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INFLUENCE OF REFERENCE RESPONSE PLACEMENT ON THE OPERATING DEFLECTION SHAPES AND TRANSMISSIBILITY SPECTRUM

Jakerson R. Gevinski*[1], Robson Pederiva[1]

[1]University of Campinas – Department of Mechanical Design
{jakerson, robson}@fem.unicamp.br

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Abstract *The transmissibility is traditionally used in the operating mode shapes identification of the structures, when the excitation cannot be measured. It is calculated the same way as the Frequency Response Functions (FRF), whereas the FRF is the ratio of the response divided by force, transmissibility is the ratio of the response divided by the reference response. An unexpected drawback of transmissibility measurements however, is that the poles of the transmissibility function do not correspond to the poles of the FRF functions. In other words, the peaks in the amplitude transmissibility spectrums are not associated with the natural frequencies of the system. Thus, this paper aims to present the relationship between the reference response placement and the peaks in the transmissibility spectrum, and its influence on the operating deflection shapes (ODS). Three experimental cases were carried out on the aluminum plate, excited by random white noise. Different reference points and excitation were assumed, with satisfactory results, for each case. Frequency of the transmissibility peaks coincided with the antiresonance frequency of the reference point. The knowledge of this relationship is important for the finite element model updating using antiresonance frequencies, and it clearly demonstrates the existence of peaks in the transmissibility spectrum.*

1 INTRODUCTION

Under operating conditions, it is common to only apply measures of system responses techniques in the evaluation of the dynamic behavior. The operating deflection shape (ODS) [1] and operational modal analysis (OMA) [2, 3] are examples of such techniques. The ODS allows the operating deflection shape identification of the machine or equipment subjected to vibration. This technique can be used in the time domain or frequency from a set of simultaneously measured data, or in cases where simultaneous measurement of multiple points of interest is difficult; at least a pair of measurements, response and fixed reference, must be measured simultaneously. The reference can be the excitation force, using frequency response function, or a reference response, using transmissibility functions.

The transmissibility is the frequency domain ratio between two response signals. It is calculated in the same way as the frequency response functions (FRF), but whereas the FRF is the ratio between the response and the excitation force; the transmissibility is the ratio between the response (acceleration, velocity, or displacement), and a reference response. As a ratio between system responses, transmissibility does not depend on the excitation force data. This allows identification of the vibration operating mode when the excitation forces cannot be measured, and their amplitudes vary with time. The effect due to force level variations will be canceled in the transmissibility calculation [4].

A peculiarity is observed during the comparison between FRF and transmissibility. The poles of the transmissibility functions do not correspond to the poles of the FRF. Therefore, the peaks in the transmissibility amplitude spectrums are generally not associated with the natural frequencies of the system. One drawback is the difficulty in identifying the natural frequencies of the system, when considering only the transmissibility functions. Analysis of the peaks and transmissibility amplitude spectrums characteristics were performed by some authors. In [1] they show that the resonance in the transmissibility spectrums is defined by a flat rather than a peaked region. Moreover, the peaks in the transmissibility are merely the result of the division of the response spectrum by the reference response spectrum at frequencies where the reference response is relatively small. In [5] it is shown that the spectrums from two transmissibilities cross each other exactly at the resonance frequency when the transmissibility is from the same responses, but with different loading conditions. Transmissibilities peaks coincide with the natural frequencies of the system when the excitation degree of freedom is constrained [6]. However, this system does not match the real system analyzed.

The peaks of the transmissibility spectrums are associated with antiresonance peaks of the reference frequency response function. As a system local characteristic, the antiresonance frequency is different for each adopted point in the structure. Accordingly, the peak frequencies in the transmissibilities depend on the reference location. Therefore, this paper aims an experimental evaluation of the relationship between the point of reference adopted in the processing of the transmissibilities, and the peaks shown in their spectrums, as well as their influence on the identification of operating modes.

