11th International Conference on Vibration Problems
Z. Dimitrovová *et al.* (eds.)
Lisbon, Portugal, 9-12 September 2013

MODELLING OF VIBRATIONS INDUCED BY TRAFFIC IN TUNNELS: FROM THE SOURCE TO THE RECEIVER

Patrícia Lopes*[1,2], Pedro Alves Costa[2], Miguel Ferraz[2], Rui Calçada[2], António Silva Cardoso[2]

[1]Polytechnic Institute of Porto, School of Engineering
cpl@isep.ipp.pt

[2]Faculty of Engineering of University of Porto
pacosta@fe.up.pt, ferraz@fe.up.pt, ruiabc@fe.up.pt, scardoso@reit.up.pt

Keywords: vibrations , 2.5D FEM-PML, tunnel-ground interaction, ground-building interaction, frequency-wavenumber

Abstract. *The simulation and study of problems related with vibrations induced by railway traffic in tunnels is a difficult and complex task. Actually, the complexity of the problem can be attributed to the vast field of analysis, which comprises not only the generation of vibrations inside the tunnel as well as its propagation through the ground and its interaction with existing buildings in surroundings. In the present paper a numerical procedure is presented in order to allow an efficient simulation of the vibrations induced by traffic from the source to the receiver (building). The numerical model is divided into three distinct parts, comprising the simulation of rolling stock, the simulation of the tunnel-ground system and the simulation of the building. The solution is obtained by a compliance formulation between the three subsystems, developed in the frequency-(wavenumber) domain. Regarding the simulation of the tunnel-ground system, which deals with unbounded domain problem, an efficient solution is developed using a 2.5D technique based on the finite elements method, and adopting perfect matched layers (PML's) for the treatment of the boundaries due to the truncation of the finite elements mesh.*

The numerical model is used to study the vibrations induced inside of a simple building close to a shallow railway tunnel. From the study performed, it was possible to conclude that the dynamic characteristics of the building, namely the natural frequencies of resonance of the slabs, play an important role in the problem.

1 INTRODUCTION

Vibrations inside buildings due to urban traffic are becoming a relevant environmental problem. Indeed, this kind of problems is especially relevant in the case of railway traffic in tunnels that cross the modern cities. Sometimes, these tunnels are shallow and there are sensitive buildings in the surrounding, which aggravates the problem.

Regarding the analytical modelling of the problem, it should be referred that several kinds of models have been proposed during the last decade. The most recent advances concern the simulation of the tunnel-ground system, which is quite complex due to the unbounded character of the ground. Semi-analytical models have been proposed by Hussein and Hunt [1], where high levels of computational efficiency can be reached. On the other hand, periodic numerical models for the dynamic simulation of tunnels have been extensively applied by Gupta et al. [2, 3]. Alternatively, for longitudinally invariant structures, a 2.5D approach, which can be extended to BEM and FEM, can be applied [4-9]. Although the advantages inherent to a BEM formulation, in some cases, the option by a 2.5D FEM approach for whole domain is somewhat more simple and efficient. However, in a FEM approach, it is necessary to adopt special procedures in order to avoid the spurious reflection of waves that impinge the truncation boundaries. Several strategies have been proposed and applied in the context of tunnel-ground interaction problems to solve that problem, as for instance a Perfect Matched Layers. In the authors opinion, this procedure is efficient, accurate and do not introduce undesirable complexity to the numerical formulation [10-12].

A comprehensive model for the prediction of vibrations induced by railway traffic in tunnels cannot be limited to the simulation of the propagating path. Actually the source (train) and the receiver (building) are relevant aspects that cannot be neglected. Regarding the source, one of the most important mechanisms of vibration generation is due to the track unevenness, which causes inertial forces on the rolling stock, giving rise to a dynamic interaction problem between the train and the track.

Contrarily to the vast majority of the studies, with some exceptions, as for instance a recent paper presented by Hussein et al. [13], in the present paper the receiver, i.e., a building close to the tunnel that is coupled to the natural ground, is also considered in the modelling strategy. The three-dimensional building is coupled to the ground by shallow foundations. The solution of the dynamic behaviour of the building is found by a conventional finite elements approach, being the coupling to the ground established by adding the impedance terms of the ground to the dynamic stiffness matrix of the building.

