11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.* (eds.)
Lisbon, Portugal, 9–12 September 2013

ITERATIVE ADJOINT-BASED CONTROL OPTIMIZATION FOR ACTIVE SUSPENSION OF QUARTER-CAR MODEL

Domenico Guida[1], Carmine M. Pappalardo*[1]

[1]Department of Industrial Engineering, University of Salerno, Italy, Via Ponte don Melillo, 84084, Fisciano, Salerno
{guida,cpappalardo}@unisa.it

Keywords: Active Control, Adjoint-Based Optimization, Vibration, Quarter-Car Model.

Abstract. *The goal of our research is to develop an effective method to design control laws for active suspension of road vehicle in order to mitigate vibrations caused by the road roughness. The system examined is a quarter-car model equipped with a control actuator interposed between the sprung mass and the unsprung mass. The suspension system is realised by a nonlinear device. The quarter-car system is subjected to a road-induced excitation whose harmonic content is close to system natural frequencies. The control law is obtained using optimal control theory and is calculated solving a constrained minimization problem. The constrained minimization problem includes a cost function which penalizes the control effort and the state deviation from a reference position. The resulting equations necessary to solve the constrained minimization problem constitute a two-point boundary-value problem. This problem is nonlinear and it can be solved numerically by using nonlinear optimization techniques. The numerical algorithm used in this paper is an iterative adjoint-based control optimization method. This method combines the nonlinear conjugate gradient algorithm with the adjoint-based gradient computation procedure. Simulations show that the designed control law significantly reduces vibrations caused by the road profile.*

1 INTRODUCTION

The main function of a road suspension system is twofold: to guarantee a good ride quality for the vehicle occupants and at the same time a good vehicle stability [1]. Since the demand of a good ride quality and the need of a good vehicle stability are two conflicting goals, in the design of a passive suspension system there is a trade-off between the two [2]. The main drawback of a passive suspension system is that the suspension force can be only a function of the relative displacement and of the relative velocity between the sprung mass and the unsprung mass. On the other hand, active suspension systems can improve the performance of vehicle suspension circumventing the design constraints imposed on the passive suspension systems.

In this paper a general and effective method to control nonlinear underactuated mechanical system disturbed by noise has been developed [3], [4], [5]. This method combines one of the most important analytical results of optimal control theory, namely the linear quadratic Gaussian regulation algorithm, with an effective procedure belonging to numerical methods for nonlinear optimal control, namely the iterative adjoint-based control optimization method [6], [7]. The proposed method has been applied to develop control laws for active suspension system of road vehicle in order to mitigate road-induced vibrations [8], [9], [10].

This paper is organized as follows. In section 2 the quarter-car system under study is examined and the equations of motion describing the system dynamical model are derived and represented in the state space. In particular, a nonlinear model for the suspension system is presented. In section 3 a detailed description of the results obtained by numerical simulations implementing the proposed methods is presented. In particular, the problem on hand was solved designing two controllers: a regulation controller and a compensation controller. In this section the development of the open-loop regulation controller by using the iterative adjoint-based control optimization algorithm and the development of the close-loop compensation controller by using the linear quadratic Gaussian regulation method are described. Finally, a conclusion section which summarises the most important results obtained in this paper is presented.

2 SYSTEM MODEL

2.1 System Description

The system examined is a quarter-car model showed in figure 1. The displacement of the unsprung mass is denoted with $x_1(t)$ whereas the displacement of the sprung mass is denoted with $x_2(t)$. The unsprung mass is denoted with m_1 whereas the sprung mass is denoted with m_2. The tyre stiffness is denoted with k_1 whereas the tyre damping is denoted with r_1. The suspension system is realised by a nonlinear device interposed between the sprung mass and the unsprung mass. The nonlinear suspension device provides a nonlinear elastic force field $f_k(x_1(t), x_2(t), t)$ and a nonlinear dissipative force field $f_r(\dot{x}_1(t), \dot{x}_2(t), t)$. The force fields produced by the nonlinear suspension device are modelled as follows:

$$f_k(x_1(t), x_2(t), t) = -k_2 (x_1(t) - x_2(t)) \left(1 + \mu(x_1(t) - x_2(t))^2\right) \tag{1}$$

$$f_r(\dot{x}_1(t), \dot{x}_2(t), t) = -r_2 (\dot{x}_1(t) - \dot{x}_2(t)) \left(1 + \eta(\dot{x}_1(t) - \dot{x}_2(t))^2\right) \tag{2}$$

where k_2 denotes the suspension stiffness and r_2 denotes the suspension damping. The parameter μ and η respectively regulate the amount of nonlinearity in the restoring force field $f_k(x_1(t), x_2(t), t)$ and in the damping force field $f_r(\dot{x}_1(t), \dot{x}_2(t), t)$. For the system under study, the nonlinear characteristic of the elastic force field $f_k(x_1(t), x_2(t), t)$ is represented in figure

