11th International Conference on Vibration Problems
Z. Dimitrovová *et.al.* (eds.)
Lisbon, Portugal, 9–12 September 2013

PASSIVE VIBRATION CONTROL IN MID-FREQUENCY REGION

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Keywords: Mid-frequency, Vibration control, Rib-stiffened plates.

Abstract. *This paper deals with passive vibration control of structures exhibiting both long- and short-wavelength deformations within the same frequency range. Such behaviour is often observed in complex build-up structures such as rib-stiffened plates, automotive vehicles, aeroplane fuselages and so on, and is inherent property of any structure comprising of both stiff and flexible structural elements. Addressing both short- and long-wavelength vibrations within the same frequency range is often referred to as the mid-frequency problem, and presents considerable difficulties since it is not amendable for deterministic (e.q. finite element method, FEM) or statistical (statistical energy analysis, SAE) analysis techniques alone. Furthermore, mid-frequency vibrations exhibit intrinsic sensitivity to system uncertainties (e.q. structure variability, uncertainty in boundary conditions). Mid-frequency vibration problem has attracted considerable interest in the past decade, and a number of hybrid FE/SAE methods for predicting the vibration response has been developed. Yet, passive vibration control of the structures exhibiting mid-frequency vibration has not been fully addressed. In this paper, we discuss an optimal passive control technique for a generic mid-frequency vibration problem via multiple tuned mass dampers (TMDs). The problem at hand is tackled as reducer-order optimal H_∞ control of uncertain system with controller order and structure constraints We address several issues of the proposed approach, such as system uncertainty modelling, applicable optimization algorithms and computational costs. To illustrate the applicability and the efficiency of the proposed approach, we present an example comprising of beam-stiffened plate with multiple TMDs. We also investigate the influence of total TMDs mass, number of TMDs and distribution of individual TMDs resonant frequencies on vibration attenuation efficiency.*

1 INTRODUCTION

Passive vibration attenuation, despite decades of research and vast amount of knowledge and published literature, still remains one of the most important problems in vibration control and a subject of permanent scientific interest [1]. Passive vibration attenuation encompasses a wide variety of methods which, by the use of passive devices, suppress structural vibrations and noise, and is in many instances preferred due to its effectiveness, low cost, reliability and robustness.

When dealing with passive control of complex built-up structures, regardless of the control strategy used, we arguably face two key challenges. The first is the passive vibration control itself — it is well known that it boils down to computationally complex, or even NP-hard problems [2]. The second, which we discuss in more specific setting in this paper, is the inherent uncertainty of complex-built up structures induced by the presence short- and long-wavelength vibrations within the same frequency range. This is often referred to as the mid-frequency problem, and presents considerable difficulties since it is not amendable for deterministic (e.q. finite element method, FEM) or statistical (statistical energy analysis, SAE) analysis techniques alone. Instead, a number of methods specifically tailored to this problem have been developed, such as the hybrid FE/SAE method [4], nonparametric probabilistic approach [3], mode-based approach [5] and so on. Most of such methods deal separately with the deterministic and the stochastic vibration system components and, by employing appropriate coupling between the subsystems, compute overall system response, usually in some average sense.

On the other hand, the intrinsic uncertainty of such complex systems suggests the use of robust control techniques when designing vibration control device. This requires the definition of (typically compact) uncertainty sets which encompass all system uncertainties and, together with the so-called nominal system, capture all possible system vibration responses [7]. In other words, uncertainty modeling within the robust control framework is based on the ”worst case” paradigm.

Yet, virtually none of the mid-frequency analysis techniques provide us with sufficient information for defining such uncertainty sets — apart from the average system response, we are at best able to obtain confidence region of the random frequency response functions [3]) or system frequency response variance [6]. Thus, we are often confined to computationally very expensive (and this is due to the fine discretization which is needed for the stochastic subsystems) techniques such as Monte Carlo for determining the uncertainty set.

In this paper, we propose optimal passive control framework applicable to the structures exhibiting both mid-frequency behavior and having well-defined and well-separated vibration modes. Such specific system behavior serves us as the basis for the system uncertainty modeling, and is the key issue affecting both the overall computational cost and the efficiency of the passive control device optimization algorithm.

