

## CONTROL OF A THREE-DIMENSIONAL STRUCTURE WITH MAGNETO-RHEOLOGICAL DAMPERS.

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**Abstract.** *In recent decades, new alternative approaches such as active, semi active and passive control have been proposed and developed to protect structures from earthquakes and severe winds. Among these control approaches semi active control devices are a very attractive technology for mitigating structural vibrations. One of the most popular semi active devices is the magneto-rheological damper (MR-MD). Magneto-rheological (MR) dampers are a very attractive alternative technology for the control of structural vibrations since they are adjustable in real time and their power requirements are relatively small compared with active control devices. They are one of the most promising control devices for mitigating structural vibrations. This paper describes a 3-DOF per floor tier building analytical model, which can incorporate models of either traditional tuned mass dampers (TMD) or MR mass dampers (MR-MD). In addition, dynamic behaviour of the 3-DOF building incorporating magneto-rheological dampers has been numerically investigated under a real earthquake excitation. The behaviour of the structure without dampers is compared to the cases of the structure with either traditional passive TMDs or with semi-active magneto-rheological dampers. The structural displacement and absolute acceleration reductions are presented. The results include graphical representations of displacement time histories.*

## 1 INTRODUCTION

The last two decades have seen the development of important new technologies for control of structures subject to dynamic loading. These include passive techniques such as base isolation, active control techniques, combinations of both (hybrid control), and semi-active techniques. Extensive review of the literature in structural hazard mitigation shows that: Passive devices have a limited capacity to adapt to changing conditions beyond the ones for which they were originally designed. For example, performance of passive tuned mass dampers degrades as de-tuning increases, or in the presence of structural nonlinearities [1]. Active devices (direct force actuators) provide significant improvement of the structure's response, but require massive and expensive actuators attached to the structure. In a hazardous event such as an earthquake, failure of the power supply lines may render these actuators useless. The cost and complexity of retro-fitting existing structures with direct force actuators seems also very high. While semi-active devices such as magneto-rheological (MR) dampers are attractive alternatives due to their low power consumption and adjustability in real time [2-3], the complexity of modelling and controlling such actuators have limited their use. MR fluids are non-colloidal suspensions of paramagnetic particles dispersed in a mineral or silicone oil. When a magnetic field is applied, particle chains form and the fluid shows viscoplastic behaviour. MR fluids can transit to rheological equilibrium in a few milliseconds, allowing fast response, and yield stresses in excess of 80kPa. The use of MR dampers in hazard mitigation of structures has been proposed mainly as direct force actuators - large components integrated as part of the structure. Researchers described a 20-ton MR damper for semi-active control of seismic vibrations [4]. In a study, a quasi-static model of the MR damper based on the Bouc-Wen hysteretic model was developed [5]. Some researchers presented a linear optimal controller combined with a force feedback loop and showed analytically that the control system could surpass the performance of active control [6]. The efficacy of MR dampers for control of seismic vibrations was confirmed experimentally, and a clipped-optimal control algorithm based on acceleration feedback was proposed [7]. Shear mode MR dampers [8] were installed as direct structural elements on a six story sub-scale structure on a small shake table. Both clipped-optimal control and a Lyapunov-based algorithm were tested, and the superiority of MR dampers compared to other passive systems was demonstrated. More recently, a phenomenological model of large scale MR dampers has been proposed [9], showing that direct current control reduces the response time of MR dampers.

The use of MR dampers for seismic mitigation has been investigated in [10]. The use of MR dampers in base isolation was studied [11]. An analytical study of the performance of MR dampers in hysteretic structures using a variety of realistic ground motion excitations was studied in [12]. The use of MR dampers as direct force actuators (i.e. as members connecting two points of the structure) has been discussed [13]. Direct force actuation requires comparatively large MR dampers installed as inter-story braces in several points of the structure. This configuration has been successfully tested in real-time hybrid simulations [14].

The growing interest in MR dampers has generated substantial efforts in understanding and modelling such devices. MR damper models have become increasingly more accurate, however:

The most accurate models available to date (variations of the Bouc-Wen hysteretic model) are complex, and require experimental identification of several parameters. This complexity renders design and estimation difficult, and makes them hardly suitable for advanced controller design. Most existing MR damper controllers are therefore forced to be on-off switching algorithms based on some variation of a Lyapunov stability condition. Such controllers provide marginal performance improvements - as a result, very few systems using MR dampers

have been deployed. There are no standard modelling techniques available to the practicing engineer that would allow her or him to incorporate MR dampers in numerical models of structures for testing the efficiency of semi-active control techniques. Stream lined models of the MR damper dynamics are not available.

