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PARAMETER IDENTIFICATION OF NONLINEAR DYNAMIC SYSTEMS WITH TIME-DELAYED FEEDBACK CONTROL

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Abstract. *There are mainly two problems lie in the researches on parameter identification of nonlinear dynamic systems. The first one is that no common identification model has been widely applied because of the complexity in nonlinear systems. The other one is that there isn't a general identification methodology employed to the system since the first one hasn't been well solved.*

Focused on these two questions, it's assumed in this paper that the nonlinearity of the system was smooth with respect to system's deformation. Then the identification model can be described by a standard Kronecker product through Taylor expansion. Its linear degenerated form is the well-known state space function. Thereafter, the identification algorithm in the form of complex exponential function based on harmonic balance principle can be constructed.

For the demonstration of this algorithm, a numerical simulation of a 2-DOF system with time-delayed control was made. The results showed that the identification algorithm can acquire accurate structural and delay parameters.

1 INTRODUCTION

Time delay appears in signal processing and transmitting. It can cause fundamental changes in dynamic characteristics when the system is controlled by close-loop mode. J. Xu [1,2] etc. certified the bifurcation and chaos characteristics deduced by time delay. The results show that the delay can be used as a 'switch' to control motions of a system. Therefore, the identification and utilizing of the delay effect is so important to the control strategy that the methods of time-delay identification are discussed intensively.

Firstly, in the linear dynamic area, the identification methods are mostly based on analytical principles. Y. Orlov [3] defined an adaptive identifier assuming that the state and delay parameters are dependent variables of time t. This identifier ensures the convergence theoretically when t is tending to infinity. However, S. V. Drakunov [4] pointed out that the adaptive identifier's accuracy depends on the number m of implemented delays and the computational effort strongly increases with m. So an improved method is formulated and called 'sliding mode observer'.

Secondly, in the nonlinear dynamic area, the differential functions of nonlinear systems do not have analytical solutions usually. So the identification methods are mostly based on optimization principles. Some are deterministic algorithms. For example, Ryan Loxton [5] presented a steepest descent algorithm in which the gradient of the cost function is computed by auxiliary functions instead of difference approach. Thus the singularity caused by numerical difference is avoided. V. S. Udaltsov [6] investigated a T-recovery algorithm for the dealing with chaotic cryptosystems.

However, there is a common drawback in the optimization algorithms. The identified results depend on the initial conditions highly and the algorithms only ensure the local optimal solutions.

We noted that in the research of nonlinear dynamics, the principle of harmonic balance (HB) is effective and has been used to identify the state parameters successfully. The identification algorithms based on HB method can deal with steady-state response [7,8], limit cycle response[9] and chaotic response[10,11]. So it's expected that the HB principle could be extended to the identification of time delay parameters. In this paper, the delay identification algorithm based on HB method will be formulated and researched.

2 IDENTIFICATION MODEL OF NONLINEAR SYSTEMS

The state space function of a MDOF (multi-degree of freedom) system without time-delayed feedback control can be written as:

$$\dot{\mathbf{y}} = \mathbf{U}(\mathbf{y}) + \mathbf{g}(t) \tag{1}$$

where $\mathbf{y} \in R^n$ is the system's state variable, $\mathbf{U}(\mathbf{y})$ is the structural function and $\mathbf{g}(t)$ is the external excitation.

However, this kind of equation is too general to extract any helpful information for parameter identification and some simplification should be determined correspondingly. For example, the structural function of Eq. (1) can be considered to be smooth with respect to $\mathbf{y}$ and the equilibrium point locates at origin. Consequently $\mathbf{U}(\mathbf{y})$ can be expanded into Taylor series:

$$\mathbf{U}(\mathbf{y}) = \frac{\partial \mathbf{U}(\mathbf{0})}{\partial \mathbf{y}}\mathbf{y} + \frac{1}{2!}\frac{\partial^2 \mathbf{U}(\mathbf{0})}{\partial \mathbf{y}^{[2]}}\mathbf{y}^{[2]} + \frac{1}{3!}\frac{\partial^3 \mathbf{U}(\mathbf{0})}{\partial \mathbf{y}^{[3]}}\mathbf{y}^{[3]} + \cdots \tag{2}$$