2 MODELING FRAMEWORK

Assume that we divide the system exhibiting mid-frequency behavior into deterministic and stochastic subsystems, and that both subsystems are spatially discretized using the finite element method. For deterministic subsystem, such procedure results in linear time invariant (LTI) second order system

$$\begin{aligned} \mathbf{M}_D\ddot{\mathbf{q}}_D + \mathbf{D}_D\dot{\mathbf{q}}_D + \mathbf{S}_D\mathbf{q}_D &= \mathbf{B}_{Du1}\mathbf{u} + \mathbf{B}_{Dw1}\mathbf{w}, \\ \mathbf{y} &= \mathbf{C}_{Dy1}\dot{\mathbf{q}}_D + \mathbf{C}_{Dy2}\mathbf{q}_D, \\ \mathbf{z} &= \mathbf{C}_{Dz1}\dot{\mathbf{q}}_D + \mathbf{C}_{Dz2}\mathbf{q}_D, \end{aligned} \tag{1}$$

where $\mathbf{M}_D, \mathbf{D}_D, \mathbf{S}_D \in \mathbb{R}^{n_D \times n_D}$ are mass, damping and stiffness matrices, respectively, $\mathbf{B}_{Dw1} \in \mathbb{R}^{n_D \times m_D}$ is input matrix, and $\mathbf{C}_{Dz1} \in \mathbb{R}^{p_D \times n_D}$ and $\mathbf{C}_{Dz2} \in \mathbb{R}^{p_D \times n_D}$ are velocity and displacement output matrices, respectively. Time-dependent vectors $\mathbf{q}_D, \dot{\mathbf{q}}_D, \ddot{\mathbf{q}}_D \in \mathbb{R}^{n_D}$, $\mathbf{w} \in \mathbb{R}^{m_D}$ and $\mathbf{z} \in \mathbb{R}^{p_D}$ are displacement, velocity, acceleration, input and output vectors, respectively. We assume that $\mathbf{M}_D \succ \mathbf{0}$ and $\mathbf{D}_D, \mathbf{S}_D \succeq \mathbf{0}$, i.e. mass matrix is positive definite and damping and stiffness matrices are positive semidefinite. To accommodate the control of the system (1), we define control input $\mathbf{u} \in \mathbb{R}^{k_D}$, control measurement $\mathbf{y} \in \mathbb{R}^{l_D}$, as well as corresponding control input matrix $\mathbf{B}_{Du1} \in \mathbb{R}^{n_D \times k_D}$ and control measurement matrices $\mathbf{C}_{Dy1}, \mathbf{C}_{Dy2} \in \mathbb{R}^{l_D \times n_D}$.

In the similar fashion, we have LTI second order system representing the stochastic subsystem

$$\mathbf{M}_S \ddot{\mathbf{q}}_S + \mathbf{D}_S \dot{\mathbf{q}}_S + \mathbf{S}_S \mathbf{q}_S = \mathbf{0}, \tag{2}$$

where $\mathbf{M}_S \in \mathcal{M}_S, \mathbf{D}_S \in \mathcal{D}_S$ and $\mathbf{S}_S \in \mathcal{S}_S$ are uncertain mass, damping and stiffness matrices, respectively, and belong to uncertainty sets $\mathcal{M}_S, \mathcal{D}_S, \mathcal{S}_S \subset R^{n_S \times n_S}$. We assume that $\mathbf{M}_S \succ \mathbf{0}$ for all $\mathbf{M}_S \in \mathcal{M}_S$. Time-dependent $\mathbf{q}_D, \dot{\mathbf{q}}_D, \ddot{\mathbf{q}}_D \in \mathbb{R}^{n_S}$ are displacement, velocity and acceleration vectors, respectively. Furthermore, we assume that the mass matrices for both deterministic and stohastic subsystems are lumped — this is important on several occasion throughout our discussion.

Subsystems (1) and (2) are connected through $\mathbf{q}_I \in \mathbb{R}^{n_I}$ degrees of freedom (DOFs), which we refer to as interface DOFs. We express this connection as

$$\mathbf{q}_I \equiv \mathbf{P}_{DI} \mathbf{q}_D \equiv \mathbf{P}_{SI} \mathbf{q}_S. \tag{3}$$

The matrix $\mathbf{P}_{DI} \in \mathbf{R}^{n_I \times n_D}$ maps n_I (out of n_D) deterministic subsystem DOFs into n_I interface DOFs. Analogously, the matrix $\mathbf{P}_{SI} \in \mathbf{R}^{n_I \times n_S}$ maps n_I (out of n_S) stochastic subsystem DOFs into n_I interface DOFs. The deterministic subsystem DOFs which are not interface DOFs we denote as $\mathbf{q}_{DD}$ and refer to as the deterministic subsystem internal DOFs. Also, the stochastic subsystem DOFs which are not interface DOF we denote as $\mathbf{q}_{SS}$ and refer to as the stochastic subsystem internal DOFs.