This study explores the feasibility and effectiveness of using magneto rheological mass dampers (MR-MD) in a 3 dimensional tier building. This is achieved by using a simpler MR model structure whose parameters can be extracted from force-velocity data by a straightforward method. A novel piece-wise invertible MR damper model will give access to robust control techniques whose action is a combination of both continuous and switching actions. The adjustability of damping force in real time can make the MR-MDs substantially more effective than optimally-tuned passive TMDs, since they can compensate for nonlinear effects in large base-motion events. In particular, the seismic transient response can be attenuated more successfully thanks to the adaptability of the semi-active device. The semi-active controller adapts to the changing characteristics as it might exhibit a non-linear behavior and can actually minimize or eliminate the incursion into the non-near range. The MR-MD approach is not based on tuning, but in calculating the forces required to correct the building's state vector under a performance metric evaluated in real-time.

The model structure which will be investigated is a 3-dimensional tier building. The analytical three-dimensional model of the structure incorporates either tuned mass dampers or magneto-rheological mass dampers. A novel MR damper model structure that is piece-wise input-invertible, and estimates both the hysteretic force-velocity behaviour as well as the dependency of damping force with input current is used to estimate the current required. This novel approach is included in the analytical simulations. To perform the numerical simulations for the three dimensional tier building model with or without magneto-rheological dampers MATLAB-SIMULINK simulations have been developed. The accuracy of these simulations was verified with ANSYS.

## 2 ANALYTICAL MODEL OF THE 3D TIER BUILDING

The matrix formulation of 3D multi-story tier buildings was developed by Weaver et al. [15], and has been used extensively over the years, more recently by Gattulli et al. [16]. In this formulation, each story is treated as a rigid body with 3 degrees of freedom (DOF) per floor. To integrate our MR model into a multi-DOF non symmetrical multi-story structural model, we propose the use of the well- developed and relatively simple matrix formulation of the tier buildings and incorporate into it the MR damper semi-active representations. This would enable inclusion of the lateral-torsional response of a prototypical frame structure with asymmetric mass distribution for different MR-MD configurations under bi-axial excitation.

The equations of motion for this structure can be expressed as;

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{D} \quad (1)$$

Where  $\mathbf{U}$  is the relative displacement vector (as a function of time),  $\dot{\mathbf{U}}$  and  $\ddot{\mathbf{U}}$  are relative velocity and acceleration vectors respectively.  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are  $3n \times 3n$  dimensional mass-inertia, damping and stiffness matrices respectively where  $n$  is the number of the stories.  $\mathbf{D}$  is the disturbance vector. The disturbance vector can either be a dynamic force or a real earthquake excitation. The formulation of the 3D analytical model of the building with or without dampers is obtained as follows.

Assume that the columns are axially infinitely rigid and ignore the torsional rigidity of the columns (negligible compared to the torsional rigidity of the floor). In that case, if all the floor columns at each story are identical, the centre of stiffness CS (aka as centre of rigidity)

will be at the centre of the floor (i.e. assume symmetry of stiffness). The centre of mass is CM with eccentricities  $x_{c1}$  and  $y_{c1}$  between CS and CM.

The schematic of the model is given in Figure 1, showing the DOF's.