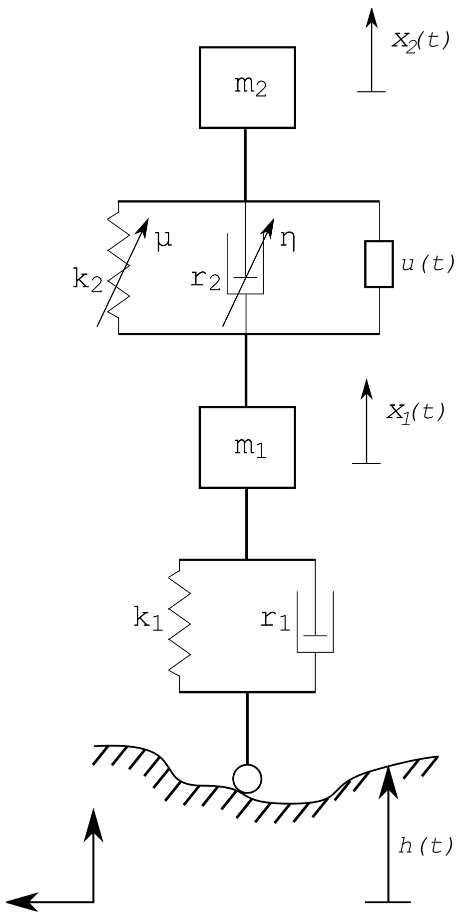

Figure 1: Quarter Car System

2 whereas the nonlienar characteristic of the dissipative force field $f_r(\dot{x}_1(t), \dot{x}_2(t), t)$ is represented in figure 3. The form of the elastic force $f_k(x_1(t), x_2(t), t)$ is often referred to as a Duffing-type restoring force [11], [12]. Since the parameter μ has been chosen positive, the elastic force field behave like an hardening spring because its stiffness increases with the relative displacement. As a result of Duffing-type hardening elastic force field $f_k(x_1(t), x_2(t), t)$, the whole mechanical system exhibits a single stable equilibrium position. Note thate the amount of nonlinearity of the suspension system quantified by the parameters μ and η can be optimised in order to mitigate road-induced vibrations. Linearising the system around its stable equilibrium configuration, the resulting system natural frequencies are denoted respectively with $f_{n,1}$ and $f_{n,2}$ whereas the resulting system damping ratios are denoted respectively with ξ_1 and ξ_2. Note that the choice of the Duffing-type form of the restoring force field $f_k(x_1(t), x_2(t), t)$ and of the damping force field $f_r(\dot{x}_1(t), \dot{x}_2(t), t)$ ensures that the linearised system natural frequencies $f_{n,1}$, $f_{n,2}$ and damping ratios ξ_1, ξ_2 are independent of the nonlinearity parameters μ and η. Considering a worst-case scenario, the quarter-car system is excited by a road profile $h(t)$ which is assumed as a superposition of two harmonic displacements whose harmonic content is close to the linearised system natural frequencies. Indeed:

$$h(t) = H_1 \sin(2\pi f_1 t) + H_2 \sin(2\pi f_2 t) \tag{3}$$

where H_1 and H_2 denote the amplitudes of the road roughness whereas f_1 and f_2 denote the frequencies of the road roughness. In order to reduce the amplitude of the road-induced vibra-

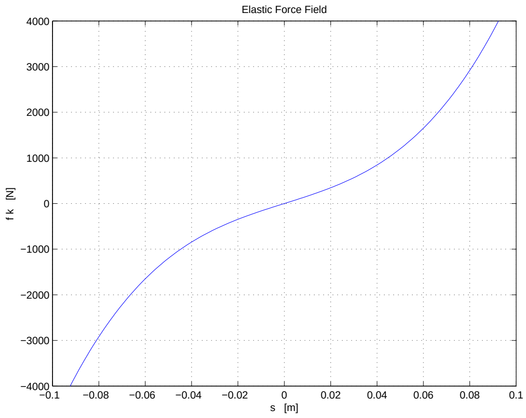

Figure 2: Elastic Force Field - $f_k(t)$

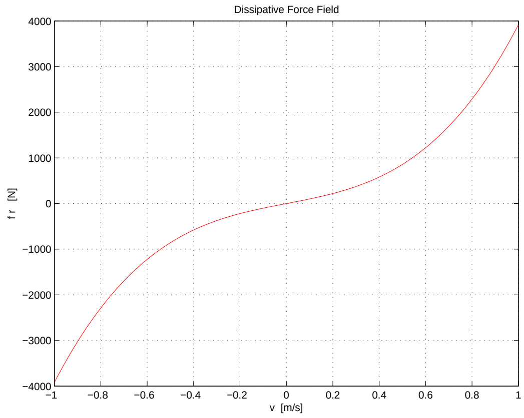

Figure 3: Dissipative Force Field - $f_r(t)$

tions, an active control system is interposed between the sprung mass and the unsprung mass. The action of the control actuator is denoted with $u(t)$. The objective of the control system is to improve the ride quality of the vehicle. A detailed list of all system data is reported in table 1.