2 THEORIC BACKGROUND

2.1 Operating Deflection Shapes

The machinery and equipment dynamic behavior depends on the properties of the structure, namely: mass, stiffness, and dumping, and the excitation characteristics, such as amplitude, frequency, and location. This behavior can be assessed by using the modal parameters: natural frequency, mode shapes, and modal damping factor. The modal parameters depend only on the structure properties, and are independent from the excitation [7]. Another way of evaluating the dynamic behavior, especially under operating conditions, is by analyzing either the deflection operating shape or the vibration operating mode, which depends on the excitation conditions [1].

The vibration operating mode is traditionally defined as the deflection shape of the structure at a specific time or frequency under any external condition or excitation sources. Generally, the operating mode can be defined as any forced movement of two or more points in the structure. By specifying the motion of these two or more points, it is possible to define the shape of vibration. These movements are vibration responses, which depend on both the dynamic properties of the system, and the excitation characteristics. Thus, it can be said that the operating modes contain mode shapes, as they depend on the dynamic properties, and the total system response will be a linear combination of the mode shapes weighed by the excitation characteristics [1]. Eq. (1) mathematically represents this linear combination.

$$\{\hat{\varphi}(\omega)\} = a_1\{\phi_1\} + a_2\{\phi_2\} + a_3\{\phi_3\} + \cdots a_M\{\phi_M\} \tag{1}$$

where the vector $\{\hat{\varphi}(\omega)\}$ is the operating mode in the angular frequency ω; a_i are the linear combination coefficients, dependent on the characteristics of the excitation, $\{\phi_i\}$ are the mode shapes of the system, and M is the total number of mode shapes.

Considering a single excitation source, close to the natural frequencies, the operating modes take the form of mode shapes associated with this frequency. In this situation, the "shape" of the operating mode will depend only on the properties of the system, regardless of location, and of the excitation force amplitude. Importantly, when referring to the operating mode shape, the amplitude of vibration is not being considered, since the mode form is represented by the relative amplitude between the degrees of freedom. Therefore, a variation of the overall vibration amplitude, i.e., across the structure analyzed, does not imply a change in the shape of vibration.

Conventionally, the technique used to identify the operating modes is the Operating Deflection Shapes (ODS). Through the ODS, the operating modes are simultaneously identified by the amplitude, and phase measurements between the degrees of freedom (points) of the structure, or the amplitude, and the relative phase of the pairs of simultaneous measurement, adopting a reference signal. The ODS can be performed in both the time, and the frequency domains. In the time domain, the ODS can be obtained from different types of time responses, either random, impulsive, or harmonics. The ODS in the frequency domain is obtained from different types of measurement, including the linear spectrum obtained by FFT (Fast Fourier Transform), from the time signals simultaneously measured, power spectral density, the FRF, the transmissibility, and from a special type of measurement known as the ODS FRF [1]. This paper will present the characteristics of the use of transmissibility in identifying the vibration operating modes.

2.2 Transmissibility

The transmissibility function is defined as the ratio of the frequency domain from two response signals:

$$T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} \tag{2}$$

where: T_{ij} is the function of transmissibility between the spectral response X_i, and the spectral response of reference X_j.

The transmissibilities are experimentally obtained by measuring the responses in different points, and directions of interest on the structure divided by the reference response of a same fixed point. This procedure is the same as that used in the FRF estimation; however, the response is divided by a reference response rather than an excitation force. As a response signal from a point *i* and a reference point *j,* the transmissibility can be estimated from the estimator H_1 using the Eq. (3) [6]:

$$T_{ij}(\omega) = \frac{S_{ij}(\omega)}{S_{jj}(\omega)} \tag{3}$$

where S_{ij} is the cross-spectral density between the response signal and the reference S_{jj} is the auto spectral density of the reference signal. The phase is preserved by the cross-spectral density from the two signals measured.

From the Eq. (3) it is possible to note that the estimation of the transmissibility does not depend on excitation force data. This is one of the advantages of using transmissibility in identifying operating modes. Whereas, in operating situations it is not always possible to measure the excitation force. Furthermore, the transmissibility can be applied in systems where the amplitude of the excitation force varies with time. In such cases, if the amplitude of the excitation force varies from one measurement to another, it is assumed that this variation effect is the same in all responses, so this effect is canceled in the calculation of the transmissibility. However, the transmissibility depends on the location of both the excitation force, and the reference response. In case of a variation in the location of the excitation, a difference will occur in the transmissibility of point *i*, for a given reference *j*.