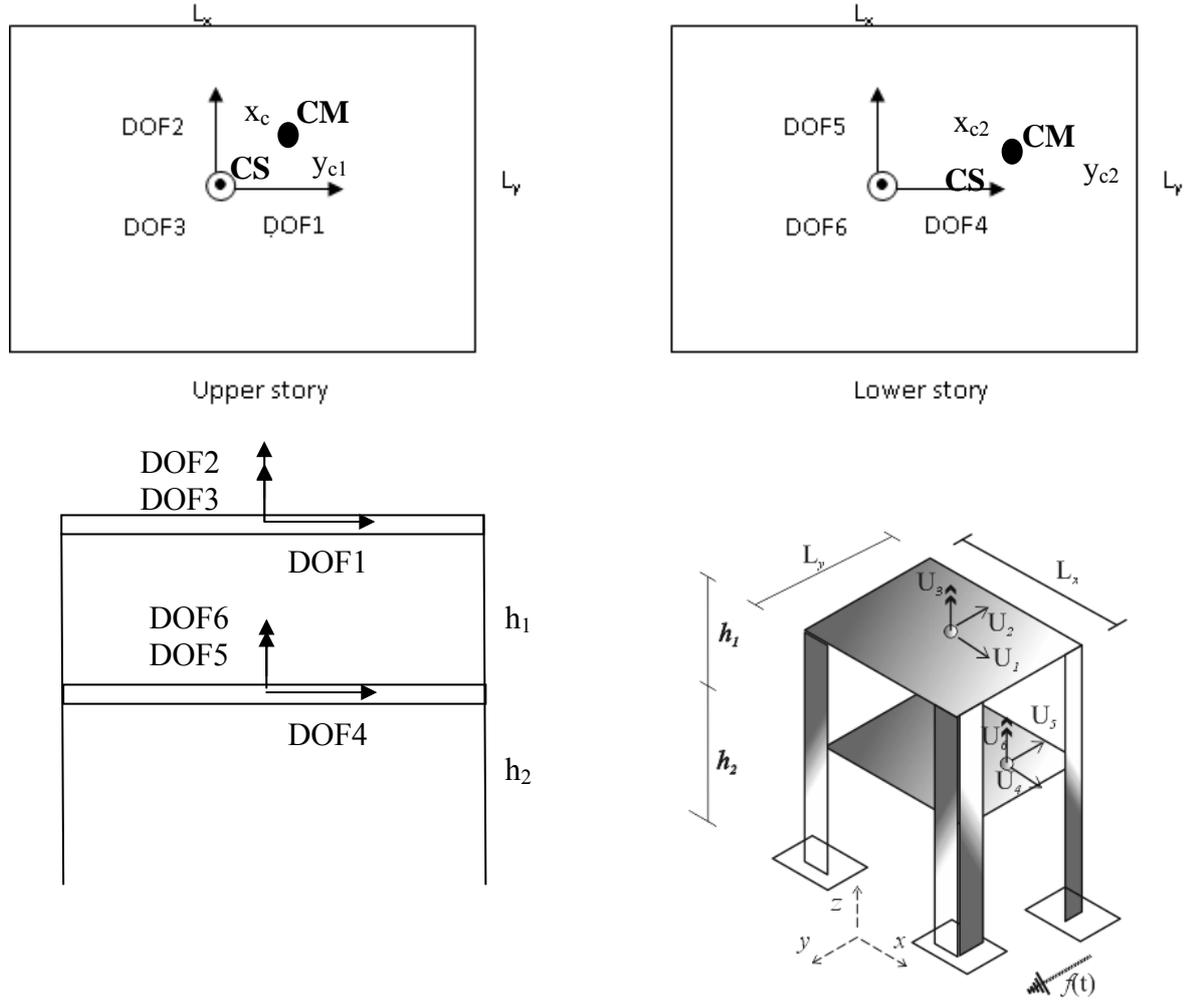


Figure 1: Tier building model of a two story building.

The mass and stiffness matrix of the two story three dimensional tier building are:

$$[\mathbf{M}] = \begin{bmatrix} m_1 & 0 & -y_{c1}m_1 & 0 & 0 & 0 \\ 0 & m_1 & x_{c1}m_1 & 0 & 0 & 0 \\ -y_{c1}m_1 & x_{c1}m_1 & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & -y_{c2}m_1 \\ 0 & 0 & 0 & 0 & m_2 & x_{c2}m_1 \\ 0 & 0 & 0 & -y_{c2}m_1 & x_{c2}m_1 & m_{66} \end{bmatrix}; [\mathbf{K}] = \begin{bmatrix} k_{11} & 0 & 0 & -k_{11} & 0 & 0 \\ 0 & k_{22} & 0 & 0 & -k_{22} & 0 \\ 0 & 0 & k_{33} & 0 & 0 & -k_{33} \\ -k_{11} & 0 & 0 & k_{44} & 0 & 0 \\ 0 & -k_{22} & 0 & 0 & k_{55} & 0 \\ 0 & 0 & -k_{33} & 0 & 0 & k_{66} \end{bmatrix} \quad (2)$$

in Eq. (2)  $m_1$  and  $m_2$  are the mass of the upper and lower story respectively with

$$m_{33} = m_1(x_{c1}^2 + y_{c1}^2 + \rho_1^2) \quad ; \quad m_{66} = m_2(x_{c2}^2 + y_{c2}^2 + \rho_2^2) \quad (3)$$