The paper presents a comprehensive model that can be used for practical purposes due to its efficiency and versatility. In the last section, an application example is shown, highlighting the influence of the building dynamic properties on the vibrations observed in its inside.

2 NUMERICAL MODEL

2.1 Generalities

The numerical model here presented is modular, based on a substructuring approach. Figure 1 shows the main parts of the numerical model, as well as, the main steps involved in the solution. As can be seen, there are three distinct parts in the modelling strategy (the train, the track-tunnel-ground; the building), corresponding to different simulation techniques, being whole of them coupled by a compliance formulation. In the following sections, a summarized description of each part of the model is presented.

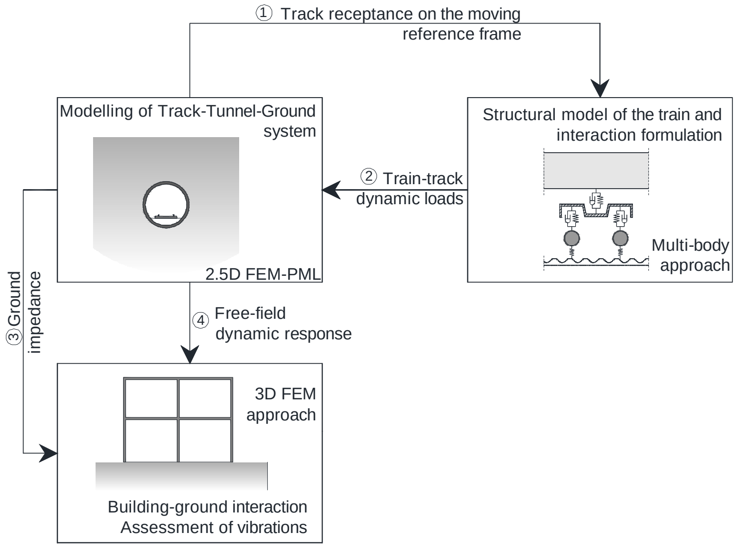


Figure 1. Numerical modeling strategy

2.2 Modelling of train-track interaction

The load applied by the train on the track can be divided into two components: i) the static load, resulting from the weight of the train; ii) the dynamic load, due to the dynamic interaction between the train and the track. The latter component requires the solution of the dynamic train-track interaction problem. This problem is solved by a compliance procedure formulated in a frame of reference that moves with the train, as suggested by several authors [4, 7, 14, 15].

The dynamic interaction loads are obtained through the compatibility of displacements between the rolling stock and the track. So, the train-track interaction force at the frequency domain is given by:

$$\mathbf{p}(\Omega) = -\left(\mathbf{F} + \mathbf{F}^{\mathrm{H}} + \mathbf{A}\right)^{-1} \boldsymbol{\Delta}\mathbf{u}(\Omega) \tag{1}$$

where **F** is the train compliance at the contact points with the track, $\mathbf{F}^{\mathrm{H}}$ is a diagonal matrix where the terms are equal to $1/k_H$, being k_H the Hertzian stifness, **A** is the compliance matrix of the track and **Ω** is the driven frequency, i.e., the frequency of oscillation of wheelset due to the unevenness with a wavelength λ ($\Omega=2\pi c/\lambda$).

All matrices of Eq. (1) are square with a dimension equivalent to the number of wheelsets. Matrix **F** is computed from the vehicle model and the terms of matrix **A** are evaluated from the track-tunnel-ground model. A detailed description of the mathematical formalism inherent to the evaluation of these matrices can be found in Alves Costa et al. [16, 17].

The influence of the vehicle properties and of the modeling strategy used only affects matrix **F**. Recent studies developed by the authors show that the dynamics of the car body can be discarded for the evaluation of the dynamic interaction loads [16].