The location of the reference response plays an important role in the estimation of the transmissibility. Usually a point of maximum response is chosen as reference, which ensures a good signal-to-noise ratio in the measurements. If the reference response shows null or near zero values in some frequencies, the identification of the operating modes from transmissibilities will be compromised. Another consequence is the appearance of peaks in these frequencies in the transmissibility spectrums. This is a drawback of the use of the transmissibility when compared to the FRF, since the peaks in the transmissibility spectrums are usually disassociated from the natural frequencies, such as in the FRF. The reference response has nulls values at the antiresonance frequencies, which will be discussed in the next section. Therefore, the peaks of the transmissibility spectrums are associated with the antiresonance frequencies of the FRF reference spectrum. A specific case of antiresonance occurs when the reference is placed in a mode node with natural frequency ω; therefore, the transmissibility spectrum will present a peak in this natural frequency ω.

2.3 Antiresonance Frequency

The peaks in the spectral representations of the frequency response functions correspond to the resonances, while the valleys correspond to antiresonance frequencies. For a conservative system of two degrees of freedom, receptance (frequency response function of displacement), considering the response and excitation at the degree of freedom *i* is given as follows:

$$\alpha_{ii}(\omega) = \frac{\phi_{i1}^2}{(\omega_1^2 - \omega^2)} + \frac{\phi_{i2}^2}{(\omega_2^2 - \omega^2)} \tag{4}$$

where ω_1 and ω_2 are the natural frequencies, φ_{i1} and φ_{i2} are the normalized modal components of displacement in the *i-th* degree of freedom for modes 1 and 2, respectively. The antiresonance frequency $\tilde{\omega}$ in the *i-th* driving point at which Eq. (4) goes to zero is expressed as [8]:

$$\tilde{\omega}_i = \gamma_i \omega_1^2 + (1-\gamma_i)\omega_2^2 \tag{5}$$

where

$$\gamma_i = \left(1 + \frac{\phi_{i1}^2}{\phi_{i2}^2}\right)^{-1} \tag{6}$$

The antiresonance frequency is a local property of the structure; since it depends on the location of both the excitation and the response. Distinctly from the resonance frequency, this is a global property of the structure, and independent of the location for both the excitation and the response. Thus, the transmissibilities will present different spectral characteristics, particularly in terms of the peak frequencies; depending on the location of both the reference and the excitation; since as previously seen, the transmissibility peaks are associated with the antiresonance of the reference.

3 EXPERIMENTAL TEST CASE

Three experimental test cases were performed on an aluminum plate of $0.150\times0.200\times0.0095$ m, with a rectangular cut of $0.150\times0.200\times0.0095$ m, illustrated in Figure 1. The plate was hanging by nylon threads (free-free condition). The plate was excited with random white noise signals with frequency ranges between 0 and 2000 Hz, using an electromagnetic exciter. The excitation force was measured using a Brüel & Kjaer® type 8200 force transducer with sensibility of 3.84 pC/N, fixed to the plate. Two Delta Tron® type 4508 piezoelectric accelerometers, sensibility 10.06 mv/ms^{-2}, was used to measure acceleration in the perpendicular direction to the plate surface.

In each experiment were adopted different points of reference and excitation. Table 1 shows the coordinate of the excitation point and reference point used in each experiment. Figure 2 shows the experimental apparatus.