where $\mathbf{y}^{[k]} := \underbrace{\mathbf{y} \otimes \mathbf{y} \otimes \cdots \otimes \mathbf{y}}_{k\ times}$ is the k^{th} order Kronecker product of $\mathbf{y}$, $\left(\dfrac{\partial^k \mathbf{U}(\mathbf{0})}{\partial \mathbf{y}^{[k]}} \right)_{i,j} = \dfrac{\partial^k \mathbf{U}_i(\mathbf{0})}{\partial \left(\mathbf{y}^{[k]} \right)_j}$ is the element of $\mathbf{y}^{[k]}$'s coefficient matrix.

Furthermore, Eq. (2) can be truncated into finite series for given tolerance. Without loss of generality, one can truncate Eq. (2) into a cubic polynomial form. Thus Eq. (1) will be rewritten as:

$$\dot{\mathbf{y}} = \mathbf{A}_1 \mathbf{y} + \mathbf{A}_2 \mathbf{y}^{[2]} + \mathbf{A}_3 \mathbf{y}^{[3]} + \mathbf{g}(t) \tag{3}$$

where $\mathbf{A}_k = \left(\dfrac{\partial^k \mathbf{U}_i(\mathbf{0})}{\partial \left(\mathbf{y}^{[k]} \right)_j} \right)$ is the constant coefficient matrix of $\mathbf{y}^{[k]}$.

On the basis of standard polynomial form like Eq. (3), the linear superposition of delayed state variables, which was mentioned in article [3], can be used to portrait multiple discrete time-delayed linear feedbacks, i.e.:

$$\dot{\mathbf{y}} = \mathbf{A}_1 \mathbf{y} + \mathbf{A}_2 \mathbf{y}^{[2]} + \mathbf{A}_3 \mathbf{y}^{[3]} + \sum_{n=1}^{L} \mathbf{D}_n \mathbf{y}(t - \tau_n) + \mathbf{g}(t) \tag{4}$$

where τ_n is time delay, $\mathbf{D}_n$ is the constant feedback gain corresponding to τ_n .

In the work of parameter identification, it's commonly accepted that the state variable $\mathbf{y}$ and the external excitation $\mathbf{g}(t)$ are prior known and what to be identified is the system's control and feedback function.

In Eq. (4), the structural function is specialized into polynomial forms and the feedback function is portrayed as linear superposition forms. Thus the identification problems come out to be the determination of constant coefficient matrices like $\mathbf{A}_k$, $\mathbf{D}_n$ and τ_n .

3 ALGORITHM OF PARAMENTER IDENTIFICATION

For the identification of system like Eq. (4), the system is assumed to be asymptotic stable. Thus when the system is excited by a homologous harmonic wave, i.e.:

$$\mathbf{g}(t) = \mathbf{p} \sin(\omega t) + \mathbf{q} \cos(\omega t) \tag{5}$$

its stable state response will be periodic and can be expanded into Fourier series:

$$\mathbf{y}(t) = \sum_{n=0}^{S} \left[\mathbf{a}_n \sin(n\omega t) + \mathbf{b}_n \cos(n\omega t) \right] \tag{6}$$

in which $\mathbf{a}_0 = \mathbf{0}$ is defined to eliminate coefficient uncertainty.

Substituting Eq. (5) and (6) into Eq. (4) and applying the principle of harmonic balance yields:

$$-\omega \mathbf{b}_l = \mathbf{A}_1 \mathbf{a}_l + \frac{1}{2} \mathbf{A}_2 \boldsymbol{\alpha}_l + \frac{1}{4} \mathbf{A}_3 \boldsymbol{\gamma}_l + \sum_{n=1}^{L} \mathbf{D}_n \left[\mathbf{a}_l \cos(l\omega\tau_n) + \mathbf{b}_l \sin(l\omega\tau_n) \right] + \mathbf{p}\delta(l-1) \tag{7}$$

for the coefficients of $\sin(l\omega t)$ and:

$$\omega \mathbf{a}_l = \mathbf{A}_1 \mathbf{b}_l + \frac{1}{2}\mathbf{A}_2 \boldsymbol{\beta}_l + \frac{1}{4}\mathbf{A}_3 \boldsymbol{\eta}_l + \sum_{n=1}^{L} \mathbf{D}_n \left[\mathbf{b}_l \cos\left(l\omega\tau_n\right) - \mathbf{a}_l \sin\left(l\omega\tau_n\right)\right] + \mathbf{q}\delta\left(l-1\right) \tag{8}$$

for the coefficients of $\cos\left(l\omega t\right)$, where

$$\boldsymbol{\alpha}_l = \pm \sum_{i+j=\pm l} \left(\mathbf{a}_i \otimes \mathbf{b}_j + \mathbf{b}_i \otimes \mathbf{a}_j\right) \pm \sum_{i-j=\pm l} \left(\mathbf{a}_i \otimes \mathbf{b}_j - \mathbf{b}_i \otimes \mathbf{a}_j\right) \tag{9}$$

$$\begin{aligned} \boldsymbol{\gamma}_l = &\pm \sum_{i+j+k=\pm l} \left(-\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k + \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k\right) \\ &\pm \sum_{i+j-k=\pm l} \left(\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k + \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k - \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k\right) \\ &\pm \sum_{i-j+k=\pm l} \left(\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k - \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k\right) \\ &\pm \sum_{-i+j+k=\pm l} \left(\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k - \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k + \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k\right) \end{aligned} \tag{10}$$

$$\boldsymbol{\beta}_l = \sum_{i+j=\pm l} \left(-\mathbf{a}_i \otimes \mathbf{a}_j + \mathbf{b}_i \otimes \mathbf{b}_j\right) + \sum_{i-j=\pm l} \left(\mathbf{a}_i \otimes \mathbf{a}_j + \mathbf{b}_i \otimes \mathbf{b}_j\right) \tag{11}$$

$$\begin{aligned} \boldsymbol{\eta}_l = &\sum_{i+j+k=\pm l} \left(-\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k - \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k - \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k\right) + \\ &\sum_{i+j-k=\pm l} \left(-\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k + \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k + \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k\right) + \\ &\sum_{i-j+k=\pm l} \left(\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k - \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k + \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k\right) + \\ &\sum_{-i+j+k=\pm l} \left(\mathbf{a}_i \otimes \mathbf{a}_j \otimes \mathbf{b}_k + \mathbf{a}_i \otimes \mathbf{b}_j \otimes \mathbf{a}_k - \mathbf{b}_i \otimes \mathbf{a}_j \otimes \mathbf{a}_k + \mathbf{b}_i \otimes \mathbf{b}_j \otimes \mathbf{b}_k\right) \end{aligned} \tag{12}$$

$$i, j, k = 0, 1, 2, \cdots, S$$

To determine system parameters, the critic work is solving matrices like $\mathbf{A}_k$, $\mathbf{D}_n$ and τ_n form Eq. (7) or (8).

Since one order of harmonic balance equations is independent from the others and the most significant response is generally on dominant frequency, i.e. the exciting frequency, one can choose only dominant harmonic balance equations from Eq. (7) and (8) to identify system parameters.

By rearranging the dominant frequency response equation from Eq. (7) and (8), one can get

$$\begin{aligned} &\left(-\omega \mathbf{b}_1 - \mathbf{p} \quad \omega \mathbf{a}_1 - \mathbf{q}\right) = \\ &\left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right) \begin{pmatrix} \mathbf{a}_1 & \mathbf{b}_1 \\ \frac{1}{2}\boldsymbol{\alpha}_1 & \frac{1}{2}\boldsymbol{\beta}_1 \\ \frac{1}{4}\boldsymbol{\gamma}_1 & \frac{1}{4}\boldsymbol{\eta}_1 \end{pmatrix} + \sum_{n=1}^{L} \mathbf{D}_n \left(\mathbf{a}_1 \quad \mathbf{b}_1\right) \begin{pmatrix} \cos\left(\omega\tau_n\right) & -\sin\left(\omega\tau_n\right) \\ \sin\left(\omega\tau_n\right) & \cos\left(\omega\tau_n\right) \end{pmatrix} \end{aligned} \tag{13}$$