2.2 Equations of Motion

The configuration of the system can be defined using a set of $n_2 = 2$ degrees of freedom. Indeed, system generalized coordinates can be grouped in a vector $q(t)$ as:

$$q(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{4}$$

where $x_1(t)$ denotes the displacement of the unsprung mass and $x_2(t)$ denotes the displacement of the sprung mass. System equations of motion can be derived using Lagrangian Dynamics [13], [14] to yield:

$$M(q(t),t)\ddot{q}(t) = Q(q(t),\dot{q}(t),t) \tag{5}$$

where $M(q(t),t)$ is system mass matrix and $Q(q(t),\dot{q}(t),t)$ is the vector of generalised forces acting on the system. For the problem on hand, the system mass matrix and the vector of generalised forces are defined as follows:

$$M(q(t),t) = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{6}$$

$$Q(q(t),\dot{q}(t),t) = \begin{bmatrix} -k_1\left(x_1(t)-h(t)\right) - k_2\left(x_1(t)-x_2(t)\right)\left(1+\mu(x_1(t)-x_2(t))^2\right) + \\ -r_1\left(\dot{x}_1(t)-\dot{h}(t)\right) - r_2\left(\dot{x}_1(t)-\dot{x}_2(t)\right)\left(1+\eta(\dot{x}_1(t)-\dot{x}_2(t))^2\right) \\ \\ -k_2\left(x_2(t)-x_1(t)\right)\left(1+\mu(x_2(t)-x_1(t))^2\right) + \\ -r_2\left(\dot{x}_2(t)-\dot{x}_1(t)\right)\left(1+\eta(\dot{x}_2(t)-\dot{x}_1(t))^2\right) \end{bmatrix} \tag{7}$$

System equations of motion (5) form a set of $n_2 = 2$ nonlinear second-order ordinary differential equations which requires a set of $2n_2 = 4$ initial conditions $q(0) = q_0$ and $\dot{q}(0) = p_0$. If a control action $Q_c(u(t),t)$ is introduced on the system, the matrix form of system equations of motion becomes:

$$M(q(t),t)\ddot{q}(t) = Q(q(t),\dot{q}(t),t) + Q_c(u(t),t) \tag{8}$$

DESCRIPTION	SYMBOLS	DATA [UNITS]
Unsprung Mass	m_1	$36\ [kg]$
Sprung Mass	m_2	$260\ [kg]$
Tyre Stiffness	k_1	$160000\ [kg \cdot s^{-2}]$
Suspension Stiffness	k_2	$16000\ [kg \cdot s^{-2}]$
Tyre Damping	r_1	$98\ [kg \cdot s^{-1}]$
Suspension Damping	r_2	$980\ [kg \cdot s^{-1}]$
Suspension Stiffness Degree of Nonlinearity	μ	$200\ [m^{-2}]$
Suspension Damping Degree of Nonlinearity	η	$3\ [m^{-2} \cdot s^{-2}]$
First Mode Natural Frequency	$f_{n,1}$	$1.2005\ [s^{-1}]$
Second Mode Natural Frequency	$f_{n,2}$	$11.0350\ [s^{-1}]$
First Mode Damping Ratio	ξ_1	$0.2093\ [\backslash]$
Second Mode Damping Ratio	ξ_2	$0.2203\ [\backslash]$
Road Profile - Amplitude 1	H_1	$0.1\ [m]$
Road Profile - Amplitude 2	H_2	$0.01\ [m]$
Road Profile - Frequency 1	f_1	$1.20\ [s^{-1}]$
Road Profile - Frequency 2	f_2	$11.03\ [s^{-1}]$
Initial Displacement of Unsprung Mass	$x_{1,0}$	$0.001\ [m]$
Initial Displacement of Sprung Mass	$x_{2,0}$	$0.001\ [m]$
Initial Velocity of Unsprung Mass	$v_{1,0}$	$0.01\ [m \cdot s^{-1}]$
Initial Velocity of Sprung Mass	$v_{2,0}$	$0.01\ [m \cdot s^{-1}]$
Time Span	T	$5\ [s]$
Time Step	Δt	$0.001\ [s]$

Table 1: System Data

For the system under study, the control input $u(t)$ is a force acting between the spung mass and the unsprung mass. Therefore, the control action $Q_c(u(t), t)$ can be written as:

$$Q_c(u(t), t) = B_2(t)u(t) \tag{9}$$

where B_2 is a Boolean matrix defined as follows:

$$B_2(t) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \tag{10}$$

Since the rank of the Boolean matrix B_2 is $b_2 = 1$ and it is lesser than the number of system degrees of freedom $n_2 = 2$, the quarter-car model is an underactuated mechanical system.

