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NONLINEAR DYNAMIC ANALYSIS OF FRAME STRUCTURES UNDER SEISMIC EXCITATION CONSIDERING SOIL-STRUCTURE INTERACTION AND ELASTO-PLASTIC SOIL BEHAVIOR

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Abstract. *Soil-structure interaction has been extensively studied over the past years, since the majority of civil structures are supported on soil. Within this area, the study of systems subjected to seismic excitation is one of the most important topics. The large frequency content of the earthquake has a major influence on the behavior of the ground, since the soil properties vary with the excitation frequency, and on the dynamic response of the structure. This work presents a methodology for the nonlinear dynamic analysis of frame structures considering the nonlinear behavior of the soil and of the structure. The soil is represented by springs with one-dimensional elastic-plastic constitutive law. The system is subjected to known seismic excitations. The soil-structure system is discretized in space considering a nonlinear finite element formulation and the resulting system of ordinary differential equations of motion are solved by the Newmark or Runge-Kutta integration method in combination with an iterative Newton-Raphson technique. A parametric analysis is performed to study the influence of the elasto-plastic behavior of the soil on the time response of the system, as well as the resonance curve.*

1 INTRODUCTION

The analysis of the response of structural systems considering soil-structure interaction has been a subject of great interest in the dynamics of structures, since most structures rests directly or indirectly on the ground. A topic of particular interest in this area is the study of the soil-structure interaction when the structure is subjected to seismic actions. Seismic events represent one of the most complex dynamic forces in civil engineering structures. The importance of the study of such problem considering the effect of soil-structure is connected to the inherent characteristic of the soil to change its properties as a function of the frequency content. The study of the geometric nonlinearity of structures has also been an important issue in structural analysis, in particular in the case of structures such as frames, arches and trusses. In the study of geometric nonlinearity, the formulations are classified according to the type of reference used. Distinct researchers have developed formulations in total or updated Lagrangian reference frame. Galvão [1,2] and Silva [3] highlight the works published by Loi and Wong [4], Torkamani et al. [5] Pacoste and Erikson [6] as significant contributions in the development of Lagrangian formulations. Comparisons among diverse geometric nonlinear formulations have also attracted the interest of many researchers. In this article only the updated Lagrangian formulation is employed.

In this context, soil-structure interaction becomes an essential topic in addressing problems of structures subjected to earthquakes. Several researchers have studied this phenomenon. The simplest hypothesis is to consider the soil as an elastic medium represented by elastic springs as in Hetenyi [7], Vlasov [8] and Aristizabal-Ochoa [9] and, recently, Nguyen [10] and Paullo [11]. A more realistic hypothesis consists in considering the soil as a continuum. Alsaleh Shahrour [12] and Clouteau et al. [13] studied problems of soil-structure interaction with the soil modeled as a continuous medium. A simplifying assumption is the consideration of soil as discrete flexible support, which is a useful hypothesis when the focus lies in the study of the behavior of structures with discrete supports such as arches and frames. Wolf [14], Halabian [15] and Ganjavi and Hao [16] studied the dynamic behavior of frames using this simplifying assumption.

In this paper, a methodology for the nonlinear dynamic analysis of planar frame structures with consideration of the effect of soil-structure interaction using simplified nonlinear spring models to represent the soil is presented. The structure-soil system is discretized using a nonlinear finite element formulation and the resulting system of equations of motion are solved by the Runge-Kutta method. Parametric analyses are performed to study the influence of the elastic-plastic behavior of the soil on the nonlinear response of structures subjected to base excitation. The implementations are in the CA-ASA program, implemented initially by Silveira [17], and modified by Silva. [3]

2 FORMULATION

2.1 Equilibrium equations.

Considering only the transversal inertia and damping associated (hypothesis consistent with the Euler-Bernoulli beam theory), the equation of motion at time t can be obtained by the principle of virtual work as:

$$\int_S \boldsymbol{f}(t)\delta\mathbf{u}(x,t)dS = \int_V \boldsymbol{\tau}(t)\delta\boldsymbol{\varepsilon}(x,t)dV + \int_V \rho\ddot{\mathbf{u}}(x,t)\delta\ddot{\mathbf{u}}(x,t)dV + \int_V c\dot{\mathbf{u}}(x,t)\delta\dot{\mathbf{u}}(x,t)dV \qquad (1)$$

where $\mathbf{u}(t)$ is the time dependent displacement vector, ρ is the material density, c is the damping coefficient and $\boldsymbol{f}$(t) is the vector of external forces.