and then right multiplying $\left(1 \quad j\right)^T$ by both side of Eq. (13) yields

$$j\omega\left(\mathbf{a}_1 + j\mathbf{b}_1\right) - \left(\mathbf{p} + j\mathbf{q}\right) = \left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right)\begin{pmatrix} \mathbf{a}_1 + j\mathbf{b}_1 \\ \dfrac{\boldsymbol{\alpha}_1 + j\boldsymbol{\beta}_1}{2} \\ \dfrac{\boldsymbol{\gamma}_1 + j\boldsymbol{\eta}_1}{4} \end{pmatrix} + \sum_{n=1}^{L} e^{-j\omega\tau_n}\mathbf{D}_n\left(\mathbf{a}_1 + j\mathbf{b}_1\right) \tag{14}$$

Let $\mathbf{G}_\omega = j\omega\left(\mathbf{a}_1 + j\mathbf{b}_1\right) - \left(\mathbf{p} + j\mathbf{q}\right)$, $\mathbf{H}_\omega = \begin{pmatrix} \mathbf{a}_1 + j\mathbf{b}_1 \\ \dfrac{\boldsymbol{\alpha}_1 + j\boldsymbol{\beta}_1}{2} \\ \dfrac{\boldsymbol{\gamma}_1 + j\boldsymbol{\eta}_1}{4} \end{pmatrix}$ and $\mathbf{c}_\omega = \mathbf{a}_1 + j\mathbf{b}_1$ corresponding to the excitation frequency ω, Eq. (14) can be simplified to the form as follows

$$\mathbf{G}_\omega = \left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right)\mathbf{H}_\omega + \sum_{n=1}^{L} e^{-j\omega\tau_n}\mathbf{D}_n\mathbf{c}_\omega \tag{15}$$

It's easy to find that the first part in Eq. (15)'s right side is linear for the system's structural function parameters and the second part is transcendental for the delayed feedback parameters.

3.1 Identification of Structural Parameters

Since linear equation assures convenience and uniqueness for its solution, which is very important for the confidence of the identified parameters, and feedback control is artificial, one can switch off the control first for the identification of structural parameters. Thus Eq. (15) without delayed feedback part can be rewritten as

$$\mathbf{G}_\omega = \left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right)\mathbf{H}_\omega \tag{16}$$

For different excitation conditions, Eq. (16) can be arrayed into an over determined matrix function, i.e.

$$\left(\mathbf{G}_{\omega_2} \quad \mathbf{G}_{\omega_2} \quad \cdots \quad \mathbf{G}_{\omega_k}\right) = \left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right)\left(\mathbf{H}_{\omega_2} \quad \mathbf{H}_{\omega_2} \quad \cdots \quad \mathbf{H}_{\omega_k}\right) \tag{17}$$

Thus the structural parameters can be solved from this equation and the solution is

$$\begin{aligned} \left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right) = {} & \left(\mathbf{G}_{\omega_2} \quad \mathbf{G}_{\omega_2} \quad \cdots \quad \mathbf{G}_{\omega_k}\right)\left(\mathbf{H}_{\omega_2} \quad \mathbf{H}_{\omega_2} \quad \cdots \quad \mathbf{H}_{\omega_k}\right)^T \\ & \cdot\left(\left(\mathbf{H}_{\omega_2} \quad \mathbf{H}_{\omega_2} \quad \cdots \quad \mathbf{H}_{\omega_k}\right)\left(\mathbf{H}_{\omega_2} \quad \mathbf{H}_{\omega_2} \quad \cdots \quad \mathbf{H}_{\omega_k}\right)^T\right)^{-1} \end{aligned} \tag{18}$$

3.2 Identification of Feedback Parameters

After the identification of structural parameters, one can switch on the feedback control for the identification of delay and feedback gain parameters. Let $\mathbf{C}_\omega = \mathbf{G}_\omega - \left(\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3\right)\mathbf{H}_\omega$, Eq. (15) can be rewritten as

$$\mathbf{C}_{\omega} = \sum_{n=1}^{L} e^{-j\omega\tau_n} \mathbf{D}_n \mathbf{c}_{\omega} \tag{19}$$

Since Eq. (19) is transcendental for delay parameters, its solving is much more difficult than Eq. (17).

