11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

TRACING THE EVOLUTION OF BRIDGE NATURAL FREQUENCIES AS A VEHICLE TRAVERSES THE BRIDGE

Daniel Cantero*[1], Eugene J. OBrien[1,2]

[1]Roughan & O'Donovan Innovative Solutions, Ireland
daniel.cantero@rod.ie

[2] University College Dublin, Ireland
eugene.obrien@ucd.ie

Keywords: Vehicle-Bridge interaction; natural frequency; wavelet; dynamics; vibrations.

Abstract. *In bridge engineering, the natural frequencies of the structure are generally considered to remain invariable. Thus, changes in natural frequencies might indicate significant structural deterioration. Other causes that influence the bridge's modal properties include ambient temperature, long term concrete hardening or repair work on the structure. However, traversing vehicle events are generally not associated with this phenomenon.*

In this paper, it is shown how the natural frequencies of a bridge change during the crossing of vehicles. Numerical models are developed to show this result. An Euler-Bernoulli beam is modelled traversed by a single DOF vehicle. The use of such a simple Vehicle-Bridge interaction model is justified by the intentions of this research which are to give additional insight into the structural dynamics of a moving load interacting with a bridge.

The numerical results indicate that the variations in natural frequencies depend greatly on vehicle to structure frequency ratio and mass ratio. In some conditions, significant variations in modal properties are observed. Additionally, it can be analysed from the passing vehicle response. Time-frequency signal analysis of the vehicle's vertical acceleration clearly shows how the frequencies evolve during the event. In the paper, the Wavelet transform is used to analyse the signal and highlight the relevant results. This is achieved due to the variable accuracy of this processing tool and the good frequency localization properties of the wavelet basis used (Modified Littlewood-Paley).

This research is particularly relevant for indirect structure monitoring techniques that use the response of traversing instrumented vehicles to obtain the modal properties of structures. It is shown that in certain circumstances a standard Fourier analysis of the accelerations is not sufficient, and a time-frequency analysis is needed.

1 INTRODUCTION

Dynamic Vehicle-Bridge Interaction (VBI) is a complex problem that has been investigated extensively. Field measurements, experimental testing, new mathematical tools and higher computing capabilities have helped to improve the understanding in this field. However, there is still some additional insight to be gained into the fundamentals of VBI. This paper aims to clarify one aspect of this problem.

In recent decades Structural Health Monitoring (SHM) has become an important area of research. In particular, for bridge assessment, over the last decade a promising indirect technique has emerged, namely, ‘drive-by’ bridge inspection [1]. It claims that an analysis of the accelerations of a passing vehicle should provide information about the bridge's natural frequencies. This idea is first presented and validated by [2] and has since then been investigated by many research groups [1, 3-6]. These studies show the potential of this new idea which offers many advantages, including periodic and more frequent monitoring, ease of sensors’ power supply and little traffic disruption.

However, most of the research fails to recognize the time-variant nature of the VBI problem. In here, by means of a simple model, it is shown how the frequencies change with changing vehicle position. It is also proposed that the extraction of frequencies should be performed by means of an adequate time-frequency signal analysis. In particular this paper briefly presents the Wavelet Transform and uses it to track the evolution of frequencies for various positions of a vehicle.

This paper presents the results with a clear correspondence to the drive-by bridge inspection technique. A reduced velocity of the vehicle is considered, which is necessary to provide results with sufficient accuracy, as shown in [7]. Additionally, for clarity, only the first natural frequency of the structure is under investigation since its contribution to the dynamic response is generally the most important. However the implications of the presented results apply to all VBI problems and for any degree of complexity.

First, the paper starts with a brief description of the numerical model used. Then it presents the problem of time-varying frequencies in VBI problems. Section 4 analyses the acceleration signal of the traversing vehicle, first using the standard Fourier analysis and then the proposed Wavelet transform. Additional comments are added in the discussion section.

2 NUMERICAL MODEL

The numerical model developed for this study represents a simply supported bridge traversed by a moving vehicle (Figure 1). The structure is modelled as an Euler-Bernoulli beam using the FEM formulation. The vehicle is approximated as a 1-DOF oscillator connected to the profile by a spring and dashpot system. The dynamic interaction between systems is solved in an iterative manner, and the particular numerical values used are listed in Table 1. Note that the vehicle's mass m and suspension stiffness k are adapted in subsequent sections. Additional information on the numerical solution and model properties can be found in [8].