Considering the Euler-Bernoulli beam-column theory, the displacements can be obtained through interpolation of nodal displacements expressed in matrix form as:

$$\mathbf{u}(x,t) = \boldsymbol{H}(x).\boldsymbol{u}(t),\ \dot{\mathbf{u}}(x,t) = \boldsymbol{H}(x).\dot{\boldsymbol{u}}(t),\quad \ddot{\mathbf{u}}(x,t) = \boldsymbol{H}(x).\ddot{\boldsymbol{u}}(t) \tag{2}$$

where $\boldsymbol{u}(t)$ is the time dependent displacements vector, H(x) is the matrix of interpolation functions. In the present work hermitian shape functions are used for the flexural displacements and linear functions for the axial displacements [3].

The kinematic strain-displacement relations are given by:

$$\boldsymbol{\varepsilon}(x,t) = \boldsymbol{B}(x).\boldsymbol{u}(t) \tag{3}$$

where $\boldsymbol{B}$(x) is calculated by differentiation of H(x), (Galvão [2]).

Introducing Eq. (3) and Eq. (2) into Eq. (1), the following system of nonlinear equations of motion is obtained in matrix form:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}.\dot{\boldsymbol{u}}(t) + \boldsymbol{Fi}(t) = \boldsymbol{Fe}(t) \tag{4}$$

where:

$$\boldsymbol{M} = \int_0^L \rho\boldsymbol{H}(x)^{\mathrm{T}}\boldsymbol{H}(x)dx\ ,\boldsymbol{C} = \int_0^L c\boldsymbol{H}(x)^{\mathrm{T}}\boldsymbol{H}(x)dx,\ \ \boldsymbol{F_{int}} = \int_0^L \boldsymbol{B}(x)^{\mathrm{T}}\boldsymbol{\tau}dx\ ,\ \ \boldsymbol{F_e}(t) = \int_0^L \boldsymbol{H}(x)^{\mathrm{T}}\boldsymbol{f}(x,t)dx \tag{5}$$

The vector of internal forces, $\boldsymbol{F_{int}}$, at a time t+Δt, is given by:

$${}^{t+\Delta t}\boldsymbol{F_{int}} = {}^{t}\boldsymbol{F_{int}} + \Delta\boldsymbol{F_{int}},\ \Delta\boldsymbol{F_{int}} = \mathbf{K}(\mathbf{U}).\Delta\mathbf{U} \tag{6}$$

where $\Delta\mathbf{U}$ is the vector incremental nodal displacements, $\mathbf{K}(\mathbf{U})$ is the nonlinear stiffness matrix.

The vector of external forces, $\boldsymbol{F_e(t)}$, is defined as the vector of force magnitude $\boldsymbol{Fr}$, multiplied by a time dependent function, *f(t)*, that is:

$$\boldsymbol{F_e}(t) = \boldsymbol{F_r}.f(t) \tag{7}$$

where $\boldsymbol{Fr}$ is given by:

$$\boldsymbol{F_r} = \int_0^L \boldsymbol{H}(x)^{\mathrm{T}}fe(x)dx \tag{8}$$

2.2 Soil-structure interaction – discrete model.

Consider initially that the soil can be represented by a discrete horizontal spring, as shown in Figure 1.

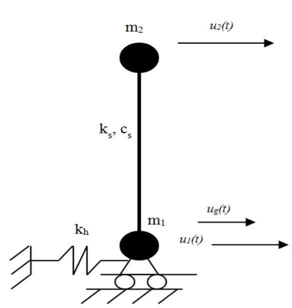

Figure 1 : Soil-structure interaction – discrete soil model.