For different excitation conditions, Eq. (19) can be arrayed into an over determined matrix function, i.e.

$$\begin{pmatrix} \mathbf{C}_{\omega_1} \\ \mathbf{C}_{\omega_2} \\ \vdots \\ \mathbf{C}_{\omega_K} \end{pmatrix} = \sum_{n=1}^{L} \begin{pmatrix} e^{-j\omega_1\tau_n} \mathbf{D}_n \mathbf{c}_{\omega_1} \\ e^{-j\omega_2\tau_n} \mathbf{D}_n \mathbf{c}_{\omega_2} \\ \vdots \\ e^{-j\omega_K\tau_n} \mathbf{D}_n \mathbf{c}_{\omega_K} \end{pmatrix} \tag{20}$$

For the solution of $\mathbf{D}_n$ and τ_n from Eq.(20), the algorithm based on least square technique is applied. For given parameters of $\mathbf{D}_n$ and τ_n, the error vector can be written as

$$\mathbf{E}\left(\mathbf{D}_n, \tau_n\right) = \begin{pmatrix} \mathbf{C}_{\omega_1} \\ \mathbf{C}_{\omega_2} \\ \vdots \\ \mathbf{C}_{\omega_K} \end{pmatrix} - \sum_{n=1}^{L} \begin{pmatrix} e^{-j\omega_1\tau_n} \mathbf{D}_n \mathbf{c}_{\omega_1} \\ e^{-j\omega_2\tau_n} \mathbf{D}_n \mathbf{c}_{\omega_2} \\ \vdots \\ e^{-j\omega_K\tau_n} \mathbf{D}_n \mathbf{c}_{\omega_K} \end{pmatrix} \tag{21}$$

By expanding this equation to the linear form at point $\left(\mathbf{D}_n^0, \tau_n^0\right)$, one can get

$$\mathbf{E}\left(\mathbf{D}_n, \tau_n\right) = \mathbf{E}\left(\mathbf{D}_n^0, \tau_n^0\right) - \sum_{n=1}^{L} \left(\begin{bmatrix} e^{-j\omega_1\tau_n^0} \mathbf{\Theta}_1 & -j\omega_1 e^{-j\omega_1\tau_n^0} \mathbf{D}_n^0 \mathbf{c}_{\omega_1} \\ e^{-j\omega_2\tau_n^0} \mathbf{\Theta}_2 & -j\omega_2 e^{-j\omega_2\tau_n^0} \mathbf{D}_n^0 \mathbf{c}_{\omega_2} \\ \vdots & \vdots \\ e^{-j\omega_K\tau_n^0} \mathbf{\Theta}_K & -j\omega_K e^{-j\omega_K\tau_n^0} \mathbf{D}_n^0 \mathbf{c}_{\omega_K} \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_1 - \boldsymbol{\lambda}_1^0 \\ \boldsymbol{\lambda}_2 - \boldsymbol{\lambda}_2^0 \\ \vdots \\ \boldsymbol{\lambda}_N - \boldsymbol{\lambda}_N^0 \\ \tau_n - \tau_n^0 \end{bmatrix} \right) \tag{22}$$

where

$\boldsymbol{\lambda}_r = \begin{bmatrix} D_{n(r,1)} & D_{n(r,2)} & \cdots & D_{n(r,N)} \end{bmatrix}^T, r \in N, n \in L$ and $\mathbf{\Theta}_i = \begin{bmatrix} \mathbf{c}_{\omega_i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_{\omega_i} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{c}_{\omega_i} \end{bmatrix}^T, i \in K$ and the number of $\mathbf{c}_{\omega_i}$ is N.