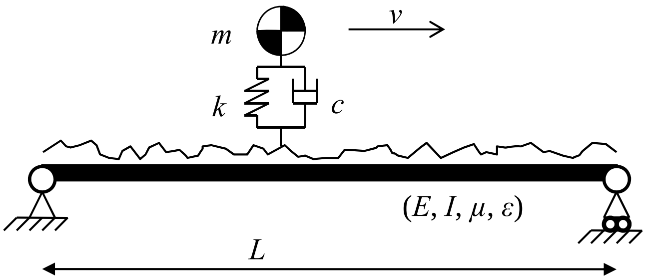

Figure 1: Sketch of Beam and moving 1-DOF oscillating mass.

Model	Variable	Description	Value	Unit
Beam	L	Span	25	m
	E	Young's modulus	35×10^9	N/m^2
	I	Second moment of area	1.3901	m^4
	μ	Mass per unit length	18 358	kg
	ε	Damping	3	%
Vehicle	v	Velocity	5	m/s
	c	Viscous damping	10 000	N·s/m

Table 1: Constant model properties.

Eqs. (1) and (2) present the natural frequencies of the vehicle and the fundamental frequency of the beam respectively, both in Hz. The structure, due to its continuous nature, has an infinite number of natural frequencies, but only the first (fundamental) frequency will be studied here, which for the particular values defined in Table 1 is 4.09 Hz.

$$f_v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

$$f_b = \frac{\pi}{2L^2}\sqrt{\frac{EI}{\mu}} \tag{2}$$

The two non-dimensional parameters used throughout this study are defined in Eqs. (3) and (4). The Mass Ratio (MR) relates the vehicle mass to total structure's mass whereas the Frequency Ratio (FR) is the ratio between vehicle and bridge frequencies.

$$MR = \frac{m}{\mu L} \tag{3}$$

$$FR = f_v/f_b \tag{4}$$

3 COUPLED SYSTEM FREQUENCIES

The complexity of the VBI problem arises mainly due to the time-variant nature of the equations of motion. When the vehicle is on the bridge their responses are coupled together and they cannot be analyzed independently. The system matrices are different for every position of the vehicle on the bridge. Thus, the natural frequencies of the combined system also vary during the vehicle crossing.

Based on the model described in section 2, the coupled equations of motion are obtained for various locations of the vehicle along the bridge span. Performing the eigenvalue analysis on these equations it is possible to extract the system's natural frequencies and their evolution with the vehicle's position. Figure 2 shows the first two system frequencies for the particular case of FR = 0.9 and MR = 0.1. The vehicle's frequency is thus 3.68 Hz and its mass (46 t) and suspension stiffness (24.5×10^6 N/m) are calculated to reach the specified MR. It is evident that while the vehicle is near the supports the frequencies correspond to those of the independent vehicle and bridge systems (Eqs. (1) and (2)). However, when the vehicle is located elsewhere on the bridge, a significant shift of the frequencies is observed and the maximum difference occurs when the oscillator is located at mid-span. In particular for the vehicle the frequency shift is δ_v = 3.68 - 3.12 = 0.56 Hz whereas for the bridge the shift is δ_b = 4.82 - 4.09 = 0.73 Hz.

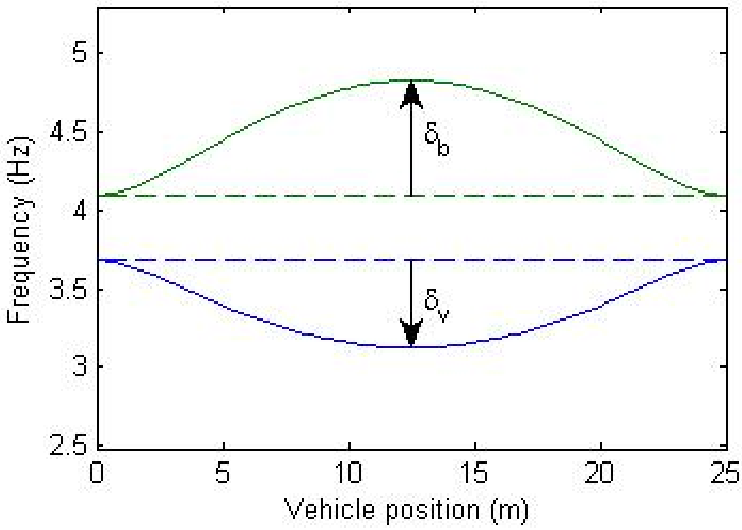

Figure 2: Vehicle (blue) and bridge (green) fundamental frequencies for coupled (solid) and uncoupled (dashed) system.