The flexibility of the soil is represented by a discrete spring system with stiffness k_h. In the case of a conventional fixed base, that is, when the value k_h tends to infinity, the displacement u1(t) =0 and the system can be reduced to a one degree of freedom model described by the following equation:

$$m_2.\ddot{u}_2(t) + c_s\dot{u}_2(t) + k_s u_2(t) = -m_2\ddot{u}_g(t) \tag{9}$$

This reduction is obtained by imposing the condition u1 (t) = 0. However, when u1 (t) is nonzero, the entire system must be considered. In such case:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix} \begin{Bmatrix} \dot{u}_2 \\ \dot{u}_1 \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} = -\begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} \ddot{u}_g \tag{10}$$

Adding the soil stiffness and damping to the corresponding degrees of freedom results in:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s + c_h \end{bmatrix} \begin{Bmatrix} \dot{u}_2 \\ \dot{u}_1 \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_h \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} = -\begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} \ddot{u}_g \tag{11}$$

Generalizing for a system with n degrees of freedom, the inclusion of the soil can be obtained by adding a diagonal stiffness and damping matrix corresponding to the degrees of freedom of the structure supports. Then the system de-scribed in (2.60) can be modified to:

$$\mathbf{M}\ddot{\overline{\mathbf{U}}}(t) + (\mathbf{C} + \mathbf{C}_s)\,\dot{\overline{\mathbf{U}}}(t) + (\mathbf{K} + \mathbf{K}_s)\overline{\mathbf{U}}(t) = -\mathbf{Mr}\ddot{u}_g(t) \tag{12}$$

where the $\mathbf{C}_s$ e $\mathbf{K}_s$ are the soil properties.

2.3 Elasto-plastic behavior of the soil

The foundation soil is represented by one-dimensional elasto-plastic springs whose evolution in the elastic domain is described by the yield function:

$$\mathbf{Fi} - (\mathbf{Fp} + \beta\Delta\mathbf{p}) = 0 \tag{13}$$

where Fi is the force in the spring, Fp is the yield strength, β is the modulus of strain hardening and Δp is the accumulated plastic strain in the spring. If β is constant, the spring stiffness in the elastic-plastic regime may be calculated as:

$$k_{elasto-plastic} = \frac{k_{elastic} \cdot \beta}{k_{elastic} + \beta} \tag{14}$$

Equation (14) indicates that the stiffness of the spring in the elasto-plastic regime can be described as a bilinear function with positive isotropic hardening, as shown in Figure 2.

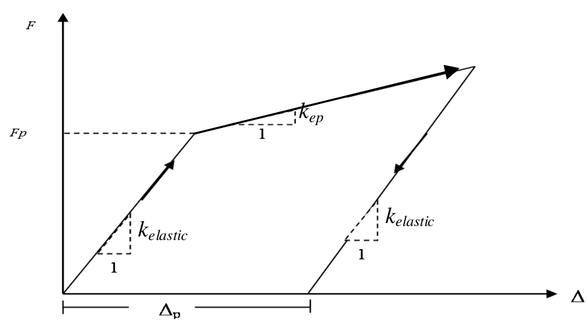

Figure 2 : Elasto-plastic behavior of the soil represented by a bilinear function.

In general, a continuous medium such as soil has a complex nonlinear behavior. Thus obtaining an explicit expression for the elasto-plastic stiffness, as shown in Equation (14), may be difficult or even impossible. To circumvent this problem, we resort to incremental analysis, through which it is possible to obtain approximations of the elastic-plastic stiffness for small load increments. In this work the calculation of the internal forces in the springs in the elasto-plastic regime is obtained by the implicit Euler algorithm. Details can be found in Souza Neto [18].

2.4 Direct integration of the equations of motion.

In the present work, the time response is obtained by direct integration of the system of equations of motion by employing the implicit to fourth order Runge-Kutta method and the two-steps procedure using the coefficients presented in Butcher [19], obtained by means of the Gaussian quadrature. The method is here adapted to nonlinear systems solved in conjunction with an iterative secant type procedure.