2.3 Track-tunnel-ground simulation

Transport infrastructures, such as railways, tunnels and roads, can be assumed as infinite and invariant domains, as can be seen in Figure 2. Three-dimensional structures with infinite development and invariable properties (geometrical and mechanical) can be dealt by a 2.5D formulation. Assuming that the response of the structure is linear, the analysis can be carried out in the wavenumber/frequency domain. All the variables, i.e., loads (action) and displacements (response), must be transformed to the wavenumber/frequency domain by means of a double Fourier transform, related with the direction along the track (x direction) and with time. Transformed quantities are functions of the Fourier images of x and t, defined as wavenumber and frequency and are represented by k_1 and ω, respectively.

However, the needs of truncation of the interest domain, implicit to a finite element formulation, give rise to a problem of boundary treatment in order to reach the Sommerfeld condition. This condition is achieved thought the adding of external layers that bound the "box domain". These external layers are formed by PML's and the Dirichlet boundary conditions are applied in the external edge of these special layers. So, this numerical device has the function of absorbing, without spurious reflection, the arbitrary direction waves that impinge the boundary between the domains described by the 2.5D FEM and by the 2.5D PML [11, 18].

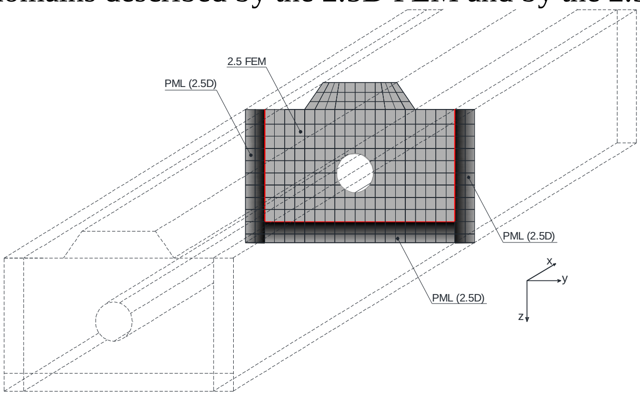


Figure 2. Infinite and invariant structure in one direction.

Since the interest domain and the PML's are formulated by a 2.5D approach in the finite elements method sense, the equations of motion are obtained by a discretization of the domain for both cases. Indeed, following the usual steps of the finite element procedure, namely the strong and weak formulations, the equilibrium equation can be derived for any point of a three dimensional domain:

$$\int_V \delta\varepsilon\,\sigma\, dV + \int_V \delta u\, \rho \ddot{u} dV = \int_S \delta u\, p\, dS \tag{2}$$

where $\delta\varepsilon$ is the virtual strain field, σ represents the stress field, δu is the virtual displacement field, u is the displacement field, ρ is the mass density and p represents the applied loads.

Considering the Fourier transformation of the time and of the development direction of the problem, as well as, the discretization of the domain into finite elements (and PML elements), the equilibrium can be described by the following equation:

$$\left(\int_z \int_y B^T(-k_1) D B(k_1)\, dy\, dz - \omega^2 \int_z \int_y N^T \rho\, N\, dydz \right) u_n(k_1,\omega) = p_n(k_1,\omega) \tag{3}$$

where: B is the matrix with the derivatives of the shape functions; N is the shape function matrix; D is the strain-stress matrix; u_n is the vector of nodal displacements; p_n is the vector of nodal loads (both in the transformed domain).

Since the direction x is transformed to the wavenumber domain, the derivatives with respect to k_1 are analytically computed. The reader can found more information about the topic in Alves Costa et al. [6, 17] beneath others.

A hysteretic damping model is considered, i.e., using complex stiffness parameters.

For the PML's domain, the same procedure can be followed, since the PML elements correspond to 2.5D finite elements affected by stretching functions that allows for the absorption of the wave field. It should be remembered that the analytically continued solution outside the interest domain satisfies the same differential equations. So, in the 2.5D context, the coordinates y and z are stretched into the complex domain. The natural and modified coordinates are related by the following relationships:

$$\tilde{y} = \int_0^y \lambda_y(y)dy \text{ and } \tilde{z} = \int_0^z \lambda_z(z)dz \quad (4)$$

where λ_y and λ_z are the stretching function in y and z directions, respectively. A detailed description of the stretching functions used can be found on Lopes et al.[11].

