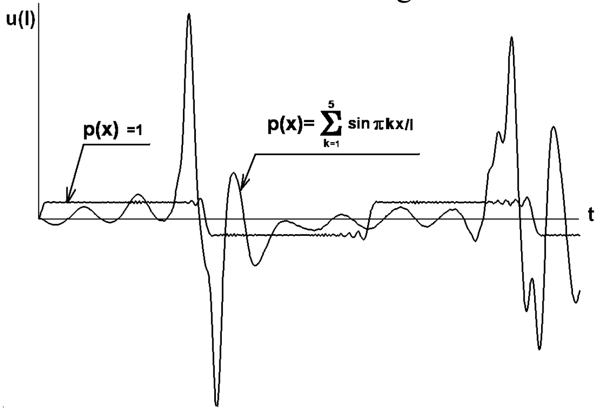

Figure 8: The displacement of the right end.

For a rod with non-uniform polarization we can observe a significant spike of displacement for the times when the wave is reflected from the right end. For a rod with a uniform polarization of such an effect is not observed.

5 MODAL ANALYSIS OF NONUNIFORMLY POLARIZED DISKS

There are many most important physical characteristics of transducers such as the eigenvalues and eigenvectors of oscillations, the amplitude-frequency characteristic of admittance, a vibration velocity on the surface of the transducer and some others. All these characteristics are calculated in the small electrical and mechanical fields, that the polarization is not unchanged. Let us consider the basic computational formulas of modal analysis for nonhomogeneously polarized piezoceramic elements. This analysis is conducted in two stages. Initially, the fields of nonhomogeneous polarization were determined, as it was described above in section 2. And then directly is conducted a modal analysis. It is noteworthy that in the second stage there is no phenomenon of depolarization. Therefore, all the calculation formulas of modal analysis are the same as in the homogeneous polarization

$$-\omega^2 \mathbf{M}\cdot\mathbf{A} + \mathbf{K}\cdot\mathbf{A} = 0$$

where

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{\mathbf{uu}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \quad \mathbf{K} = \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}^{\mathbf{T}} & -\mathbf{K}_{\phi\phi} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} \mathbf{U} \\ \phi \end{pmatrix}.$$

- the mass matrix, stiffness matrix and generalized vector of unknown. The elements of these matrices are not given here due to cumbersome formulas. We note only that the stiffness matrix has a saddle structure and slightly different from the corresponding matrix for body with a uniform polarization. All this suggests, all the theorems of spectral analysis for electroelastic uniformly polarized continua are also valid for nonuniformly polarized continua. Leaving aside the mathematical aspect of the issue, we present some of their own forms for nonuniform polarized disc shown in Figure 2 on the right.

Figure 9 presents eight forms of high-frequency oscillations of the disc. First, at lower frequencies, you can see a series of dominated thickness oscillations with frequency increasing on the surface. However, these modes are useless in practice, because the transducer oscillates in antiphase and being loaded on the acoustic environment can not create a useful acoustic pressure. However, further increase in the frequency is characterized the thickness fluctuations in-phase oscillations by a thickness of the transducer, that is quite closely to the fluctuations of the sample with uniformly polarized. At the same time, the resonances are greater than about 15% □ 20% as compared with resonances of uniformly polarized transducer.