Note that we assume that each deterministic subsystem interface DOF is connected to only one stochastic subsystem interface DOF, i.e. the matrices $\mathbf{P}_{DI}$ and $\mathbf{P}_{SI}$ have exactly one entry 1 in each row and each column and zeros elsewhere. This allows us to augment the matrices $\mathbf{P}_{DI}$ and $\mathbf{P}_{SI}$, and obtain the permutation matrices

$$\mathbf{P}_D = \begin{pmatrix} \mathbf{P}_{DD} \\ \mathbf{P}_{DI} \end{pmatrix}, \; \mathbf{P}_S = \begin{pmatrix} \mathbf{P}_{SI} \\ \mathbf{P}_{SS} \end{pmatrix}. \tag{4}$$

Although there is no unique way to define (4), we choose $\mathbf{P}_{DD}$ and $\mathbf{P}_{SS}$ according to the natural ordering of $\mathbf{q}_{DD}$ and $\mathbf{q}_{SS}$ in $\mathbf{q}_D$ and $\mathbf{q}_S$, respectively.

We reorder the rows and columns of the mass, damping and stiffness matrices in (1), and partition the resulting matrices according to the q_{DD} and q_I DOFs, which results in

$$\widetilde{\mathbf{M}}_D = \mathbf{P}_D \mathbf{M}_D \mathbf{P}_D^T, \; \widetilde{\mathbf{D}}_D = \mathbf{P}_D \mathbf{D}_D \mathbf{P}_D^T, \; \widetilde{\mathbf{S}}_D = \mathbf{P}_D \mathbf{S}_D \mathbf{P}_D^T. \tag{5}$$

We also reorder the rows/columns in the respective input and output matrices for the system (1), and obtain

$$\begin{aligned} \widetilde{\mathbf{B}}_{Du1} &= \mathbf{P}_D \mathbf{B}_{Du1}, \; \widetilde{\mathbf{B}}_{Dw1} = \mathbf{P}_D \mathbf{B}_{Dw1}, \\ \widetilde{\mathbf{C}}_{Dy1} &= \mathbf{C}_{Dy1} \mathbf{P}_D^T, \; \widetilde{\mathbf{C}}_{Dy2} = \mathbf{C}_{Dy2} \mathbf{P}_D^T, \\ \widetilde{\mathbf{C}}_{Dz1} &= \mathbf{C}_{Dz1} \mathbf{P}_D^T, \; \widetilde{\mathbf{C}}_{Dz2} = \mathbf{C}_{Dz2} \mathbf{P}_D^T. \end{aligned} \tag{6}$$

In the similar fashion, we reorder the rows and columns of the mass, damping and stiffness matrices in (2), and partition the resulting matrices according to the q_I and q_{SS} DOFs

$$\widetilde{\mathbf{M}}_D = \mathbf{P}_D \mathbf{M}_D \mathbf{P}_D^T,\ \widetilde{\mathbf{D}}_D = \mathbf{P}_D \mathbf{D}_D \mathbf{P}_D^T,\ \widetilde{\mathbf{S}}_D = \mathbf{P}_D \mathbf{S}_D \mathbf{P}_D^T. \tag{7}$$

Taking into the account (5), (6) and (7), we assemble the deterministic and stohastic subsystems according to (3), and obtain

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{S}\mathbf{q} &= \mathbf{B}_{u1}\mathbf{u} + \mathbf{B}_{w1}\mathbf{w}, \\ \mathbf{y} &= \mathbf{C}_{y1}\dot{\mathbf{q}} + \mathbf{C}_{y2}\mathbf{q}, \\ \mathbf{z} &= \mathbf{C}_{z1}\dot{\mathbf{q}} + \mathbf{C}_{z2}\mathbf{q}, \end{aligned} \tag{8}$$

where $\mathbf{q} = \begin{pmatrix} \mathbf{q}_{DD}^T & \mathbf{q}_I^T & \mathbf{q}_{SS}^T \end{pmatrix}^T \in \mathbb{R}^{n_D + n_D - n_I}$ is DOFs vector for the assembled system.