	Excitation	***Coordinate***	***Reference***	***Coordinate***
Case 1	Excitation 1	$0.200\times0.200\times0.000$	Reference 1	$0.150\times0.200\times0.0095$
Case 2	Excitation 2	$0.050\times0.450\times0.000$	Reference 1	$0.150\times0.200\times0.0095$
Case 3	Excitation 1	$0.200\times0.200\times0.000$	Reference 2	$0.025\times0.025\times0.0095$

Table 1: Experiments configurations

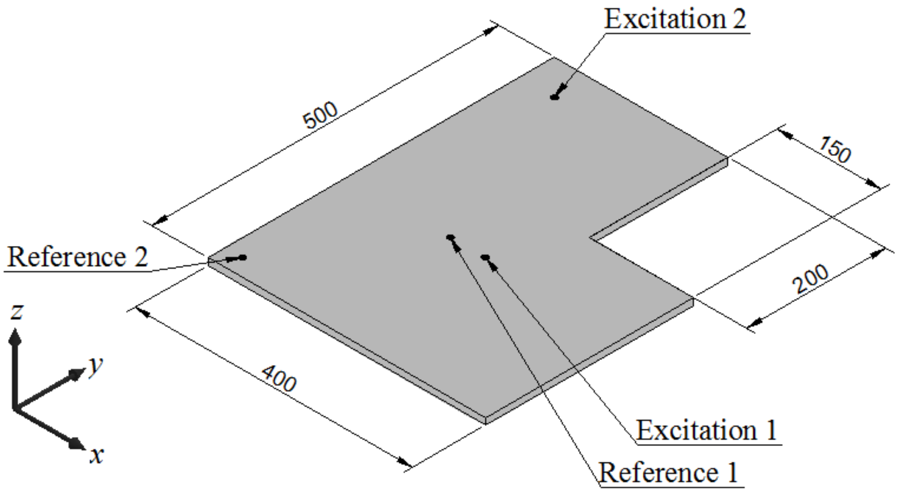

Figure 1 - Plate used in the experiment

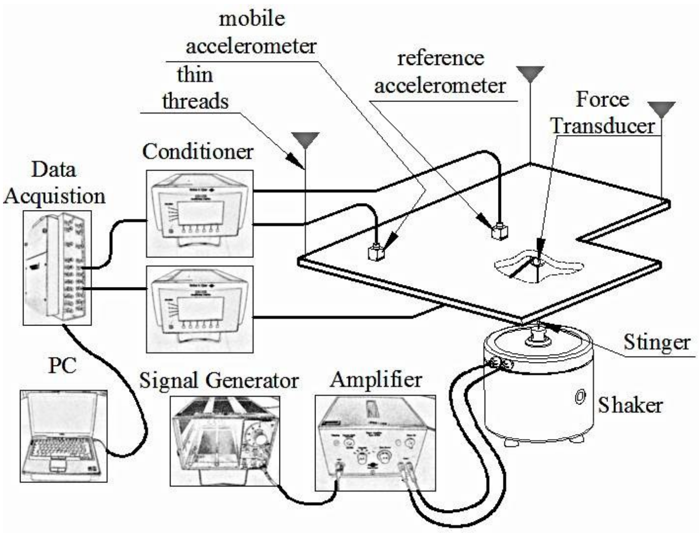

Figure 2 - Experimental apparatus

It was verified the influence of the reference location in the operating deflection shapes, and in the transmissibility amplitude spectrums.

4 RESULTS

The FRF of the reference point, and the transmissibility of an arbitrarily chosen point with the $0.200 \times 0.05 \times 0.0095$ coordinates, were estimated using the estimator H_1 in the three evaluated cases. Below are comparisons between the FRF and transmissibilities, and operating modes, identified by the transmissibilities, compared to the simulated modes.

4.1 Case 1

Figure 3 shows the reference FRF (dashed line), and the transmissibility (solid line) of the chosen point. For each antiresonance (valley) in the reference FRF there is a peak in the transmissibility.