2.3 State Space Model

System equations of motion (8) can be represented in the state space form defining a state vector as follows:

$$z(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \tag{11}$$

Indeed, system state space model can be derived rewriting system equations of motion (8) in terms of the state vector (11) to yield:

$$\dot{z}(t) = f(z(t), t) + f_c(z(t), u(t), t) \tag{12}$$

where $f(z(t), t)$ is system state function and $f_c(z(t), u(t), t)$ is system control function which can be respectively computed as:

$$f(z(t), t) = \begin{bmatrix} \dot{q}(t) \\ a(q(t), \dot{q}(t), t) \end{bmatrix} \tag{13}$$

$$f_c(z(t), u(t), t) = \begin{bmatrix} 0 \\ a_c(q(t), u(t), t) \end{bmatrix} \tag{14}$$

where $a(q(t), \dot{q}(t), t)$ is system generalised acceleration when there is no control action and $a_c(q(t), \dot{q}(t), u(t), t)$ is system generalised acceleration induced by the control action. These generalised accelerations can be computed as follows:

$$a(q(t), \dot{q}(t), t) = M^{-1}(q(t), t)Q(q(t), \dot{q}(t), t) \tag{15}$$

$$a_c(q(t), u(t), t) = M^{-1}(q(t), t)Q_c(u(t), t) \tag{16}$$

In addition, for the system under study the system control function $f_c(z(t), u(t), t)$ is a linear function of control input $u(t)$ which can be expressed as:

$$f_c(z(t), u(t), t) = B(z(t), t)u(t) \tag{17}$$

where $B(q(t), t)$ is the state influence matrix which can be computed as follows:

$$B(q(t), t) = \begin{bmatrix} 0 \\ M^{-1}(q(t), t)B_2(t) \end{bmatrix} \tag{18}$$

System state space model (12) constitutes a set of $n = 2n_2 = 4$ nonlinear first-order ordinary differential equations and requires the identification of the initial state $z(0) = z_0$ which arise from the set of $n = 2n_2 = 4$ initial conditions $q(0) = q_0$ and $\dot{q}(0) = p_0$.

2.4 Measurement Equations

In practical application, it is common that there are not enough sensors to completely measure the state vector $z(t)$ and even the system initial state z_0 is unknown [15], [16], [17]. The available measurements can be collected in a measurement vector $y(t)$ which can be computed as a function of the state $z(t)$ and of the input $u(t)$. Indeed:

$$y(t) = f_m(z(t), u(t), t) \tag{19}$$

where $f_m(z(t), u(t), t)$ is system measurement function which can be expressed as a linear combination of system generalised displacement, velocity and acceleration [3] . Indeed:

$$f_m(z(t), u(t), t) = C_d(t)q(t) + C_v(t)\dot{q}(t) + C_a(t)\left(a(q(t), \dot{q}(t), t) + a_c(q(t), u(t), t)\right) \tag{20}$$

where $C_d(t)$, $C_v(t)$ and $C_a(t)$ are respectively system displacement, velocity and acceleration sensing matrices. For the problem on hand, it is assumed that only the difference between the displacement of the sprung mass and the displacement of the unsprung mass can be measured. Therefore, the sensing matrices can be computed as:

$$C_d(t) = \begin{bmatrix} -1 & 1 \end{bmatrix} \tag{21}$$

$$C_v(t) = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{22}$$

$$C_a(t) = \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{23}$$

The measurement equations represent a fundamental tool to design an observer in order to solve the state estimation problem.

3 RESULTS AND DISCUSSION

3.1 Control Development

The technique utilised to synthesise the control system consider two steps. The first step consists in the design of a regulation controller by using the iterative adjoint-based control optimization method. The second step consists in the design of a compensation controller by using the linear quadratic Gaussian regulation method. The purpose of the regulation controller is to reduce the vibration amplitudes of the sprung and unsprung masses induced by the ground excitation. The synthesis of the regulation controller provides a feedforward control action and the corresponding evolution of the system state. The purpose of the compensation controller is to stabilise the system around the designed trajectory coping with process disturbances, measurement uncertainty and incomplete state information. The synthesis of the compensation controller provides a feedback control action which has to be added to the feedforward control action in order to effectively control the system.