Without the loss of generality, we transform the second order system (8) into the first companion form and reorder the state variables such that $\mathbf{x}^T = \begin{pmatrix} \mathbf{x}_D & \mathbf{x}_I & \mathbf{x}_S \end{pmatrix}^T$, where $\mathbf{x}_D^T = \begin{pmatrix} \dot{\mathbf{q}}_{DD} & \mathbf{q}_{DD} \end{pmatrix}^T$, $\mathbf{x}_I^T = \begin{pmatrix} \dot{\mathbf{q}}_I & \mathbf{q}_I \end{pmatrix}^T$, $\mathbf{x}_S^T = \begin{pmatrix} \dot{\mathbf{q}}_{SS} & \mathbf{q}_{SS} \end{pmatrix}^T$, and obtain the first order descriptor LTI system

$$\begin{aligned} \begin{pmatrix} \mathbf{E}_S & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E}_S \end{pmatrix} \begin{pmatrix} \dot{\mathbf{x}}_D \\ \dot{\mathbf{x}}_I \\ \dot{\mathbf{x}}_S \end{pmatrix} &= \begin{pmatrix} \mathbf{A}_{DD} & \mathbf{A}_{DI} & \mathbf{0} \\ \mathbf{A}_{ID} & \mathbf{A}_{II} & \mathbf{A}_{IS} \\ \mathbf{0} & \mathbf{A}_{SI} & \mathbf{A}_{SS} \end{pmatrix} \begin{pmatrix} \mathbf{x}_D \\ \mathbf{x}_I \\ \mathbf{x}_S \end{pmatrix} + \\ &\quad \begin{pmatrix} \mathbf{B}_{Du} \\ \mathbf{B}_{Iu} \\ \mathbf{0} \end{pmatrix} \mathbf{u} + \begin{pmatrix} \mathbf{B}_{Dw} \\ \mathbf{B}_{Iw} \\ \mathbf{0} \end{pmatrix} \mathbf{w}, \\ \mathbf{y} = \begin{pmatrix} \mathbf{C}_{Dy} & \mathbf{C}_{Iy} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{x}_D \\ \mathbf{x}_I \\ \mathbf{x}_S \end{pmatrix}&,\ \mathbf{z} = \begin{pmatrix} \mathbf{C}_{Dz} & \mathbf{C}_{Iz} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{x}_D \\ \mathbf{x}_I \\ \mathbf{x}_S \end{pmatrix}. \end{aligned} \tag{9}$$

2.1 Feedback connection of deterministic and stochastic subsystem

Note that, in our modeling framework, we have introduced the following assumptions:

1. the assembled system input $\mathbf{w}$, output $\mathbf{z}$, control input $\mathbf{u}$ and control output $\mathbf{y}$ are defined for deterministic subsystem internal DOFs $\mathbf{q}_{DD}$ and/or interface DOFs $\mathbf{q}_I$, i.e. there are no inputs/outputs acting on stochastic subsystem internal DOFs $\mathbf{q}_{SS}$,

2. both stochastic and deterministic subsystems mass matrices $\mathbf{M}_D$ and $\mathbf{M}_S$ are positive definite and lumped, which implies that the matrices $\mathbf{E}_D$, $\mathbf{E}_D$ and $\mathbf{E}_D$ in (9) are also positive definite and diagonal,

3. stochastic and deterministic subsystems interact only through the interface DOFs $\mathbf{q}_I$, which results in the structured system (9).

The structure of the system (9), imposed by the above assumptions, allows us to reformulate its mathematical model as feedback connection of the nominal (deterministic) system, parametric uncertainty and frequency-weighted dynamic uncertainty, as follows.

Note that, by assembling the subsystems, we add (essentially uncertain) values to the elements of the deterministic subsystem matrices which correspond to the interface DOFs. Thus, we consider the elements of the assembled system matrices which correspond to the interface DOFs to be parametric uncertainty, which we may model by the appropriate technique. We choose nominal values of the uncertain parameters, which are now the part of the nominal system $\mathbf{G}_D$, followed by "pulling out" the parametric uncertainties via linear fractional transformation (LFR) ([8]) into the uncertainty block Δ_p which is feedback-connected with the nominal system $\mathbf{G}_D$, as shown in Figure 1.