It is convenient to normalize the frequency shifts with respect to the bridge's fundamental one, as seen in Eqs. (5) and (6). While it is not shown here, this normalization generalizes the results making them applicable to any vehicle and bridge properties, including span length.

$$\delta_v^N = \delta_v / f_b \tag{5}$$

$$\delta_b^N = \delta_b / f_b \tag{6}$$

Regarding SHM it is more relevant to focus only on the bridge frequency shift and its maximum values, that occur when the vehicle is exactly at mid-span. Figure 3 presents the variation of δ_b^N for a range of mass and frequency ratios. It shows that, in general, higher MR values produce greater frequency shifts. For instance, in the situation where MR = FR = 0.75, Figure 3 gives a δ_b^N= 0.46. This means that the fundamental frequency of the beam increases by a factor of 1+0.46 when the appropriate vehicle is at mid-span.

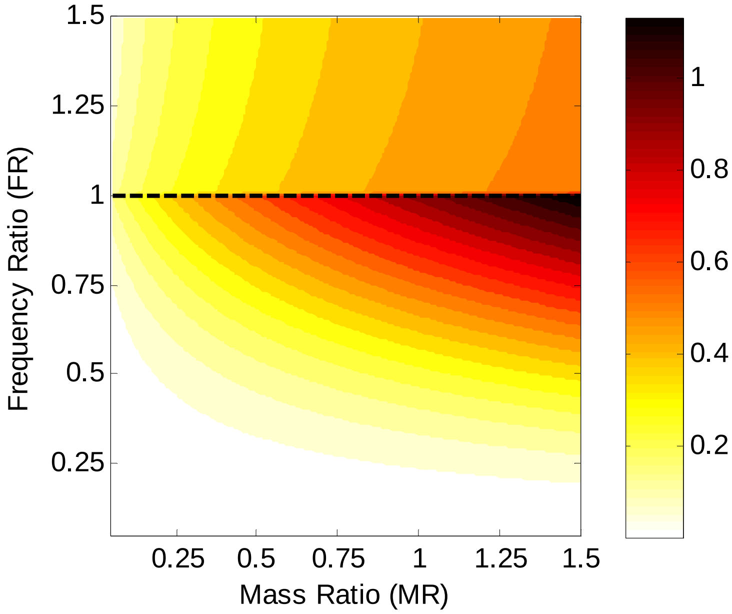

Figure 3: Normalized beam's frequency shift (δ_b^N), in absolute value, for vehicles located at mid-span.

Figure 3 shows a clear distinction for cases above or below the FR = 1 line. When FR < 1 the bridge frequencies increase in value (shifts up as in Figure 2), whereas for FR > 1 the beam frequencies reduce (shift down) following a different pattern. However, the most important fact about the results presented in Figure 3 is that δ_b^N is never zero. This means that, in every VBI problem, the presence of a vehicle on the bridge will produce some change in the bridge's fundamental frequency.

4 SIGNAL PROCESSING

The frequency content analysis of the vehicle's vertical accelerations should give information about both the vehicle and the traversed structure. For this purpose the dynamic interaction model presented in Section 2 is solved for the same mass and frequency ratios as in Section 3 (MR = 0.1 and FR = 0.9). Figure 4 presents the vertical acceleration for the 25 m long bridge traversed by the vehicle moving at 5 m/s over a perfectly smooth profile.

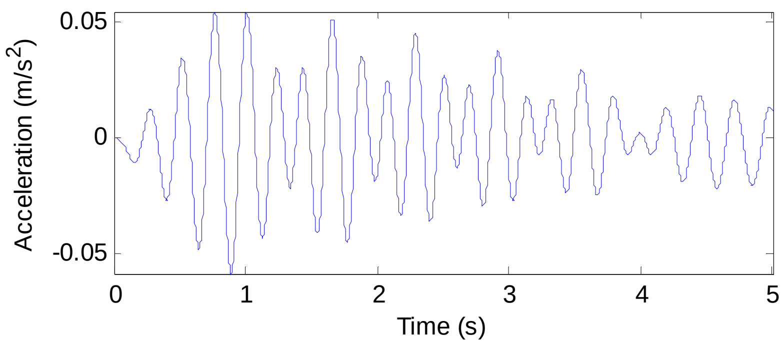

Figure 4: Vertical acceleration of vehicle while traversing the bridge.