Furthermore, Eq.(22) can be simplified to the following form

$$\mathbf{E} = \mathbf{E}^0 - \begin{bmatrix} \mathbf{J}_1^0 & \mathbf{J}_2^0 & \cdots & \mathbf{J}_L^0 \end{bmatrix} \left(\begin{bmatrix} \mathbf{\Lambda}_1 \\ \mathbf{\Lambda}_2 \\ \vdots \\ \mathbf{\Lambda}_L \end{bmatrix} - \begin{bmatrix} \mathbf{\Lambda}_1^0 \\ \mathbf{\Lambda}_2^0 \\ \vdots \\ \mathbf{\Lambda}_L^0 \end{bmatrix} \right) := \mathbf{E}^0 - \mathbf{J}^0 \cdot \left(\mathbf{\Lambda} - \mathbf{\Lambda}^0\right) \tag{23}$$

by remarking $\mathbf{J}_n^0 = \begin{bmatrix} e^{-j\omega_1\tau_n^0}\mathbf{\Theta}_1 & -j\omega_1 e^{-j\omega_1\tau_n^0}\mathbf{D}_n^0\mathbf{c}_{\omega_1} \\ e^{-j\omega_2\tau_n^0}\mathbf{\Theta}_2 & -j\omega_2 e^{-j\omega_2\tau_n^0}\mathbf{D}_n^0\mathbf{c}_{\omega_2} \\ \vdots & \vdots \\ e^{-j\omega_K\tau_n^0}\mathbf{\Theta}_K & -j\omega_K e^{-j\omega_K\tau_n^0}\mathbf{D}_n^0\mathbf{c}_{\omega_K} \end{bmatrix}, \mathbf{\Lambda}_n = \begin{bmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \\ \vdots \\ \boldsymbol{\lambda}_N \\ \tau_n \end{bmatrix}$.

From the theory of least square technique, the variance of error would be minimal when parameters like $\mathbf{D}_n$ and τ_n touch the real ones, i.e.

$$\frac{\partial\sigma}{\partial\mathbf{\Lambda}} = 2\frac{\partial\mathbf{E}^H}{\partial\mathbf{\Lambda}}\mathbf{E} = \mathbf{0} \tag{24}$$

where $\sigma = \mathbf{E}^H\mathbf{E}$ and H means the conjugate transpose. Substituting Eq. (23) into Eq. (24) yields

$$\mathbf{\Lambda} = \left(\left(\mathbf{J}^0\right)^H \mathbf{J}^0\right)^{-1}\left(\mathbf{J}^0\right)^H \mathbf{E}^0 + \mathbf{\Lambda}^0 \tag{25}$$

From the recursion of Eq.(25), one can get the iteration form of parameter $\mathbf{\Lambda}$, which is

$$\mathbf{\Lambda}^{i+1} = \left(\left(\mathbf{J}^i\right)^H \mathbf{J}^i\right)^{-1}\left(\mathbf{J}^i\right)^H \mathbf{E}^i + \mathbf{\Lambda}^i \tag{26}$$

Thus for a given tolerance, the delay and feedback gain parameters like τ_n and $\mathbf{D}_n$ can be identified.

4 PARAMETER IDENTIFICATION SIMULATIONS

4.1 Simulation model

A 2-DOF Duffing system with coupled time-delay feedbacks is constructed to demonstrate the identification method. The model of Duffing system is shown as follows:

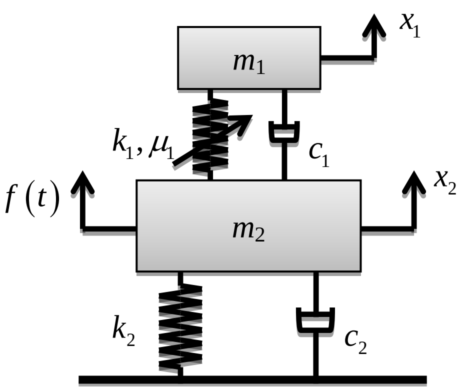

Figure 1: Model of 2DOF Duffing System

Thus its dynamic differential equation can be written by Lagrange function as:

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix}+\begin{pmatrix} c_1 & -c_1 \\ -c_1 & c_1+c_2 \end{pmatrix}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}+\begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \mu_1 & 0 \\ -\mu_1 & 0 \end{pmatrix}\begin{pmatrix} (x_1-x_2)^3 \\ x_2^3 \end{pmatrix}=\begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} \tag{27}$$

Letting $(y_1 \quad y_2 \quad y_3 \quad y_4)^T=(x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2)^T$ and assuming that $g_1(t)$ and $g_2(t)$ are homologous sine excitations. Thus Eq. (27) can be rewritten into the form of state-space equation:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix}=\begin{pmatrix} y_3 \\ y_4 \\ \frac{c_1}{m_1}(y_4-y_3)+\frac{k_1}{m_1}(y_2-y_1)+\frac{\mu_1}{m_1}(y_2-y_1)^3 \\ \frac{c_1}{m_2}(y_3-y_4)+\frac{k_1}{m_2}(y_1-y_2)+\frac{\mu_1}{m_2}(y_1-y_2)^3-\frac{c_2}{m_2}y_4-\frac{k_2}{m_2}y_2 \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ \frac{q_1}{m_1} \\ \frac{q_2}{m_2} \end{pmatrix}\sin(\omega t+\varphi)$$

By setting the structural parameters as $m_1=0.25$, $m_2=0.37$, $c_1=0.02\pi$, $c_2=0.08\pi$, $k_1=1.2\pi^2$, $k_2=0.8\pi^2$, $\mu_1=2$ and feedback parameters as $\tau_1=0.3s$, $\tau_2=0.5s$ and

$$\mathbf{D}_1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -3.1 & 1.8 & 0 & 0 \\ 2.2 & 1.9 & 0 & 0 \end{pmatrix}, \quad \mathbf{D}_2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.14 & 0.09 \\ 0 & 0 & 0.07 & -0.11 \end{pmatrix},$$

the frequency response curve of x_1 and x_2 can be shown as Fig. 2 and Fig. 3.