Since the solution inside the PML domain satisfies the same differential equation as in the interest domain, the change of coordinates in Eq. (4) is enough to create a 2.5D PML approach. So, the following stiffness, $[K^*]$ and mass, $[M^*]$, matrices can be derived for the PML region:

$$\left[K^*\right] = \int_z \int_y B^{*T}(-k_1) D\, B^*(k_1) \lambda_y \lambda_z dy\, dz \quad (5)$$

and

$$\left[M^*\right] = \int_z \int_y N^T \rho\, N \lambda_y \lambda_z dy\, dz \quad (6)$$

As usual, the matrix B^* is derived by the product of the differential operator matrix $[L^*]$ (in the transformed and stretched domain) and matrix [N] [11].

After the assemblage of the matrices of the PML layers to the remaining domain, the solution, in the frequency-wavenumber domain, is obtained by solving the system of equations. The terms of matrix A, corresponding to the compliance of the track, are obtained assuming a unitary load applied on the rails. The solution on the space-time domain can be obtained by a double inverse Fourier transformation regarding the wavenumber k_1 and the frequency ω.

2.4 The building and its interaction with the remaining system

In the present paper, the building is modeled using a homemade code able to simulate a 3D building through a finite elements approach [19]. Thereby, the motion of the building, in the frequency domain, is given by:

$$\left(K^b + i\omega C^b - \omega^2 M^b\right)u^b = f^b \quad (7)$$

where K^b, C^b and M^b are the stiffness matrix, the Rayleigh damping matrix and the mass matrix, respectively; u^b is the vector of the displacements and f^b is the load vector.

The foundations of the building are impinged by a displacement field generated by the traffic in the tunnel, so, the displacements of the degrees of freedom of the building connected to the ground can be written as:

$$u_s^b = u_0 + \Delta u^b \quad (8)$$

where u_0 is the displacement field evaluated by the 2.5D FEM-PML model at the foundations positions and assuming free-field conditions. On the other hand Δu^b is the increment of displacement of the degree of freedom generated by the inertial forces on the flexible ground. This increment of displacement can be related with the load transmitted by the building to the ground, f_s, as follows:

$$K_s \Delta u^b = f_s \quad (9)$$

where K_s is the dynamic stiffness of the foundation.

Considering the coupling between both systems, i.e., the building and the ground, the following system of equations can be derived after some mathematical manipulation:

$$\begin{bmatrix} K^{bb} & K^{bs} \\ K^{bs} & K^{bb} + K_s \end{bmatrix} \begin{bmatrix} u^b \\ \Delta u^b \end{bmatrix} = -\begin{bmatrix} K^{bb} & K^{bs} \\ K^{bs} & K^{bb} \end{bmatrix} \begin{bmatrix} 0 \\ u_0 \end{bmatrix} \quad (10)$$

where K^{bb} and K^{bs} are terms of the dynamic stiffness matrix of the building.

Regarding the evaluation of matrix Ks, it can be obtained through inversion of a flexibility matrix generated by means of the application of the 2.5D FEM-PML model described above. However, a simplification can be here introduced in order to save time of computation: if the influence of the tunnel could be neglected on the computation of the dynamic stiffness of the ground, a layered infinite and invariant model of the ground can be adopted. Assuming reasonability on this assumption, the solution can be obtained by an integral transform method, as for instance the stiffness matrices or by the Haskell-Thomson transfer matrices.

The compatibility between both systems, i.e., the ground and the footings of the building is achieved by the introduction of transformation matrices, so:

$$K_s = RG^{-1}R^T \tag{11}$$

where G is flexibility matrix of the ground on the frequency domain and R is a transformation matrix which attends to the compatibility between the degrees of freedom of the ground and the degrees of freedom of the footings.

Once the footings of a building can be dealt as rigid interfaces, attention should be putted on the discretization of the interface between the ground and the footings. In the present version of the model it is assumed that the traction field inside each discretized area of the ground is constant being the displacements computed in the center of each discretized region. Obviously, the accuracy of the solution is so much higher as finer is the ground interface discretization.