Due to the above introduced assumptions, the part of the stochastic subsystem that does not directly contribute to the deterministic subsystem admits the representation

$$\begin{aligned} \mathbf{E}_S \dot{\mathbf{x}}_S &= \mathbf{A}_{SS}\mathbf{x}_S + \mathbf{A}_{SI}\mathbf{x}_I, \\ \mathbf{f}_I &= \mathbf{A}_{IS}\mathbf{x}_S, \end{aligned} \tag{10}$$

i.e. as the LTI descriptor system with $2n_S$ state variables and $2n_I$ inputs and outputs. The inputs are the interface DOF velocities and displacements $\mathbf{x}_I$ and the outputs contain forces acting on the interface DOFs. The system (10) has the transfer function $\mathbf{G}_S$, and is feedback-connected to the nominal system $\mathbf{G}_D$, as shown in Figure 1. This rather general modeling framework decomposes the assembled system (9) into the nominal (deterministic) system $\mathbf{G}_D$, which is feedback-connected to the uncertainty block Δ_p and the stochastic subsystem $\mathbf{G}_S$.

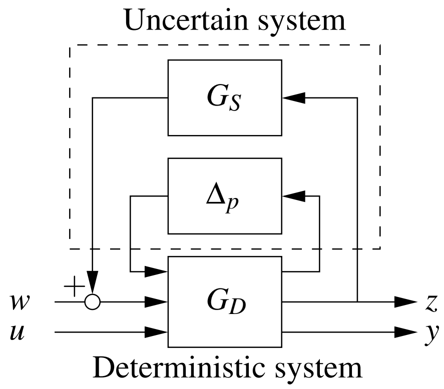

Figure 1: System decomposition into the nominal system, uncertainty block and the stochastic subsystem

2.2 Reduced-order stochastic subsystem uncertainty modeling

Complex assembled structures such as ship hulls, aeroplane fuselages, or more generally rib-stiffened shell-like structures, despite exhibiting mid-frequency behaviour in the wide frequency range, often have several well-defined and well-separated modes. Moreover, such mode shapes are uncertain to some degree (due to the intrinsic uncertainty of the overall structure) and usually dominate the structure vibration response. To model the stochastic system uncertainty, we confine our discussion to the above described vibration modes which are closely related to the deterministic subsystem vibration modes.

To be more specific, we assume that the vibration modes of the nominal system $\mathbf{G}_D$ dominate the structure vibration response. To calculate such modes, we observe that the LTI representation of the system $\mathbf{G}_D$ can be obtained by taking into the account (10) and rewriting the first equation in (9) as

$$\begin{aligned} \begin{pmatrix} \mathbf{E}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{I,n} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{x}}_D \\ \dot{\mathbf{x}}_I \end{pmatrix} &= \begin{pmatrix} \mathbf{A}_{DD} & \mathbf{A}_{DI} \\ \mathbf{A}_{ID} & \mathbf{A}_{II,n} \end{pmatrix} \begin{pmatrix} \mathbf{x}_D \\ \mathbf{x}_I \end{pmatrix} + \\ & \begin{pmatrix} \mathbf{0} \\ \mathbf{f}_I \end{pmatrix} + \begin{pmatrix} \mathbf{B}_{Du} \\ \mathbf{B}_{Iu} \end{pmatrix} \mathbf{u} + \begin{pmatrix} \mathbf{B}_{Dw} \\ \mathbf{B}_{Iw} \end{pmatrix} \mathbf{w}, \end{aligned} \tag{11}$$

where $\mathbf{E}_{I,n}$ and $\mathbf{A}_{II,n}$ correspond to the nominal system $\mathbf{G}_D$. Since (11) is the first companion form of the LTI second order system with reordered state variables, its i-th eigenvector has the form

$$\phi_i^T = \begin{pmatrix} \lambda_i \mathbf{v}_{D,i} & \mathbf{v}_{D,i} & \lambda_i \mathbf{v}_{I,i} & \mathbf{v}_{I,i} \end{pmatrix}^T, \tag{12}$$

where $\mathbf{v}_i^T = \begin{pmatrix} \mathbf{v}_{D,i} & \mathbf{v}_{I,i} \end{pmatrix}^T$ is the i-th eigenvector of the equivalent LTI second order system, and λ_i is the corresponding eigenvalue.

Furthermore, we assume that the deterministic system vibration modes remain uncoupled after the deterministic and stochastic systems assembly. This serves as the basis for the following stochastic subsystem decomposition and uncertainty modeling procedure.