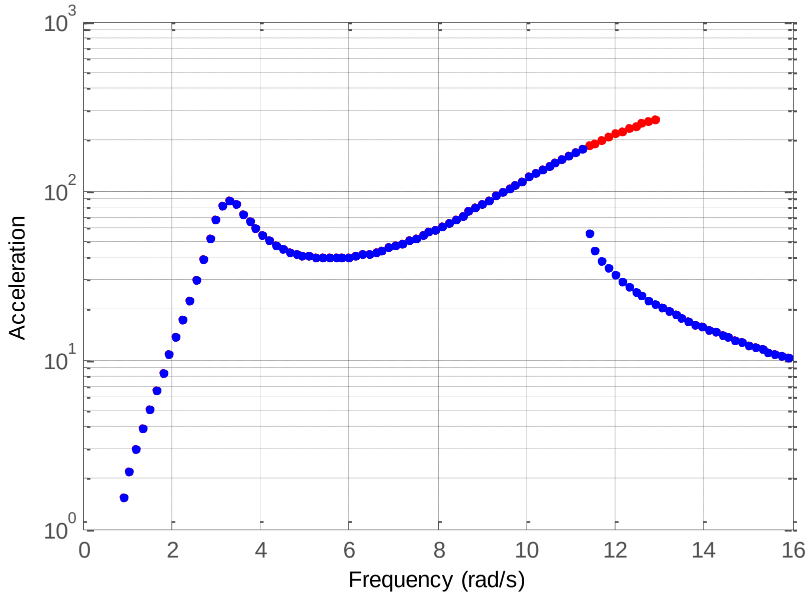

Figure 2: Frequency Response of x_1

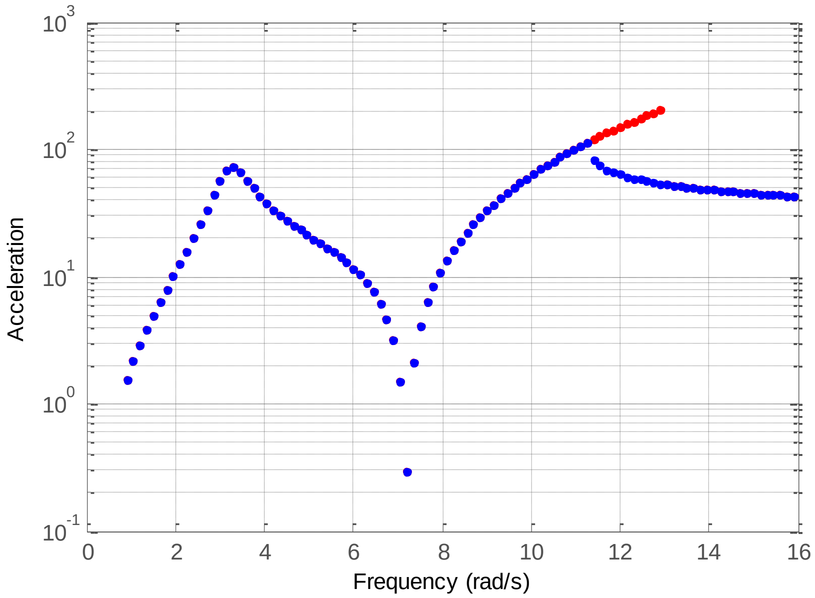

Figure 3: Frequency Response of x_2

4.2 Parameter identification

Then, the simulated response data can be introduced into the identification algorithm, i.e. Eq. (26).

To inspect the algorithm's noise resistance, some white noise was added into the response signals. The level of noise is described by the Signal to Noise Ratio (SNR). Besides, the simulation was made for 5 times for each different level of SNR for statistics.

The identified results are shown in the following tables:

Table 1: Identified Parameters in Different SNR Settings

Noise Level	$D_{1(3,1)}$	$D_{1(3,2)}$	$D_{1(4,1)}$	$D_{1(4,2)}$	τ_1	$D_{2(3,3)}$	$D_{2(3,4)}$	$D_{2(4,3)}$	$D_{2(4,4)}$	τ_2
No Noise	-3.099	1.799	2.199	1.901	0.300	0.140	0.090	0.070	-0.110	0.500
SNR18dB	-3.085	1.789	2.203	1.893	0.299	0.138	0.090	0.071	-0.111	0.501
SNR15dB	-3.069	1.770	2.182	1.913	0.299	0.143	0.092	0.074	-0.117	0.499
SNR12dB	-3.080	1.784	2.208	1.889	0.299	0.138	0.088	0.072	-0.110	0.502
SNR9dB	-3.089	1.779	2.165	1.938	0.301	0.147	0.093	0.076	-0.118	0.495

From this table, one can find that this identification algorithm insures accurate results and possesses a good stability for noise disturbance. For the results of identified delays, the error maintains less than 1% even when SNR mounts up to 9dB.

Furthermore, the standard variance of the error was calculated to demonstrate the algorithm's stability.

Table 2: Standard Variance of the Identified Parameters' Errors

Noise Level	$D_{1(3,1)}$	$D_{1(3,2)}$	$D_{1(4,1)}$	$D_{1(4,2)}$	τ_1	$D_{2(3,3)}$	$D_{2(3,4)}$	$D_{2(4,3)}$	$D_{2(4,4)}$	τ_2
SNR18dB	0.019	0.013	0.010	0.010	0.001	0.003	0.001	0.002	0.004	0.002
SNR15dB	0.071	0.066	0.034	0.028	0.002	0.004	0.003	0.004	0.009	0.005
SNR12dB	0.045	0.043	0.045	0.039	0.002	0.010	0.004	0.007	0.011	0.004
SNR9dB	0.093	0.080	0.062	0.065	0.003	0.011	0.005	0.009	0.013	0.011

5 CONCLUSIONS

In this paper, the principle of harmonic balance was introduced to the identification of nonlinear time-delayed systems. The structural parameters are linear with respect to the harmonic coefficients so that the parameter uniqueness is insured. The delay parameters are contained in the complex exponential functions with respect to the frequency. Thus the curve fitting algorithm based on least square estimation was introduced.

The identified results show that this algorithm gets good accuracy and satisfying stability. It can be used into the identification of multi-DOF systems with or without time-delays.

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