3.2 Regulation Controller Design

The regulation controller is an open-loop controller which has been designed using the iterative adjoint-based control optimization method. The control scheme which describes how the regulation controller acts on the system is represented in figure 4. In this case the weight

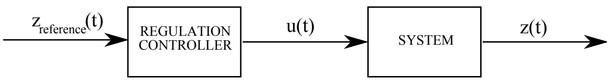

Figure 4: Control Scheme for Regulation Controller

matrices which characterise the cost function have been defined as follows:

$$Q_T(T) = diag\left(10^4, 10^4, 10^4, 10^4\right) \tag{24}$$

$$Q_z(t) = diag\left(10^4, 10^4, 10^4, 10^4\right) \tag{25}$$

$$Q_u(t) = 10^{-4} \tag{26}$$

where $Q_T(T)$ denotes the final state cost matrix, $Q_z(t)$ denotes the state cost matrix and $Q_u(t)$ is the input cost matrix. For the design of the regulation controller, the reference trajectory and the reference control action have been set as follows:

$$\bar{z}(t) = 0 \tag{27}$$

$$\bar{u}(t) = 0 \tag{28}$$

where $\bar{z}(t)$ denotes the reference trajectory and $\bar{u}(t)$ denotes the reference control action. In this case the final state cost matrix $Q_T(T)$ and the state cost matrix $Q_z(t)$ impose a heavy penalization respectively on the deviation of the final state $z(T)$ from the final reference trajectory $\bar{z}(T)$ and on the deviation of the state $z(t)$ from the reference trajectory $\bar{z}(t)$. On the other hand, the effort of the control action is relatively less penalised by the input cost matrix $Q_u(t)$. The iterative convergence towards the minimum of the cost function is represented in figure 5 whereas the resulting regulation controller is represented in figure 6. Figure 7 repre-

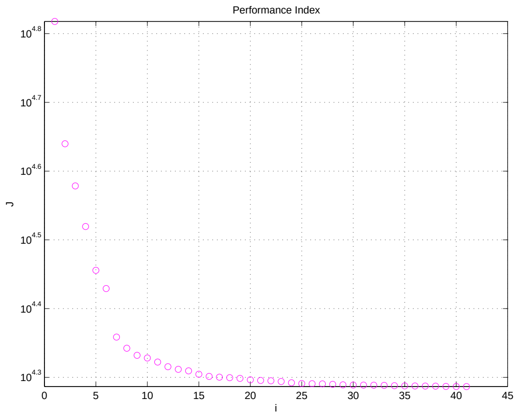

Figure 5: Cost Function - J

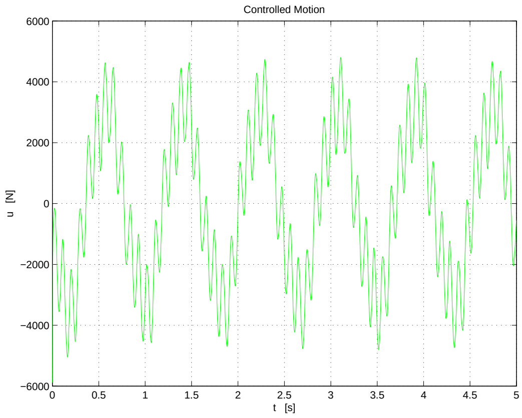

Figure 6: Regulation Controller - $u(t)$

sents the displacement of the unsprung mass when the system is uncontrolled whereas figure 8 represents the displacement of the unsprung mass when the regulation controller acts on the system. Figure 7 represents the displacement of the sprung mass when the system is uncon-