We select r nominal system vibration modes having $\lambda_i | i = 1, \ldots, r$ eigenvalues and corresponding $\mathbf{v}_i | i = 1, \ldots, r$ eigenvectors. Then, for each $(\lambda_i, \mathbf{v}_i) | i = 1, \ldots, r$, we proceed as follows:

1. Determine the corresponding parametric uncertainty $\Delta_{p,i}$ by projecting the parametric uncertainty set Δ_p to the subspace spanned by the $\mathbf{v}_i$.

2. Select an instance of the stochastic subsystem, which we refer to as the nominal i-th mode stochastic subsystem and construct its reduced order model by means of interpolatory projection method [9]. When doing so, use λ_i as the interpolation point and $\mathbf{v}_i$ as the interpolation direction and construct the projection matrix $\mathbf{W}_i$ of sufficiently high order.

3. Construct reduced order models of arbitrary many instances of the stochastic subsystem by projecting them onto the subspace spanned by $\mathbf{W}_i$, and calculate corresponding error systems with respect to the nominal i-th mode stochastic subsystem by means of Monte Carlo procedure.

4. From the frequency response functions of such error systems, design the i-th vibration mode dynamic uncertainty model comprised of the frequency weight $\mathbf{W}_{f,i}$ and norm-bounded dynamic uncertainty $\|\boldsymbol{\Delta}_{d,i}\| < 1$. This is commonly used and straightforward procedure [7], and we do not give the details here.

Essentially, we model each vibration mode of the assembled system (8) as the independent single-input single-output (SISO) uncertain LTI system, comprised of the nominal vibration mode $\mathbf{G}_{D,i}$ and the uncertainty, as shown in Figure 2. Nominal vibration mode is composed of the i-th vibration mode for the system (11), as well as the reduced-order nominal i-th mode stochastic subsystem. The uncertainty consists of the parametric uncertainty $\Delta_{p,i}$, suitably chosen frequency weight $\mathbf{W}_{f,i}$ and norm-bounded dynamic uncertainty $\boldsymbol{\Delta}_{d,i}$.

2.3 Optimal passive H_∞ control

The above described mathematical model of the vibration system exhibiting mid-frequency behavior can be readily used for robust controller synthesis procedure. In the case of passive vibration control, such procedure boils down to robust optimal H_∞ controller synthesis problem with controller rank and structure constraints. Unfortunately, even without the controller rank and structure constraints, this problem is recognized as being NP-hard [2] and admits computationally tractable solution only for a few special cases. To tackle this rather difficult controller synthesis problem, we employ the following procedure:

1. Perform D-K iteration procedure to design H_∞ controller [7]. The resulting controller is dynamic (e.q. not passive) and usually has large order.

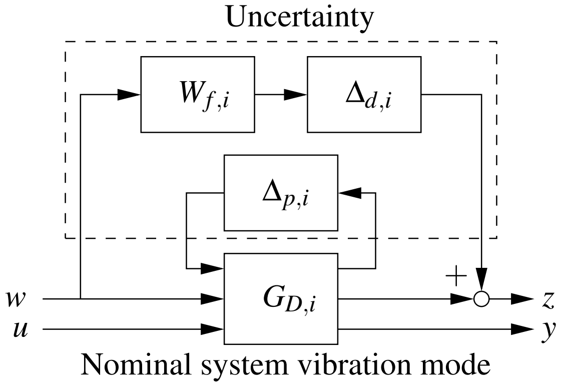

Figure 2: Uncertain LTI system representing i-th vibration mode

2. Tune the parameters of the passive controller such that the error system between the passive and dynamic controller is minimized in its H_∞ norm. This is done by appropriate local optimization procedure [10].

Since we are constrained by the article length limitations, we leave out the details of the procedure. Although there is no guarantee that the proposed procedure will converge to the (locally) optimal passive H_∞ controller, the numerical results presented in the next section indicate satisfactory performance.

3 NUMERICAL EXAMPLE

To illustrate the applicability and the efficiency of the proposed approach, we revisit the numrical example presented in [5] comprised of simply supported beam-stiffened plate, as shown in Figure 3. The plate dimensions are: length 2 m, width 0.9 m, thickness may vary between 3 and 7 mm. The beam is 2 m long and has rectangular cross-section with 59 mm width and 68 mm height. Both the beam and the plate are made of the material with Young modulus 4400 N mm^{-2}, density 1152 kg m^{-3}, Poisson's ratio 0.38. The other dimensions are: $x_1 = 30$ mm, $y_1 = 0.3$ m, $\phi = 10°$.