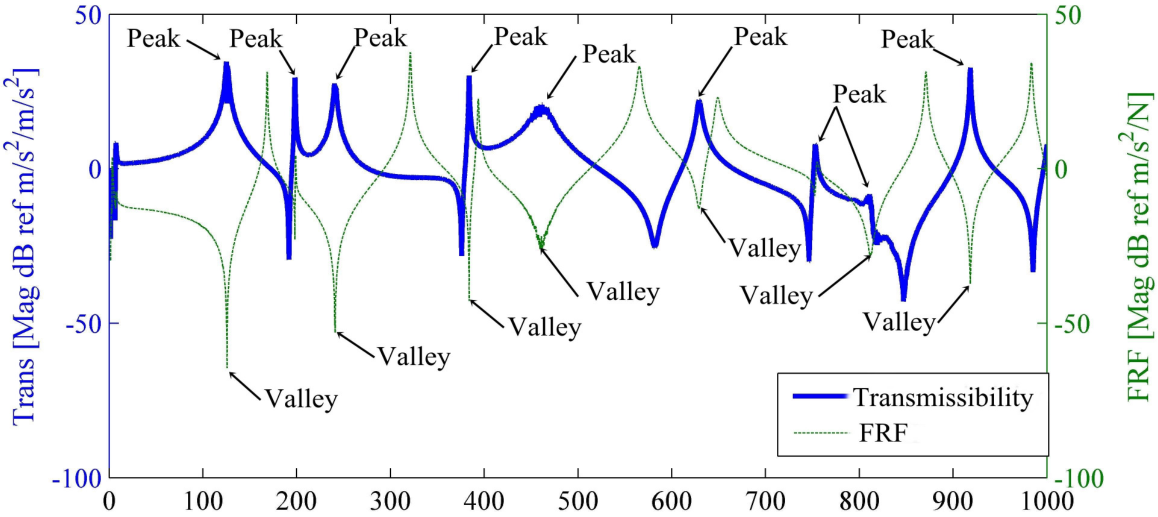

Figure 3 - Transmissibility function peaks coinciding with the antiresonance frequencies of the FRF of the reference adopted in the calculation of transmissibility.

The peak near 200 Hz practically coincides for both the function of transmissibility and for the FRF, and presents a narrow characteristic. The second mode shape of the evaluated aluminum plate is 196 Hz. In this situation, the reference was located near the modal node of the second mode. Figure 4 shows the comparison between the operating mode at a frequency of 198 Hz, which was identified using the transmissibilities, estimated by using the reference in the node from this mode, and the simulated mode.

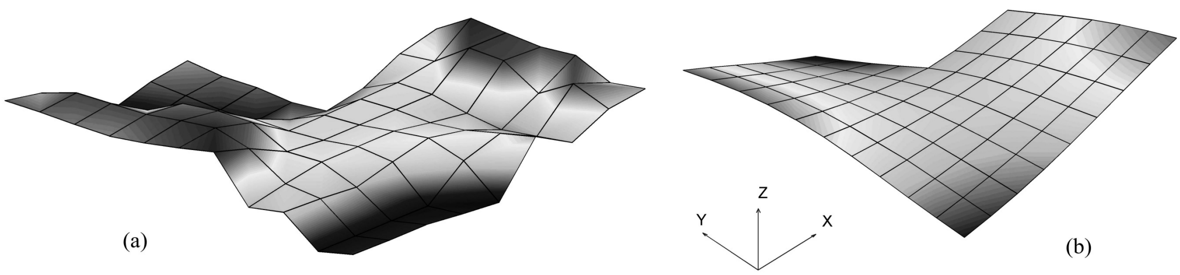

Figure 4 - Comparison between the operating mode at 198 Hz (a), and the simulated mode (b).

The operating mode identified by the transmissibility is distorted. In this mode, the reference and the excitation force are closely positioned to the node, causing this effect.

4.2 Case 2

The reference FRF (dashed line), and the transmissibility (solid line) of the chosen point are illustrated in Figure 5.