The standard approach for analysing signals in the frequency domain is the Fast Fourier Transform (FFT). This provides excellent information for stationary signals by means of a very efficient algorithm. The disadvantage of this tool is that the information obtained is for the whole signal and localized variations in time vanish within the results. Figure 5 gives the outcome of applying the FFT to the acceleration signal and presents it in terms of Power Spectral Density (PSD). Clearly the theoretical system frequencies and those extracted from the signal analysis differ significantly. This difference can be explained by the shift in frequencies due to changing position of the vehicle on the bridge. This example highlights the need for a time-frequency analysis.

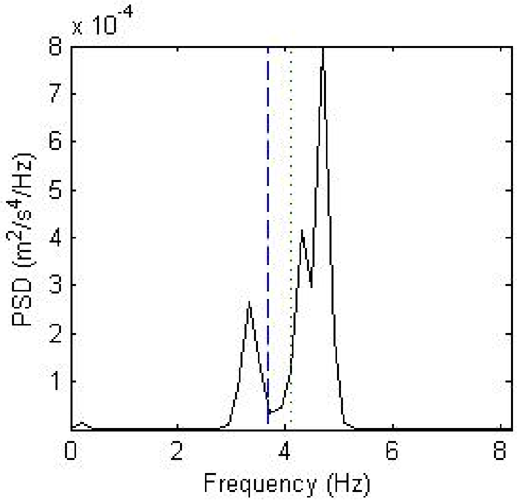

Figure 5: Power spectra density of acceleration signal. Vertical lines are vehicle (blue dotted) and bridge (green dashed) frequencies.

An excellent time-frequency tool is the Continuous Wavelet Transform (CWT) that has been developed over the last three decades. This transform decomposes the analysed signal into a set of coefficients in two dimensions, shift and scale, where scale is inversely proportional to frequency. A basis function is translated (shift) and stretched (scale) and compared against the signal. High coefficients indicate a good match between signal and wavelet at a particular instant in time and associated frequency. This tool offers variable resolution providing a map of energy content of the signal in time and frequency. A mathematical description of the CWT has been presented in many publications, and the reader can refer to [9] amongst many others.

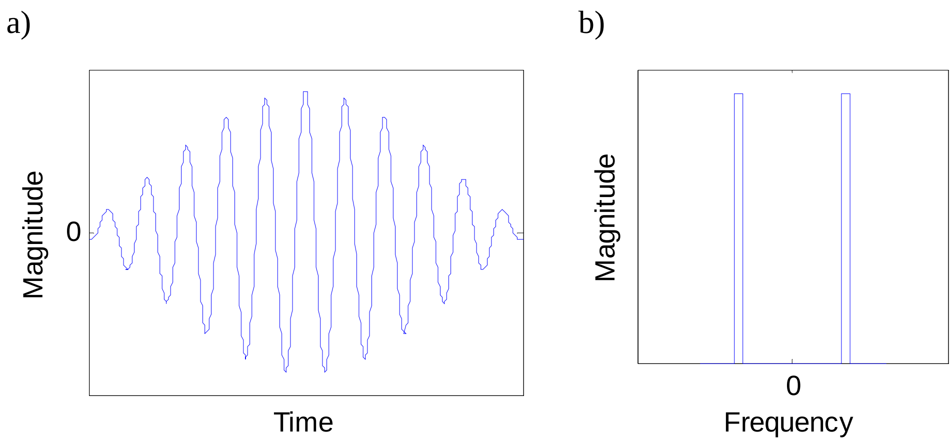

Figure 6: Modified Littlewood-Paley wavelet basis in a) time domain; b) frequency domain.

To obtain the maximum benefit of the CWT it is crucial to select the most suitable basis for each particular problem. There are many possibilities and here the orthogonal wavelet basis called Modified Littlewood-Paley (MLP) is recommended to study the frequency evolution of VBI problems. This basis, proposed by [10], features excellent localization properties in the frequency domain. Figure 6 presents the MLP basis in time and frequency domains. For a full description of the wavelet basis, refer to [10].

Figure 7 shows the results of the CWT on the vehicle's accelerations (Figure 4) with the described MLP basis. The time-varying theoretical values of the system frequencies (Figure 2) are superimposed on the wavelet coefficients. It is possible to clearly identify the evolution of the frequencies while the vehicle traverses the bridge.