3 NUMERICAL EXAMPLES

3.1 Shallow circular arch resting on a nonlinear soil.

Fist a shallow circular arch with flexible supports is considered, whose geometric properties are taken from [3]. The arc is subjected to vertical time-varying base displacements and a static force applied on its top. The arc is discretized using twenty equal nonlinear bar elements. The geometrical and material properties are shown in Table 1.

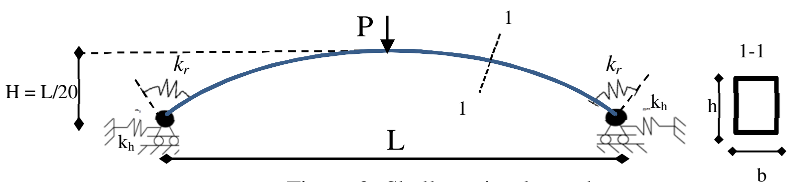

Figure 3: Shallow circular arch.

Parameter	Symbol	Unit	Value
Modulus of Elasticity	E	MPa	2000
Cross sectional area	A	dm^2	1.00
Moment of inertia	I	dm^4	1.00
Length	L	m	10.0
Material density	ρ	dg/cm^3	0.0240

Table 1: Arch's proprieties.

Figure 4 shows the vertical displacement of the center of the arch as a function of time, considering a sinusoidal base excitation pulse with duration of Tg = 25sec and acceleration amplitude A of 0.8 times the acceleration of gravity. In this example, the arch is rigidly supported. After the base excitation, the damped free vibration response converges to a nonlinear equilibrium position corresponding to the adopted static load level, P. For P=0.80 the response converges to a pre-buckling configuration. For P=1.27 and P=1.75 the response converges to a post-buckling configuration. These values agree with those obtained in the static analysis of the arch under vertical concentrated load.

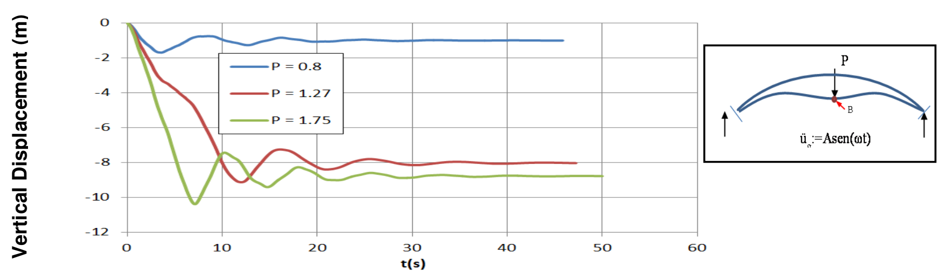

Figure 4 : Response of the arch under a short sinusoidal base pulse. Vertical displacement vs. time. C=0.75M, ω=0.80rad/s. Base excitation: Tg = 25s, A=0.8g.

Figure 5 shows the final equilibrium position for increasing static load level P for the arch starting from rest considering two values of the base excitation magnitude. When the amplitude of the base excitation increases from 0.8g to 2.2g, buckling occurs for very low load levels. The final configuration of the arch for load level between the highest and the lowest limit points depends on the initial conditions.

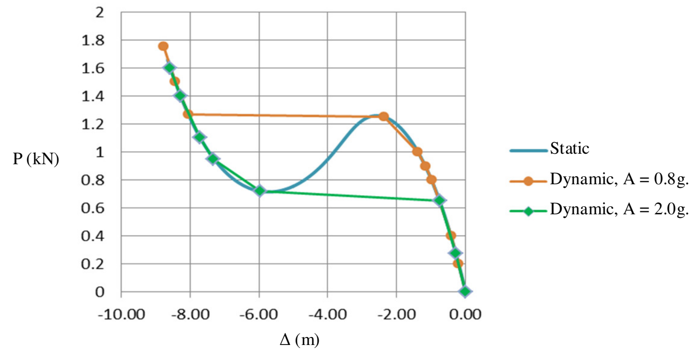

Figure 5: P vs. Vertical displacement at B in steady state. C=0.75M, ω=0.80rad/s. pulse duration Tg = 25s.