3 APPLICATION EXAMPLE

3.1 Model description

Despite of the ability of the model to simulate quite complex systems, namely in what concerns to the geometry of the building, a quite simple example was chosen for the present study in order to avoid undesirable complexity.

As shown in Figure 3a, it is assumed a shallow tunnel embedded in a halfspace with the properties indicated in the figure. The tunnel, with circular geometry presents a diameter of 6.0 m, being lined in the inner side with 0.3 m thickness of concrete. The properties of the tunnel liner and of the invert are also indicated in the Figure 3a. The FEM-PML mesh adopted is also shown in Figure 3b.

Regarding the track, it is assumed a continuous concrete slab track with 0.3 m of thickness and 2.5 m of width. The slab track remains on a resilient mat with a stiffness of $2\text{x}10^9$ N/m^3 and a viscous damping of $2.25\text{x}10^4$ Ns/m^3. The rails, materialized with UIC60 profiles are supported continuously by railpads with a stiffness of $2.5\text{x}10^8$ N/m^2 and a damping coefficient of $6\text{x}10^4$ Ns/m^2.

The geometry of the building is illustrated in Figure 4. The footings of the building are square with an area of 2x2 m^2. Regarding to the damping, Rayleigh damping factors were adopted in order to address a damping coefficient around 1-2% in the frequency range between 5 Hz and 80 Hz. The coordinates of the center of the footings are: A(-2.5;16;0); B(-2.5;20;0); C(2.5;20;0); D(2.5;16;0).

In what concerns to the rolling stock, it was assumed the passage of the Alfa-Pendular train at a running speed of 40 m/s. The main geometrical and mechanical properties of the train Alfa-Pendular can be found in Alves Costa et al. [16].

The source of vibration is due to the track unevenness. An artificial unevenness profile was generated taking into account the power spectral density of amplitude of the track unevenness for a range of wavelengths between 28 m and 0.55 m. The following equation was used for address the PSD of the track unevenness:

$$S(k_1) = S(k_{1,0})\left(\frac{k_1}{k_{1,0}}\right)^{-w} \tag{12}$$

where, $k_{1,0}$=1 rad/s, w=3.5 and $S(k_{1,0})$ was assumed equal to $1\text{x}10^{-8}$ m^3.
Once the train speed was assumed to be 40 m/s, the unnevess profile considered excitates the train in the frequency range between 1.4 Hz to 72 Hz.

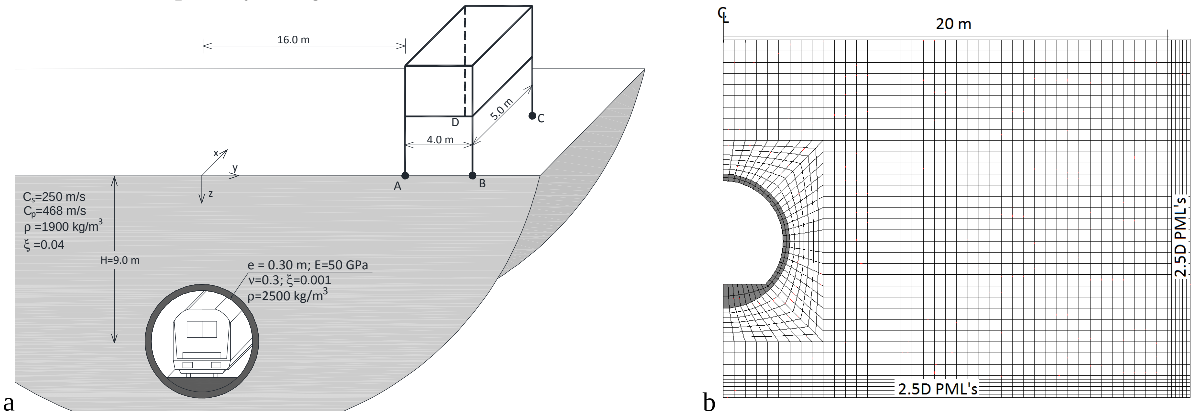


Figure 3. Geometry and properties of the example of application: a) Schematic representation; b) 2.5D FEM-PML mesh.