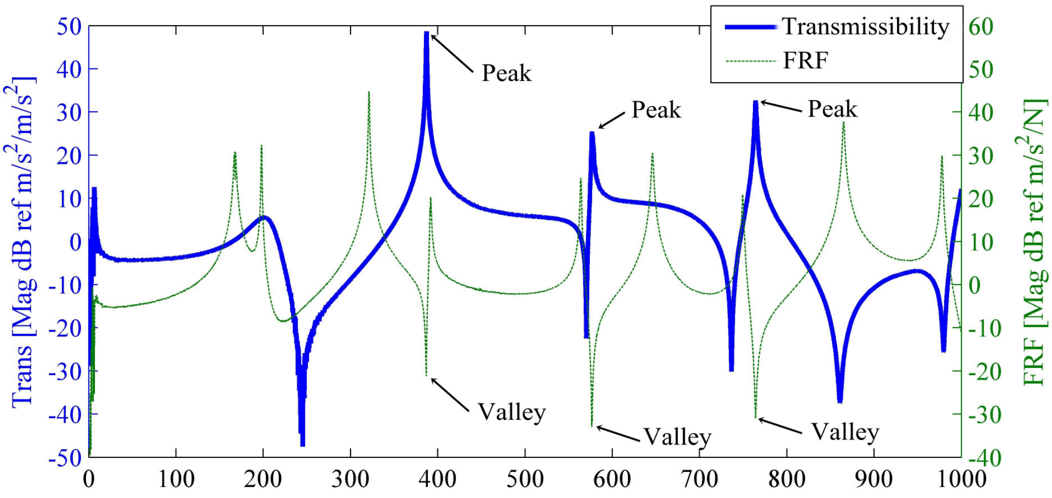

Figure 5 - Transmissibility function peaks coinciding with the antiresonance frequencies of the reference FRF adopted in the calculation of transmissibility.

The location of the reference in Case 2 is the same as in case 1; however, the location of the excitation force is different. In the FRF spectrum of the Figure 5, the antiresonance close to 200 Hz isn't as well defined as in the case 1. In the transmissibility spectrum, the peak at this frequency appears, though in a more attenuated form. Since the reference is positioned in a region of low amplitude, the appearance of a peak in the transmissibility for the mode close to the frequency of 200 Hz is expected, but the characteristic of this peak depends on the anti-resonance peak of the reference FRF. Figure 6 shows the comparison between the operating mode at a frequency of 198 Hz, which was identified by using the estimated transmissibilities, and the simulated mode.

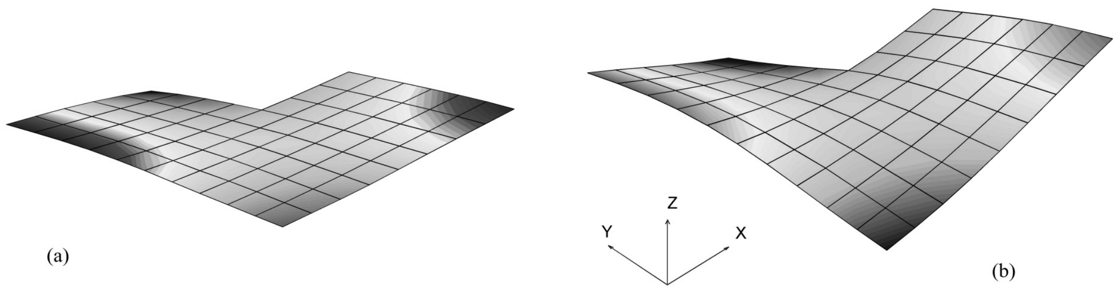

Figure 6 - Comparison between the operating mode at 198 Hz (a), and the simulated mode (b).

The operating mode identified with the configurations of the case 2, shows a good correlation with the simulated mode.

4.3 Case 3

The reference FRF (dashed line), and the transmissibility (solid line) of the chosen point are illustrated in Figure 7.