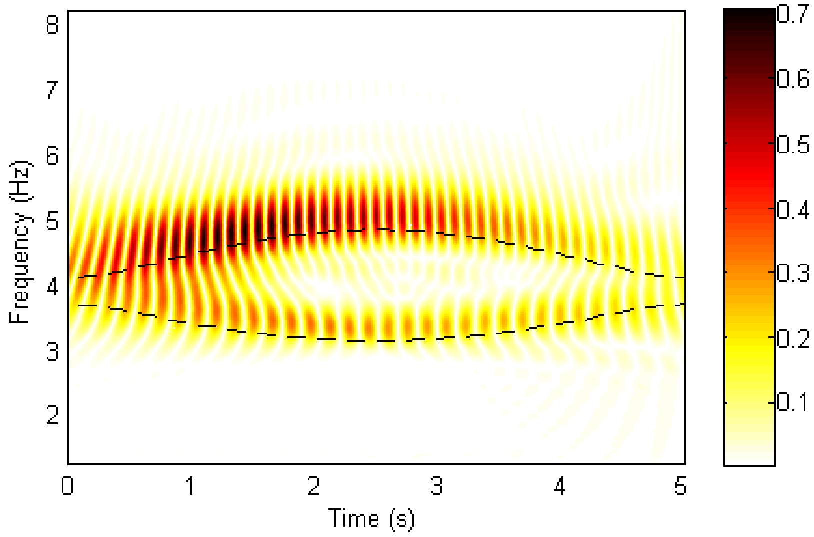

Figure 7: Wavelet transform coefficients of acceleration signal. Theoretical frequency values (Dashed lines).

It is not shown here, but similar studies have been done using alternative wavelet basis, including, Haar, Morlet, Mexican Hat and Littlewood Paley among others. It was found that only MLP provides clear frequency evolution results.

4.1 Rough profile example

The exact same configuration is studied again considering now some profile roughness. A Class A profile [11] has been included in the VBI model. Additionally a 100 m approach was incorporated to allow the vehicle to reach dynamic equilibrium before traversing the structure.

Figure 8(a) shows the PSD of the vehicle's vertical acceleration while traversing the bridge. A high energy concentration is observed near the vehicle's frequency due to the profile excitation that dominates the dynamic response of the system. However there is a small peak near the bridge's frequency that could be associated with the structure. As in Figure 5, theoretical and measured frequencies differ significantly. When the same signal is analysed using CWT with the MLP basis, Figure 8(b) shows that the frequency evolution becomes clear again. Now high energy concentrations are observed near the vehicle's frequency, but most importantly, the evolution in time can be clearly appreciated. It is also possible to identify the bridge's time-varying fundamental frequency.

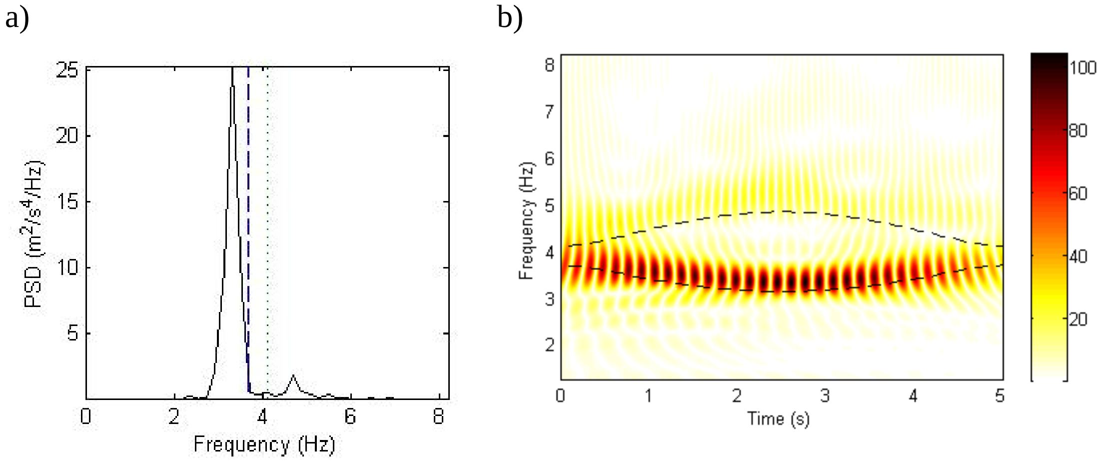

Figure 8: Vehicle vertical acceleration for vehicle traversing bridge on a Class A profile a) Power Spectral Density; b) Continuous Wavelet Transform with MLP basis.