Elements	Properties (E(GPa),ν,ρ (kg/m^3))	Dimensions
Slabs	30, 0.2, 2500	Thickness: 0.3 m
Beams	30, 0.2, 2500	0.25 x0.50 m^2
Columns	30, 0.2, 2500	0.25x0.25 m^2

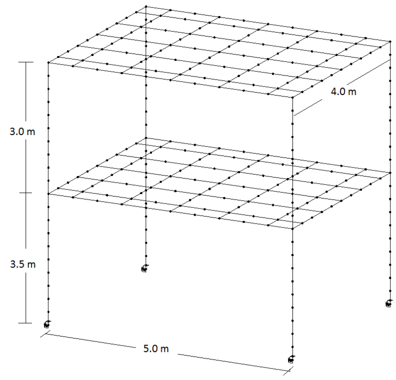


Figure 4.Properties of the building.

3.2 The dynamic response of the building

The dynamic response of the building is evaluated taking into account the dynamic response of the “fictitious” free-field at the location points of the footings, the impedance of the ground and the dynamic characteristics of the structure itself. The dynamic displacements at the free-field surface induced by the train passage are obtained by the approach presented on the previous section. Due to limitations of length of the paper, the present analysis is mainly focused on the dynamic response of the building on the vertical direction.

The dynamic properties of the building affect the dynamic response of the system, mainly at the points located close to the building’s footings. This effect is well evidenced on Figure 5, where the frequency content of the vertical velocity of the footing A is compared with the vertical velocity evaluated at the same location but assuming a free-field condition. From the analysis of the figure it can be seen that for frequencies up to 17 Hz, the vertical velocity of the footing is almost equal to the vertical velocity of the free-field. However, for frequencies above the mentioned limit, the building has a “filter” effect being responsible for a decrease of the dynamic response when compared with the free-field condition.

Regarding the dynamic response of the slabs, it can be seen that the peak value of the vertical velocity at the slabs is higher than the value reached at the footings, as can be observed in Figure 6. Moreover, the peak value of the vertical velocity reaches higher values with the increasing of the level of the floors.

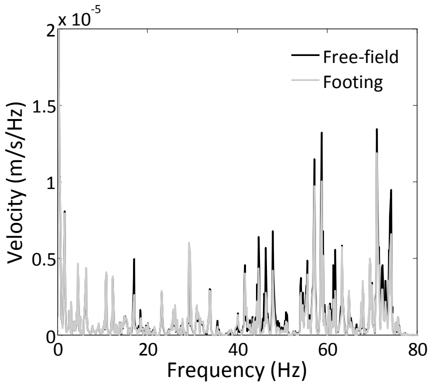


Figure 5. Vertical velocity of footing A on the frequency domain.