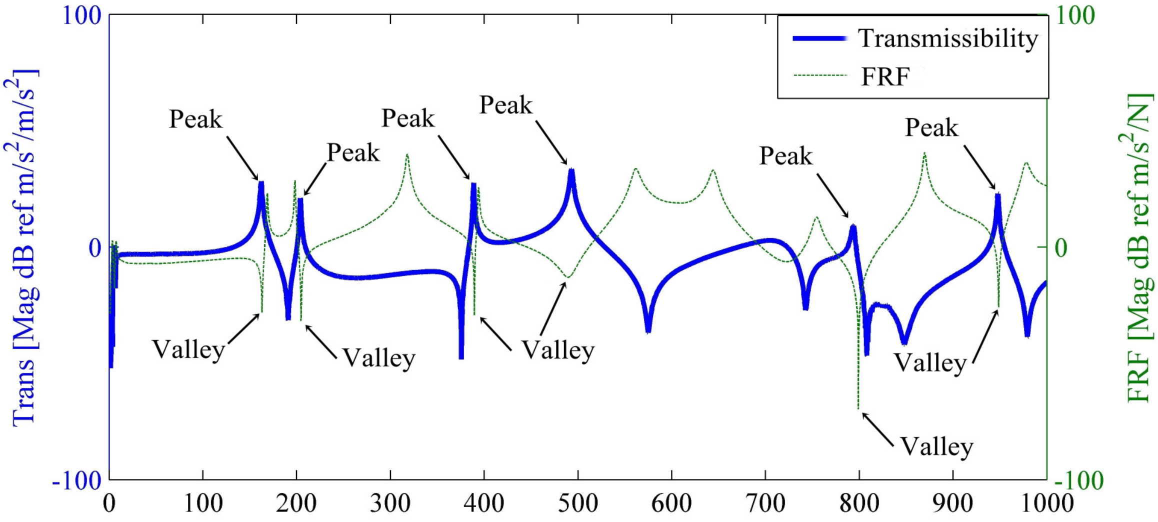

Figure 7 - Transmissibility function peaks coinciding with the antiresonance frequencies of the reference FRF adopted in the calculation of transmissibility.

Figure 8 shows the comparison between the operating mode at a frequency of 198 Hz, which was identified by using the estimated transmissibilities and the simulated mode.

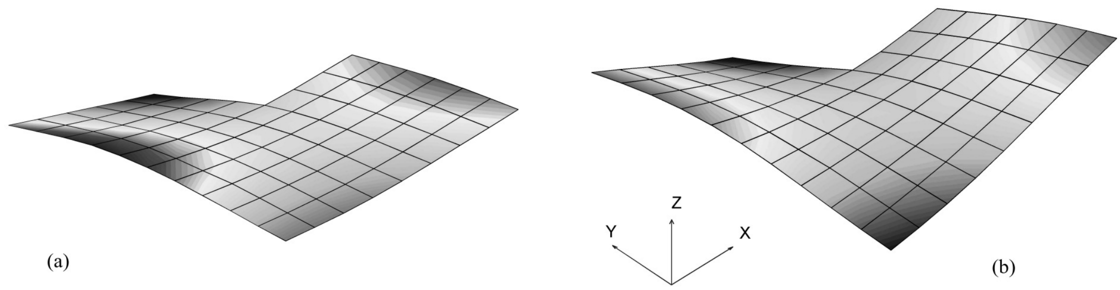

Figure 8 - Comparison between the operating mode at 198 Hz (a), and the simulated mode (b).

In case 3, although the excitation force is located next to a region of modal node of the second mode shape; the reference is located in a large amplitude region of this vibration mode. Operating modes identified in this case showed better correlation with the simulated mode.

5 CONCLUSIONS

The relationship between the transmissibilities peaks and the antiresonance frequencies of the reference point was evaluated in three test cases. The transmissibility peaks coincided with the antiresonance frequencies of the reference point in all transmissibilities. Owing to this relationship, when one adopts different locations of the reference, the transmissibilities peaks occur at different frequencies. As a result of the antiresonance's local characteristic, i.e., the antiresonance frequency is different depending on point location of the structure. Unlike in the FRF, where the peaks (resonance) must occur at the same frequency, regardless of the location of both the force and the response; since the natural frequency is a global characteristic of the structure. These findings clarify the existence of transmissibilities peaks unrelated to the natural frequency in structures excited with random white noise signals. As well as, in this paper it was verified the influence of the reference location in the identification of operating modes by the Operation Deflection Shapes using the transmissibility. Depending on the location of the reference, it is not possible to identify certain operating modes. References positioned in areas of low amplitude vibration compromise the noise signal ratio, and the identification of operating modes at certain frequencies.

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