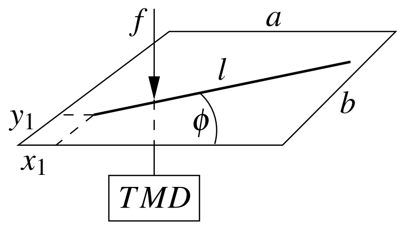

Figure 3: Simply-supported rib-stiffened plate

The beam is excited by a point force, which we refer to as the system input, at 0.73 m from its lower-left end. The system output is displacement of the same point. Detailed FEM models for the beam, and for 9 plates with varying thickness are constructed. After assembling the models and calculating frequency response functions (FRFs) (see Figure 4), we observe mid-frequency behavior across a wide frequency range, as well as several modes (i.e. beam rigid body modes and beam first flexural mode) dominating the frequency response.

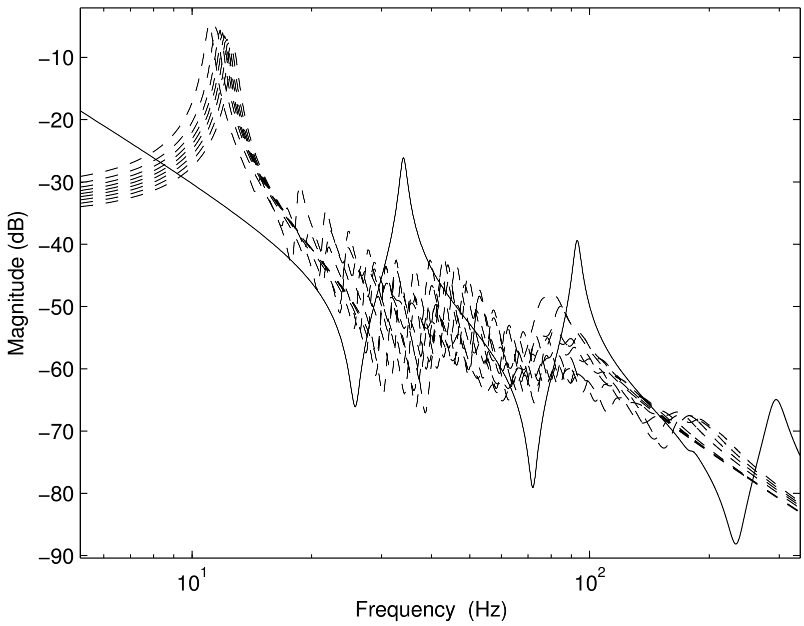

Figure 4: Frequency response functions for the beam (solid line) and the assembled structure (dashed line)

Thus, we focus on the uncertainty modeling for the first vibration mode of the assembled structure, as described in Section 2.2. The resulting SISO uncertain LTI model is comprised of nominal LTI system with 2 state variables with parametric uncertainty block, third-order type 1 Chebyshev bandpass filter as frequency weight and norm-bounded dynamic uncertainty. Respective FRFs, as well as FRFs for the error systems are shown in Figure 5.

Displacements at the position of the point force are controlled by means of single DOF tuned mass dampers (TMDs) attached to the same point. Configurations with 1, 3, 5, 7 and 13 independent TMDs are optimized using the optimization algorithm described in Section 2.2. When doing so, total mass of TMDs is constrained to 5% of the overall mass of the structure and TMDs individual damping ratios are 0.02. Thus, mass distribution among the TMDs and individual tuning frequencies are optimized.

Number of TMDs	1	3	7	13
Maximum frequency response, dB	-5.42	-13.8	-13.3	-13.0
Minimum frequency response, dB	-43.1	-34.5	-31.4	-31.0

Table 1: Minimum and maximum frequency response for different number of TMDs.

By comparing FRFs for different TMD configurations (see Figure 6), we notice that single TMD achieves the best vibration attenuation (-43.1 dB) in the narrow frequency range, yet its minimal performance (-5.42 dB) is close to the FRF maximum (-4.36 dB) of the structure without the TMDs. Configuration with 13 TMDs is much more robust, at the expense of maximum performance (-31.0 dB). The comparison of maximum and minimum FRFs in the frequency band of the first vibration mode for different TMD configurations are given in Table 1. We notice obvious performance/robustness trade off, which is commonly observed in all robust control applications.