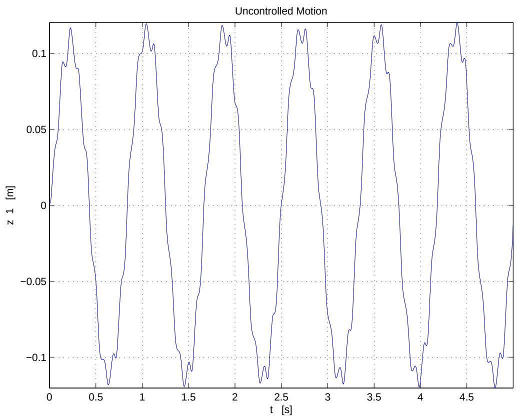

Figure 7: Uncontrolled Motion - $x_1(t)$

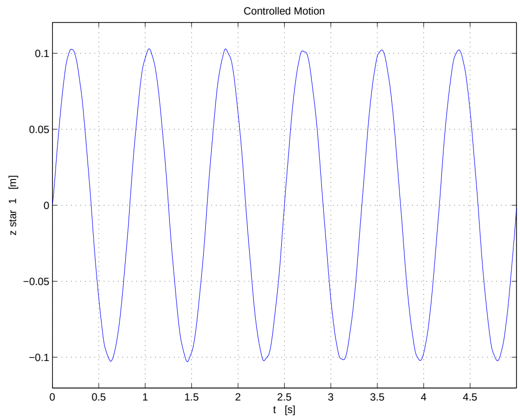

Figure 8: Controlled Motion - $x_1(t)$

trolled whereas figure 10 represents the displacement of the sprung mass when the regulation controller acts on the system. These figures show that the effect of the action of the regulation controller is a considerable amplitude reduction of the sprung mass displacement whereas the amplitude of the unsprung mass displacement is relatively less decreased. The amplitude reductions can be quantified comparing the maximum amplitudes of system motion with and without the controller as follows:

$$\varepsilon_{x_1} = \frac{X_{1,u} - X_{1,c}}{X_{1,u}} = 14.36\ \% \tag{29}$$

$$\varepsilon_{x_2} = \frac{X_{2,u} - X_{2,c}}{X_{2,u}} = 88.01\ \% \tag{30}$$

$$\delta_{x_1} = 20 \log \left(\frac{X_{1,u}}{X_{1,c}} \right) = 1.3465\ dB \tag{31}$$

$$\delta_{x_2} = 20 \log \left(\frac{X_{2,u}}{X_{2,c}} \right) = 18.4215\ dB \tag{32}$$

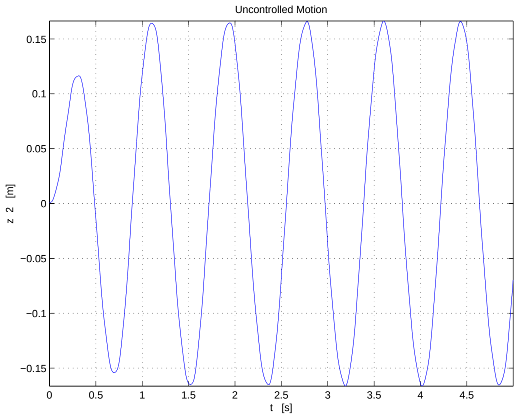

Figure 9: Uncontrolled Motion - $x_2(t)$

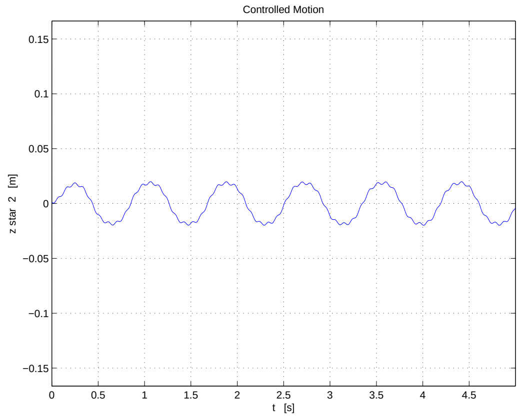

Figure 10: Controlled Motion - $x_2(t)$

where $X_{1,u}$ and $X_{2,u}$ denote the maximum amplitudes of system steady-state displacements when there is no control action whereas $X_{1,c}$ and $X_{2,c}$ denote the maximum amplitudes of system steady-state motion when the regulation controller acts on the system. On the other hand, figure 11 represents the velocity of the unsprung mass when the system is uncontrolled whereas figure 12 represents the velocity of the unsprung mass when the regulation controller acts on the system. Figure 13 represents the velocity of the sprung mass when the system is uncontrolled