Now the horizontal flexibility of the foundation soil represented by horizontal springs with stiffness k_h is considered. Figure 6 displays the time response of the arch considering a sinusoidal base excitation pulse with a time duration Tg = 15sec for increasing values of the foundation stiffness k_h. Comparing the results with those in Fig. 4, a significant difference is observed due to the consideration of a flexible base. For values of $k_h > 10^6$kN/m response of the system response is very close to that of rigid base.

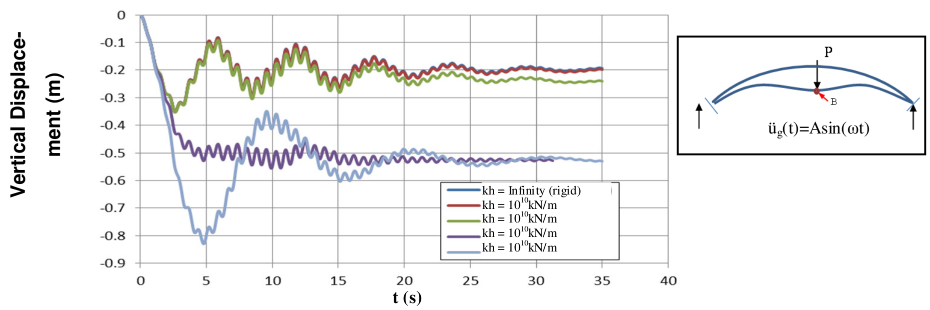

Figure 6 – Time response of the pre-loaded arch under base excitation. C=0.75M, ω=5. 0rad/s, P=0.2. Tg = 15s.

Figure 7 shows the resonance curve (vertical displacement versus forcing frequency) for an arch with a flexible support with rotational linear stiffness k_r. It is observed that when the stiffness k_r decreases, the resonant peak moves to the left due to a decrease in the natural frequency and the value of maximum displacement increases.

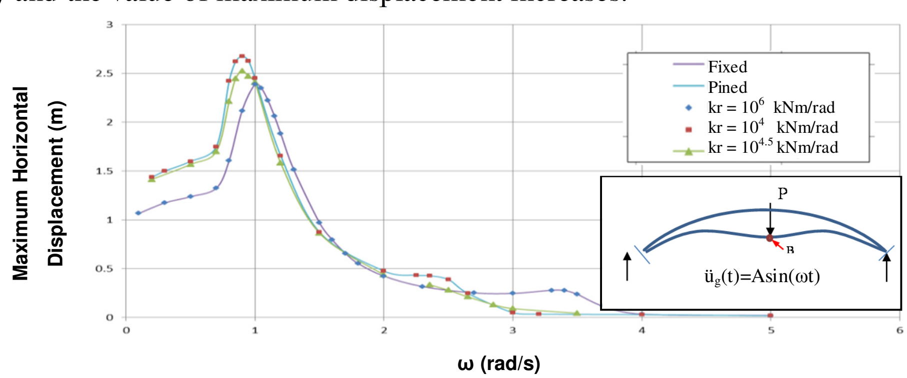

Figure 7: Arch with flexible supports under harmonic base excitation. Maximum displacement as a function of the forcing frequency. A=0.4g, C=0.25M, P=0, k_h=infinity.

3.2 Slender tower with a concentrated mass at the top and elasto-plastic soil behavior.

Here a model of a concrete water tower simulated by Halabian and Naggar [15] is considered. In this paper the structure is modeled as a column of constant cross section and with a concentrated mass at the top and it is subjected to a horizontal base displacement, as illustrated in Figure 8. The structure is discretized using ten elements of equal length. The model properties are shown in Table 2. A flexible support with rotational elastic stiffness k_r and plastic spring stiffness kep is considered. Plastic deformations occur when the moment at the base is equal to or greater than Mp.