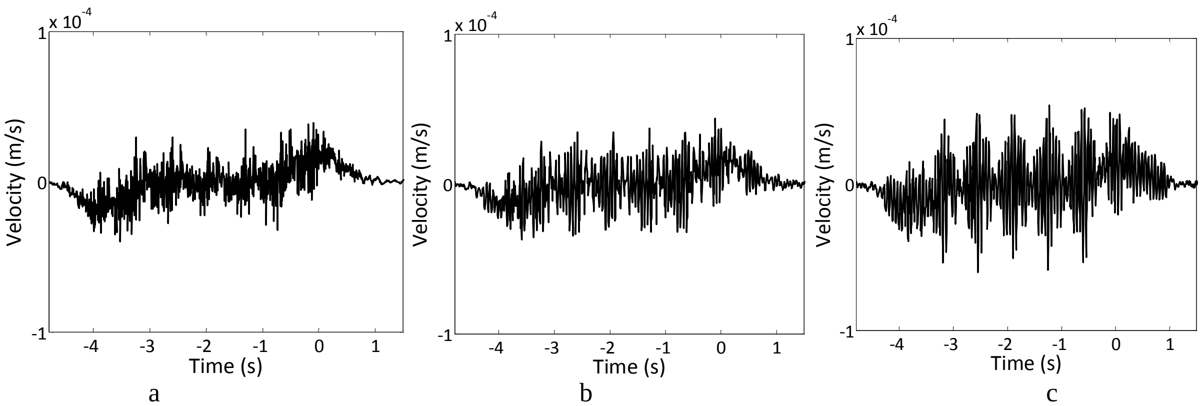


Figure 6. Vertical velocity at distinct points of the building: a) footing A; b) central point of the first slab; c) central point of the second slab.

Analyzing the frequency content of the dynamic response of the floors, which is illustrated in Figure 7, it is possible to conclude that the vertical velocity of the first and second floors is dominated by a frequency around 17 Hz and 18 Hz, respectively. This aspect is justified by the dynamic behavior of the building considered. Performing a modal analysis of the building and assuming that the degrees of freedom of the footings are constrained, it was found that the modes 6 and 7 accomplish the vertical movement of the slabs and have natural frequencies of 17.54 Hz and 19.16 Hz. The configurations of these modes are depicted on Figure 8.

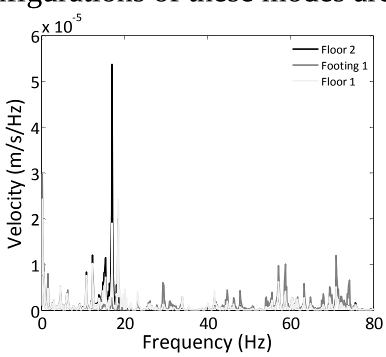
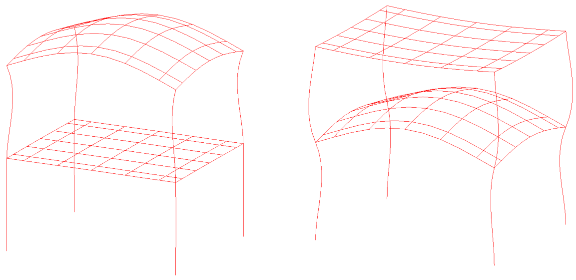


Figure 7. Vertical velocity of distinct points of the building on the frequency domain.

As expected, the inclusion of the foundation flexibility provides to a slight shift of these frequencies for lower values. So, the amplification of the dynamic response around these frequencies is explained by the dynamic characteristics of the building.

Comparing the results of the vertical velocity of the footing with the response of the floors it is also possible to observe that the frequency content for frequencies above 20 Hz is attenuated, which is also consequence of the justification is expressed above.



a b

Figure 8. Mode shapes of the building concerning vertical movement of the slabs: a) Mode 6; b) Mode 7.

4 CONCLUSIONS AND ONGOING RESEARCH

A comprehensive approach for the simulation of vibrations induced by traffic in tunnels was presented. The proposed methodology comprises the simulation of the vibrations generation, of the propagation mechanism and of the receiver. Due to the complexity of the problem a substructuring approach was followed, using distinct models for the simulation of each part of the domain. Regarding to the simulation of the propagation medium a 2.5D FEM-PML approach was followed, which revealed to be efficient and accurate for the simulation of wave propagation on three-dimensional domains. On the other hand, the coupling of the building to the remaining ground is one of the key aspects of the model proposed. The analysis performed, although the simplicity assumed for the geometry of the building, allowed to conclude that the dynamic characteristics of the building have a considerable effect on the vertical vibration of the slabs.

The research is ongoing and it is expected, in the near future, to study the effect of floating slab systems and ballast mats on the vibrations perceived inside of buildings surrounding railway tunnels.

Acknowledgements

This paper reports research developed under the financial support of "FCT - Fundação para a Ciência e Tecnologia", Portugal, namely from the research project - PTDC/ECM/114505/2009. First author wish also acknowledge to FCT for the financial support provided by the grant SFRH/BD/69290/2010.