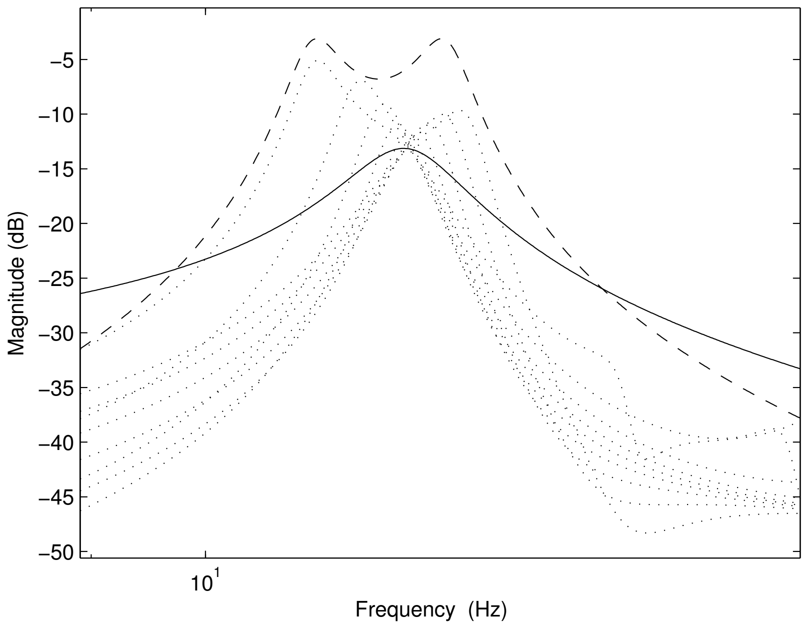

Figure 5: Frequency response functions for the nominal LTI system (solid line), frequency weight (dashed line) and error systems (dotted lines)

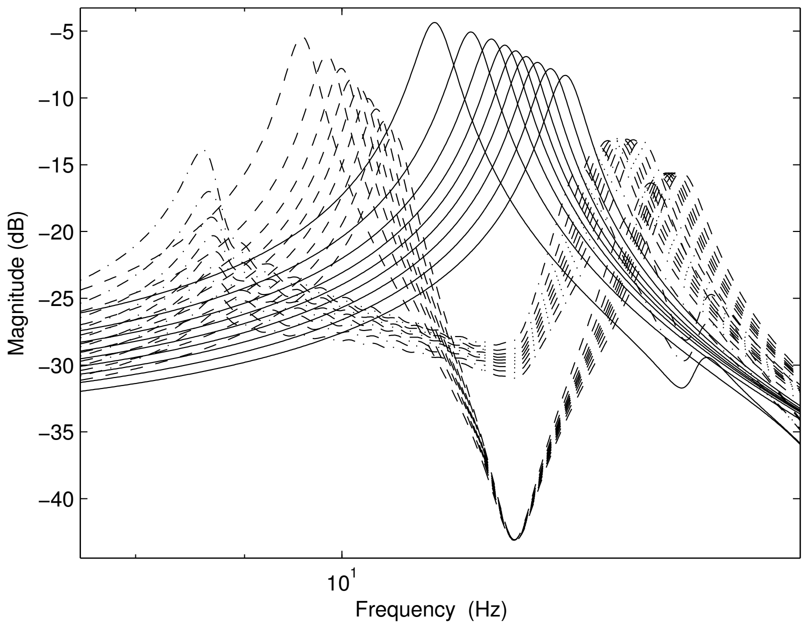

Figure 6: Frequency response functions for the structure alone (solid line), structure with 1 TMD (dashed line) and structure with 13 TMDs (dash-dot line)

4 CONCLUSIONS

In this paper, we have discussed passive optimal control framework applicable to the structures exhibiting both mid-frequency behavior and having well-defined and well-separated vibration modes. Such behavior is often observed in the lower frequency range in structures comprising of both stiff and flexible structural elements. The cornerstone of the proposed approach is the approximation of the stochastic subsystem with a series of reduced-order stochastic subsystems. This effectively decouples the complex assembled system LTI model into a series of small independent SISO uncertain LTI systems. This procedure significantly reduces the computational cost of the dynamic uncertainty modeling and simplifies the uncertainty modeling procedure — in particular frequency weights can be independently computed for each vibration mode in straightforward fashion. Finally, the resulting uncertain model is small, which is very important in the subsequend robust controller synthesis procedure.

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