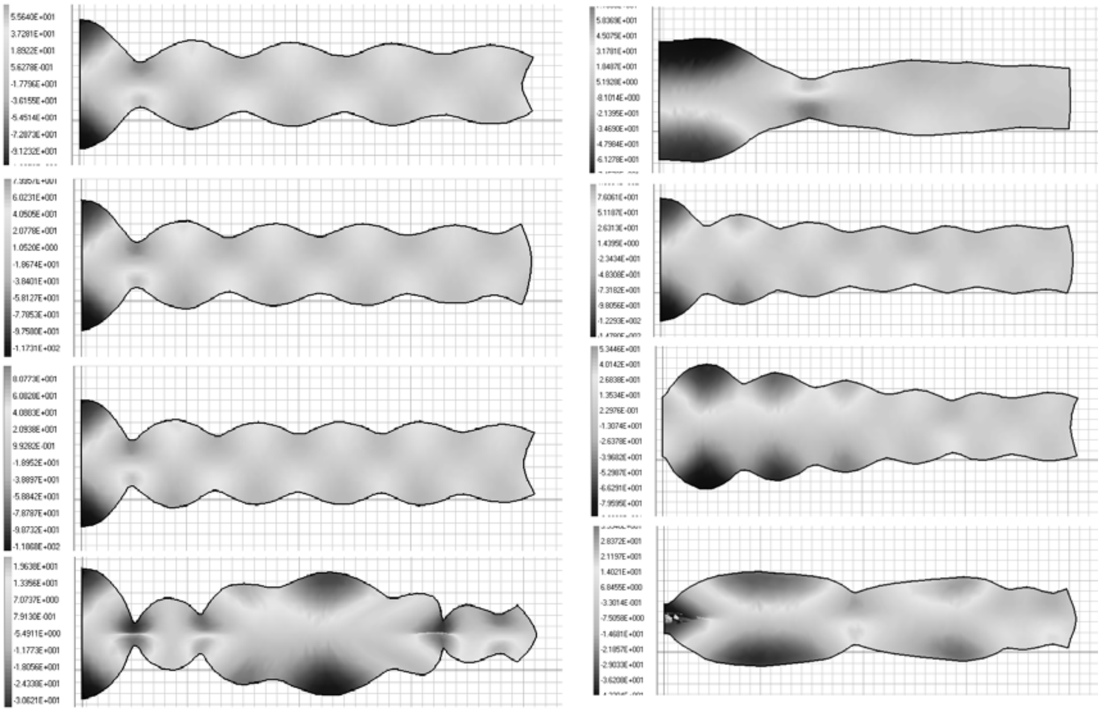

Figure 9: Eight forms of high-frequency oscillations.

6 DISCUSSION

Creating a non-uniform polarization of piezoceramic bodies is an interesting problem in a practical way. The ceramic elements with non-uniform polarization behave under mechanical and electrical loads very differently than the same elements with homogeneous polarization. This behavior can be explained in terms of mechanics, as if the body connected together some parts whith piecewise homogeneous properties. In such bodies, the physical properties of the individual constituent parts play a certain role in the behavior of the whole structure. But unlike the piecewise constant properties, nonuniformly polarized ceramic bodies vary continuously its mechanical, piezoelectric and dielectric properties, although the gradient of the changes can be quite large. On the example of the longitudinal oscillations of a piezoceramic rod with a non-uniform polarization, there are significant differences in it behavior compared to the same rods that have a constant polarization. The most remarkable characteristic of the inverter is that it can resonate at only one frequency. But this is possible only if in it the required field of polarization is created. In non-stationary problems such properties are of even greater importance. There may be cases of increasing the amplitude of displacement several times compared with the analogy of a permanent pre-polarization. But most strikingly nonuniformity polarization properties are disclosed in the bulk solids. It brought to the forefront the problem of optimization problems and the technology plan. You want to create a regime of polarization to get one or the other sensor with previously introduced desirable properties. But along with the technological challenges of the polarization you necessarily face the problem of mathematical modeling of such converters.

7 CONCLUSIONS

- The work is devoted to the study of the dynamic behavior of nonuniformly polarized piezoceramic elements.
- The field of preliminary polarization for disk converters is calculated with help of previously developed algorithms.
- The formulas of the elastic, piezoelectric and dielectric properties of ceramics uniformly polarized in the local axes connected with the polarization vector are presented.
- The harmonic longitudinal oscillations of the rod with a cross inhomogeneous polarization was considered; the cases dealt with the influence of heterogeneity of the polarization on piezoelectric properties and the set of elastic, piezoelectric and dielectric properties are treated separately.
- The influence of the nature of the internal damping of oscillations of an inhomogeneously polarized rod is considered and noted that the damping can also be irregular as the physical properties of material.
- We considered the transient problem; we show that if the function of polarization is represented by a segment of the harmonics series, then in the time moments of reflection of the passing wave from the end of the rod it has a displacement in a few times greater than the displacement in the rod with uniformly polarized.
- An algorithm for solving dynamic problems for uniformly polarized disks was developed; the analysis of solutions of some problems gives us the opportunity to judge the form of the natural oscillations.
- It is concluded that the properties of piezoelectric bodies can be changed by the prepolarization.

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