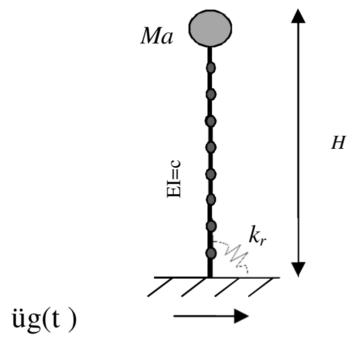

Figure 8: Tower model with elasto-plastic support

Parameter	Symbol	Unit	Value
Modulus of elasticity	E	GPa	31.00
Cross sectional area	A	m^2	6.28
Moment of inertia	I	m^4	39.52
Tower height	H	m	70.00
Material density	ρ	$kg/\ m^3$	2400.00
Concentrated mass	Ma	kg	150000.00

Table 2: Tower proprieties.

Figure 9 shows the time response of the horizontal displacement at the top of the tower, considering a rotational elasto-plastic spring with plastic moment Mp and stiffness in the elasto-plastic regime kep. It can be seen that when an elasto-plastic rotational spring is considered the maximum steady-state amplitude decreases due to dissipation of energy in the spring. In this case it is not possible to consider perfect plasticity since the system would be hypostatic. It can also be observed that when the stiffness decreases in elasto-plastic regime the maximum displacement decreases.

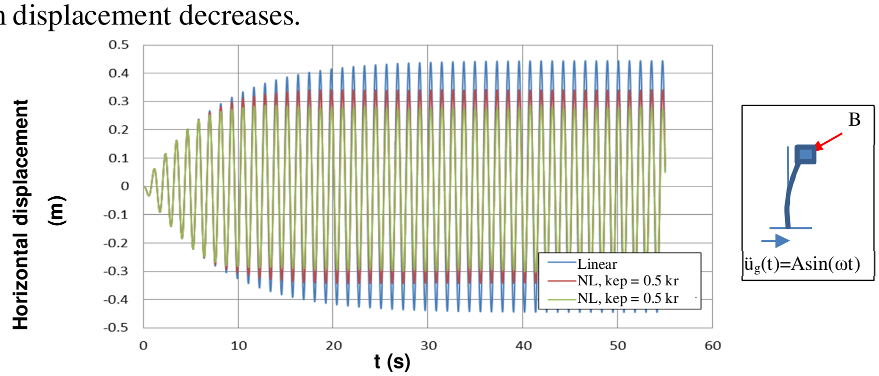

Figura 9: Time response of the horizontal displacement at the top of the tower. A=0.4g, C=0.25M, $k_r=10^{11}$kNm/rad, ω=5.40rad/s, Mp=160MNm.

Figure 10 shows the hysteretic behavior of the elastic-plastic rotational spring for the two elasto-plastic stiffness values considered. The reduction of the stiffness in the elasto-plastic regime entails an increase in the accumulated plastic strain.

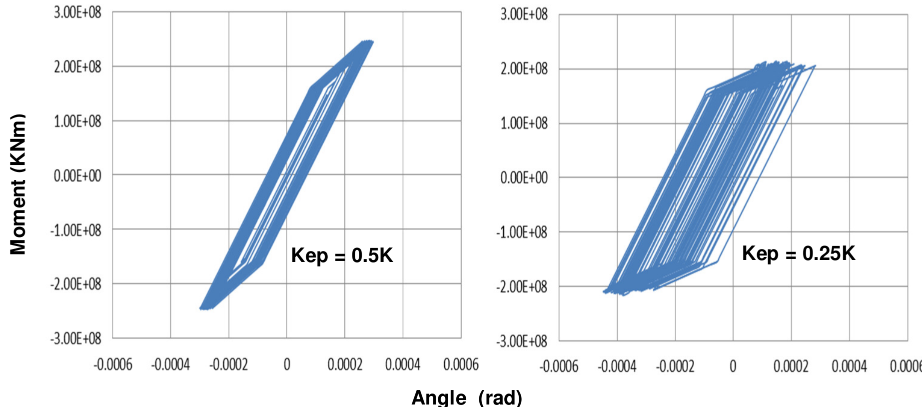

Figure 10: Moment vs. rotation at the base. A=0.4g, C=0.25M, kr=1011kNm/rad, ω=5.40rad/s, Mp=160MNm.