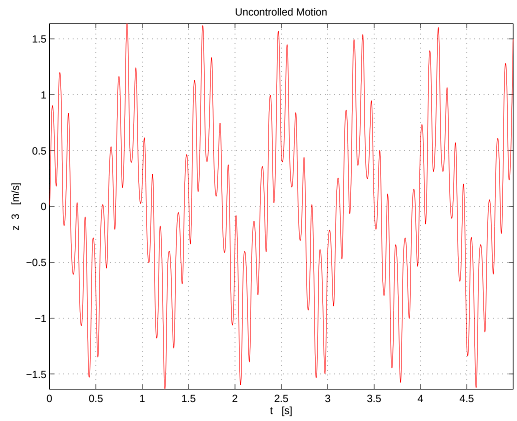

Figure 11: Uncontrolled Motion - $\dot{x}_1(t)$

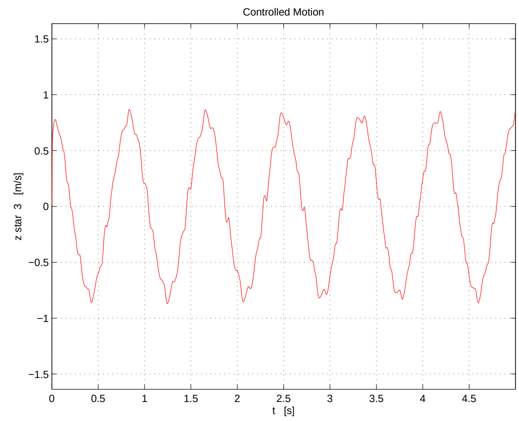

Figure 12: Controlled Motion - $\dot{x}_1(t)$

whereas figure 14 represents the velocity of the sprung mass when the regulation controller acts on the system. These figures show that the effect of the action of the regulation controller is a considerable amplitude reduction of the sprung mass velocity whereas the amplitude of the unsprung mass velocity is relatively less decreased. The amplitude reductions can be quantified comparing the maximum amplitudes of system motion with and without the controller as follows:

$$\varepsilon_{\dot{x}_1} = \frac{\dot{X}_{1,u} - \dot{X}_{1,c}}{\dot{X}_{1,u}} = 46.77\ \% \tag{33}$$

$$\varepsilon_{\dot{x}_2} = \frac{\dot{X}_{2,u} - \dot{X}_{2,c}}{\dot{X}_{2,u}} = 80.56\ \% \tag{34}$$

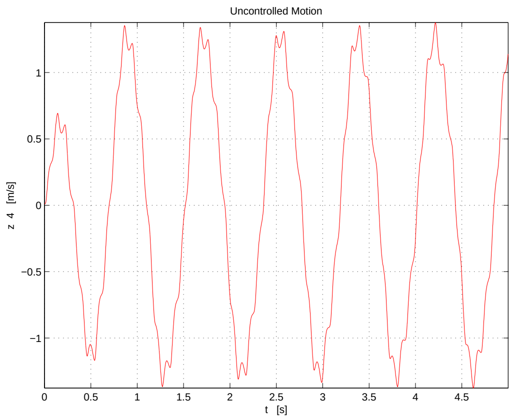

Figure 13: Uncontrolled Motion - $\dot{x}_2(t)$

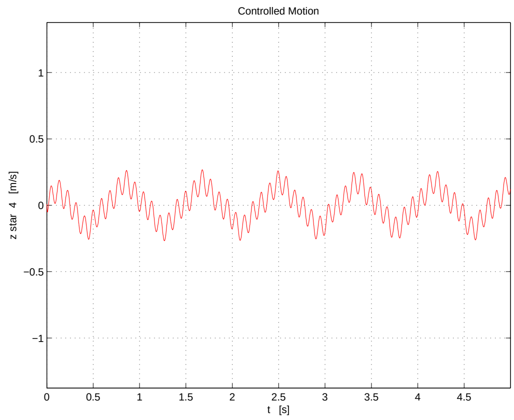

Figure 14: Controlled Motion - $\dot{x}_2(t)$

$$\delta_{\dot{x}_1} = 20 \log \left(\frac{\dot{X}_{1,u}}{\dot{X}_{1,c}} \right) = 5.4772 \; dB \tag{35}$$

$$\delta_{\dot{x}_2} = 20 \log \left(\frac{\dot{X}_{2,u}}{\dot{X}_{2,c}} \right) = 14.2283 \; dB \tag{36}$$

where $\dot{X}_{1,u}$ and $\dot{X}_{2,u}$ denote the maximum amplitudes of system steady-state velocities when there is no control action whereas $\dot{X}_{1,c}$ and $\dot{X}_{2,c}$ denote the maximum amplitudes of system steady-state motion when the regulation controller acts on the system.

4 CONCLUSIONS

In this paper a general and effective method to control nonlinear underactuated mechanical system disturbed by noise has been developed. This method combines one of the most important analytical results of optimal control theory, namely the linear quadratic Gaussian regulation algorithm, with an effective procedure belonging to numerical methods for nonlinear optimal control, namely the iterative adjoint-based control optimization algorithm. The proposed method has been applied to the design of control laws for active suspension systems of road vehicles. The system analysed is a quarter-car model of the vertical vehicle dynamics. The suspension system of the quarter-car model is realised with a nonlinear device interposed between the sprung mass and the unsprung mass which provides a nonlinear elastic force field and a nonlinear dissipative force field. Considering a worst-case scenario, the quarter-car system is excited by a ground motion which is assumed as a superposition of two sinusoid functions whose harmonic content is close to the linearised system natural frequencies. In order to reduce the amplitude of the vibrations caused by the road roughness, an active control system has been collocated between the sprung mass and the unsprung mass. The objective of the control system is to improve the ride quality of the vehicle. The problem on hand has been solved designing two controllers: a regulation controller and a compensation controller. The regulation controller is an open-loop controller which has been designed using the iterative adjoint-based control optimization method. The objective of the regulation controller is to reduce the vibration amplitudes of the sprung and unsprung masses induced by the ground excitation. The synthesis of the regulation controller provides a feedforward control action and the corresponding evolution of the system state. Authors believe that the combination of the iterative adjoint-based control optimization method with the linear quadratic Gaussian regulation algorithm for designing a feedforward plus feedback control architecture represents a viable solution to control nonlinear underactuated mechanical systems coping with process disturbances, measurement uncertainty and incomplete state information.