REFERENCES

[1] Hussein, M. and H. Hunt, A numerical model for calculating vibration from a railway tunnel embedded in a full-space. Journal of Sound and Vibration, 2007. 305(3): p. 401-431.

[2] Gupta, S., et al., Numerical modelling of vibrations from a Thalys high speed train in the Groene Hart tunnel Soil Dynamics and Earthquake Engineering, 2010. 30(3): p. 82-97.

[3] Gupta, S., et al., A comparison of two numerical models for the prediction of vibrations from underground railway traffic. Soil Dynamics and Earthquake Engineering, 2007. 27(7): p. 608-624.

[4] Lopes, P., et al., Análise numérica de vibrações induzidas por tráfego ferroviário em túneis baseada em modelos 2.5D, in 12º Congresso Nacional de Geotecnia, G. Correia, Editor. 2010: Guimarães. p. CD-ROM publication.

[5] Galvín, P., et al., A 2.5D coupled FE-BE model for the prediction of railway induced vibrations. Soil Dynamics and Earthquake Engineering, 2010. 30(12): p. 1500-1512.

[6] Alves Costa, P., et al., A 2.5D finite element model for simulation of unbounded domains under dynamic loading, in 7th European Conference on Numerical Methods in Geotechnical Engineering, T. Benz and S. Nordal, Editors. 2010: Trondheim. p. 782-790.

[7] Lombaert, G. and G. Degrande, Ground-borne vibration due to static and dynamic axle loads of InterCity and high-speed trains. Journal of Sound and Vibration, 2009. 319(3-5): p. 1036-1066.

[8] Sheng, X., C. Jones, and D. Thompson, Prediction of ground vibration from trains using wavenumber finite and boundary element method. Journal of Sound and Vibration, 2006. 293: p. 575-586.

[9] Yang, Y. and H. Hung, Soil Vibrations Caused by Underground Moving Trains. Journal of Geotechnical and Geoenvironmental Engineering, 2008. 134(11): p. 1633-1644.

[10] Johnson, S., Notes on Perfectly Matched Layers (PMLs), L. notes, Editor. 2010, Massachusetts Institute of Technology.

[11] Lopes, P., et al., Modeling of infinite structures by 2.5D FEM-PML. Application to the simulation of vibrations induced in tunnels, in Railways 2012. The First International Conference on Railway Technology: Research, Development and Maintenance, J. Pombo, Editor. 2012: Tenerife, Canarias.

[12] Lopes, P., et al., An efficient numerical model for the simulation of vibrations induced by railway traffic in tunnels, in ISEV 2013 - International Symposium on Environmental Vibrations. 2013: Shangai, China.

[13] Hussein, M., et al., The dynamic effect of a piled-foundation building on an incident vibration field from an underground railway tunnel, in ICSV20 - International Congress on Sound & Vibration. 2013: Bangkok, Thailand.

[14] Sheng, X., C. Jones, and D. Thompson, A comparison of a theoretical model for quasi-statically and dynamically induced environmental vibration from trains with measurements. Journal of Sound and Vibration, 2003. 267(3): p. 621-635.

[15] Lombaert, G., et al., The experimental validation of a numerical model for the prediction of railway induced vibrations. Journal of Sound and Vibration, 2006. 297: p. 512-535.

[16] Alves Costa, P., R. Calçada, and A. Cardoso, Influence of train dynamic modelling strategy on the prediction of track-ground vibrations induced by railway traffic. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 2012. 226(4): p. 434-450.

[17] Alves Costa, P., R. Calçada, and A. Cardoso, Track–ground vibrations induced by railway traffic: In-situ measurements and validation of a 2.5D FEM-BEM model. Soil Dynamics and Earthquake Engineering, 2012. 32: p. 111-128.

[18] Lopes, P., et al., Numerical Modeling of Vibrations Induced in Tunnels: A 2.5D FEM-PML Approach, in Traffic Induced Environmental Vibrations and Controls: Theory and Application, H. Xia and R. Calçada, Editors. 2013, Nova.

[19] Ferraz, M., A model for the assement of the strutural behaviour of bridges, in Civil Engineering. 2010, Faculty of Engineering of University of Porto.