Figure 11 shows the time response of the displacement at the top of the tower when it is subjected to the base acceleration signal of the east-west component of the El Centro earthquake (see Fig. 11.b), with a maximum acceleration of 0.39g. Na Figure 12 shows the hysteretic behavior of the elasto-plastic rotational spring for two values of plastic stiffness.

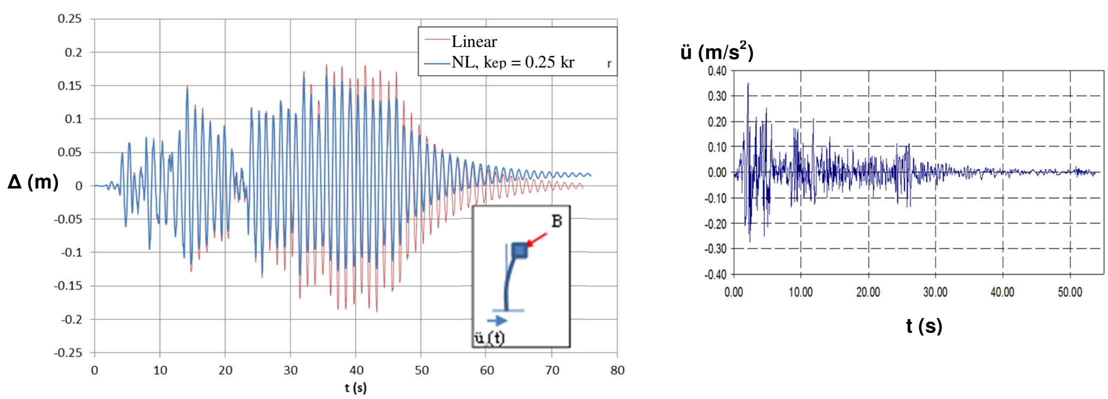

Figure 11: (a) Time history of the horizontal displacement at the top of the tower. (b) El Centro, E-W component of the ground motion acceleration. C=0.25M, kr=10^{11}kNm/rad. Mp=80MNm.

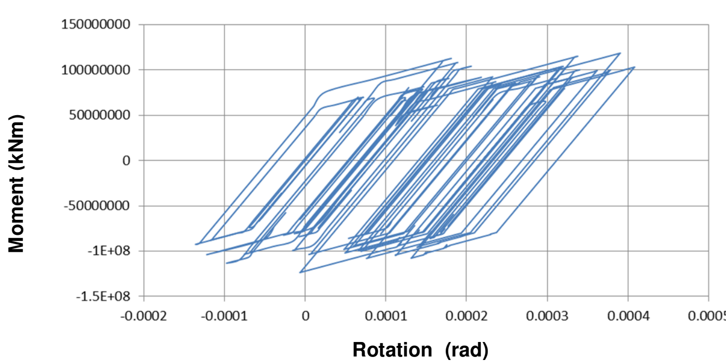

Figure 12: Moment vs. rotation at the base. El Centro. C=0.25M, k_r=10^{11}kNm/rad. Mp=80MNm.

Figures 11 and 12 show that the consideration of the elasto-plastic behavior of the soil modifies the response of the system, reducing the displacements, highlighting the dissipation associated with plasticity as well as the occurrence of a dislocation due to the continuing accumulation of plastic strain.

4 CONCLUSIONS

• The present numerical implementations agree with known results found in the literature, confirming its correctness and accuracy.

• The consideration of soil as elastic supports modifies the response of the studied systems. In all cases the consideration of flexible support leads to a decrease of the natural frequencies as well as an increase in the value of the maximum displacements.

• The plasticity of soil reduces the response of the tower subjected to seismic forces due to dissipation.

• The plasticity of the soil may produce a permanent dislocation after the end of the earthquake.

This is a work in progress, future work will include a detailed analysis of the nonlinear vibrations of frame structures under harmonic and seismic base excitation.

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