REFERENCES

[1] M. Blundell, D. Harty, *The Multibody Systems Approach to Vehicle Dynamics*. Elsevier, 2004, ISBN-10: 0750651121, ISBN-13: 978-0750651127.

[2] D. J. Ewins, S. S. Rao, S. G. Braun, *Encyclopedia of Vibration*. Academic Press, ISBN: 0122270851.

[3] C. M. Pappalardo, *Dynamics, Identification and Control of Multibody Systems*. Ph.D. Thesis, Department of Industrial Engineering, University of Salerno, Italy, 2012.

[4] D. Guida, C. M. Pappalardo, A New Control Algorithm for Nonlinear Underactuated Mechanical Systems *International Journal of Mechanical Engineering and Industrial Design (JMEID)*, ISSN: 2280-6407, Volume 1, 2012, 1(4): 61-82.

[5] C. M. Pappalardo, Combination of Extended Udwadia-Kalaba Control Algorithm with Extended Kalman Filter Estimation Method *International Journal of Mechanical Engineering and Industrial Design (JMEID)*, ISSN: 2280-6407, Volume 1, 2012, 1(1): 1-18.

[6] D. Guida, F. Nilvetti, C. M. Pappalardo, Optimal Control Design by Adjoint-Based Optimization for Active Mass Damper with Dry Friction. *Programme and Proceedings of COMPDYN 2013 4th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, 12-14 June 2013 - Kos Island, Greece - To be Published.

[7] D. Guida, F. Nilvetti, C. M. Pappalardo, Adjoint-based Optimal Control Design for a Cart Pendulum System with Dry Friction. *Programme and Proceedings of ECCOMAS Thematic Conference on Multibody Dynamics 2013*, 1-4 July 2013 - Zagreb, Croatia - To be Published.

[8] D. Guida, F. Nilvetti, C. M. Pappalardo, Parameter Identification of a Full-Car Model for Active Suspension Design. *Journal of Achievements in Materials and Manufacturing Engineering*, Vol. 40, n. 2, pag. 138-148, 2010, ISSN: 1734-8412.

[9] D. Guida, C. M. Pappalardo, Sommerfeld and Mass Parameter Identification of Lubricated Journal Bearing. *Applied and Theoretical Mechanics*, ISSN: 1991-8747, Issue 4, Volume 4, October 2009, Pages 205-214.

[10] S. DAmbrosio, D. Guida, C.M. Pappalardo, J. Quartieri, Nonlinear Identification Method of Multibody System Parameters. *IPMM-2007 The 6th International Conference*, 2007, Volume 1, pag. 27-31.

[11] G. Genta, *Vibration Dynamics and Control*. Springer, 2009, ISBN-10: 0387795790, ISBN-13: 978-0387795799.

[12] D. J. Inman, *Vibration with Control*. John Wiley and Sons, 2006, ISBN-10: 0470010517, ISBN-13: 978-0470010518.

[13] A. A. Shabana, *Computational Dynamics*. Cambridge, 2010, ISBN-10: 0470686154, ISBN-13: 978-0470686157.

[14] A. A. Shabana, *Dynamics of Multibody Systems*. Cambridge, 2010, ISBN-10: 0521154227, ISBN-13: 978-0521154222.

[15] D. Guida, F. Nilvetti, C. M. Pappalardo, Parameter Identification of a Two Degrees of Freedom Mechanical System. *International Journal of Mechanics*, ISSN: 1998-4448, Issue 2, Volume 3, 2009, Pages 23-30.

[16] D. Guida, F. Nilvetti, C. M. Pappalardo, Modelling, Identification and Control of a Three-Story Building System. *International Journal of Mechanical Engineering and Industrial Design (JMEID)*, ISSN: 2280-6407, Volume 1, 2012, 1(3): 36-60.

[17] D. Guida, F. Nilvetti, C. M. Pappalardo, Dry Friction Influence on Cart Pendulum Dynamics. *International Journal of Mechanics*, ISSN: 1998-4448, Issue 2, Volume 3, 2009, Pages 31-38.