5 DISCUSSION

The results presented in this paper are very relevant for the correct interpretation of the VBI phenomenon. For road traffic, the vehicle's mass is orders of magnitude less than the bridge's total weight, resulting in small MR that leads to small frequency shifts. However, there may be situations where the structure's mass is small (steel bridges) or multiple vehicles are present on the bridge where higher frequency shifts are predicted. This is also particularly important for railway bridges that generally feature high vehicle to structure mass ratios.

It is important to note that the presented study focuses only on the fundamental frequency of the structure. This allowed for a clear presentation of the idea providing additional insight to the fundamentals of VBI. However, frequency shifts occur on higher modes too, particularly if the vehicle frequency matches any other of the natural frequencies of the bridge. In reality, a vehicle has several frequencies, and when positioned along different locations of the span, it will affect (shift) many of the structure's frequencies in a complex manner.

The change of system frequencies with the vehicle position is an intrinsic feature of VBI that affects the response of the whole system, not only the vehicle's acceleration. Bridges when excited by passing vehicles will vibrate at varying frequencies which will be reflected in the frequency content of strain, displacements and accelerations of the structure. This is of relevance for direct SHM systems that use traversing vehicles as the source of excitation.

6 CONCLUSIONS

This paper has highlighted the time-varying nature of the VBI problem and its implications for SHM. The presence of a vehicle on the bridge will inevitably change the natural frequencies of the system. Using a simple numerical model the shift in bridge's fundamental frequency is given using a non-dimensional representation. Then the vertical acceleration signal of a traversing vehicle is studied which exposes the need of an adequate time-frequency analysis tool. The paper proposes the use of the Wavelet transform in combination with the MLP basis to accurately track and extract the structure's frequencies.

ACKNOWLEDGMENTS

The research leading to these results has received funding from the European Union's Seventh Framework Programme (FP7/2007-2013) under the Marie Curie, Industry and Academia Partnerships and Pathways (IAPP) call, Grant agreement no. 286276.

REFERENCES

[1] A. Gonzalez, E.J. OBrien, P.J. McGetrick. Identification of damping in a bridge using a moving instrumented vehicle. *Journal of Sound and Vibration*; **331**, 4115-4131, 2012.

[2] Y.B. Yang, C.W. Lin, J.D. Yau. Extracting bridge frequencies from the dynamic response of passing vehicle. *Journal of Sound and Vibration*; **227**, 471-493, 2004.

[3] D.M. Siringoringo, Y. Fujino. Estimating bridge fundamental frequency from vibration response of instrumented passing vehicle: Analytical and experimental study. *Advances in Structural Engineering*; **15**, 443-459, 2012.

[4] C.W. Kim, R. Isemoto, T. Toshinami, M. Kawatani, P.J. McGetrick, E.J. OBrien, Experimental investigation of drive-by bridge inspection. *Structural Health Monitoring of Intelligent Infrastructure*, SHMII-5, Cancun, Mexico, Paper No. 004, 2011.

[5] F. Cerda, J. Garrett, J. Bielak, R. Bhagavatula, J. Kovacevic, Exploring indirect vehicle-bridge interaction for bridge SHM. D.M. Frangopol, R. Sause and C.S. Kusko eds. *Bridge Maintenance, Safety and Management*, IABMAS 10, Philadelphia, USA, 2010.

[6] J. Kim, J.P. Lynch, J.J. Lee, C.G. Lee. Truck-based mobile wireless sensor networks for the experimental observation of vehicle-bridge interaction. *Smart Materials and Structures*; **20**,1-14, 2011.

[7] A. Gozalez, E. Covian, J. Madera. Determination of bridge natural frequencies using a moving vehicle instrumented with accelerometers and a geographical positioning system. B.H.V. Topping and M. Papadrakakis Eds. 9th International Conference on Computational Structures Technology, Stirlingshire, UK, Paper 281, 2008.

[8] D. Cantero, A. Gonzalez, E.J. OBrien. Comparison of bridge dynamic amplification due to articulated 5-axle trucks and large cranes. *Baltic Journal of Road and Bridge Engineering*; **6**, 39-47, 2011.

[9] A. Cohen, R.D. Ryan, Wavelets and multiscale signal processing. Chapman & Hall, Boundary Row, London, 1995.

[10] B. Basu, V.K. Gupta. Seismic response of SDOF systems by wavelet modeling of nonstationary processes. *Journal of Engineering Mechanics*; **124**, 1142-1150, 1998.

[11] International Organization for Standardization. ISO8608. Mechanical vibration - Road surface profiles - Reporting of measured data. 1995.