in Eq. (3)  $\rho_1$  and  $\rho_2$  are the radius of gyration of upper and lower story respectively. The stiffness parameters in Eq. (2) can be expressed as

$$\begin{aligned}
 k_{11} &= 4 \times \frac{12 E_{top} I_{1top}}{h_1^3} ; k_{22} = 4 \times \frac{12 E_{top} I_{2top}}{h_1^3} ; k_{33} = 2 \times \left( \frac{12 E_{top} I_{1top} L_y^2}{h_1^3} + \frac{12 E_{top} I_{2top} L_x^2}{h_1^3} \right) \\
 k_{44} &= 4 \times \left( \frac{12 E_{top} I_{1top}}{h_1^3} + \frac{12 E_{bot} I_{1top}}{h_2^3} \right) ; k_{55} = 4 \times \left( \frac{12 E_{top} I_{2top}}{h_1^3} + \frac{12 E_{bot} I_{2top}}{h_2^3} \right) \\
 k_{66} &= 2 \times L_x^2 \left( \frac{12 E_{top} I_{2top}}{h_1^3} + \frac{12 E_{bot} I_{2top}}{h_2^3} \right) + 2 \times L_y^2 \left( \frac{12 E_{top} I_{1top}}{h_1^3} + \frac{12 E_{bot} I_{1top}}{h_2^3} \right)
 \end{aligned} \tag{4}$$

where  $E_{top}$ ,  $E_{bot}$  are the elasticity modulus of the columns in upper and lower stories,  $I_{1top}$ ,  $I_{2top}$ ,  $I_{1bot}$ ,  $I_{2bot}$  are the moment of inertias of the columns in the direction of DOF1, DOF2, DOF4 and DOF5 respectively.  $L_x$  and  $L_y$  are the length of the floors in  $x$  and  $y$  direction respectively and  $h_1$  and  $h_2$  are the story heights of upper and lower stories respectively (see Figure 1). The damping of the tier building model is assumed to be proportional to both to the mass and stiffness matrices which can be expressed as

$$\mathbf{C} = a_0 \mathbf{M} + a_1 \mathbf{K} \tag{5}$$

$a_0$  and  $a_1$  in Eq. (5) can be obtained by using natural frequencies of the building and the damping coefficient as

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = 2 \frac{\omega_m \omega_n}{\omega_n^2 - \omega_m^2} \begin{bmatrix} \omega_m & -\omega_n \\ -1/\omega_n & 1/\omega_m \end{bmatrix} \begin{Bmatrix} \xi_m \\ \xi_n \end{Bmatrix} \tag{6}$$

where  $\omega_m$  and  $\omega_n$  are the corresponding natural frequencies  $\xi_m$  and  $\xi_n$  are the viscous damping coefficients of the corresponding natural frequencies. Although this formulation is given for a 2 story 3 dimensional tier building it can be generalized to  $n$  story buildings.

### 3 MR-MD STRATEGY

The MR damper model used in this study is piece-wise input-invertible, and estimates both the hysteretic force-velocity behavior as well as the dependency of damping force with input current [17].

In the MR damper, algebraic input invertibility means a model where the input current required for achieving a desired damper force can be calculated from the present values of the state vector and model parameters estimated off-line:

$$\begin{aligned}
 F_d^+(\dot{x}, i) &= \frac{f(i)}{\pi} [\text{arc tan}(M(\dot{x} - \dot{x}_0)) + N(\dot{x} - \dot{x}_0)] + g(i), \quad \ddot{x} > 0 \\
 F_d^+(\dot{x}, i) &= \frac{f(i)}{\pi} [\text{arc tan}(M(\dot{x} + \dot{x}_0)) + N(\dot{x} + \dot{x}_0)] - g(i), \quad \ddot{x} < 0
 \end{aligned} \tag{7}$$

In Eq. (7)  $i$  is the input current and  $\ddot{x}, \dot{x}$  are the acceleration and velocity of the MR damper respectively. A study showed that the parameters in Eq. (7) of the piece-wise invertible damper force model ( $M$ ,  $N$  and the coefficients in  $f(i)$  and  $g(i)$ ) can be determined from force-velocity data at several input current levels by a prediction error method, in a minimum mean-square error sense [18]. The resulting parameters were experimentally validated by comparing measured MR damper force to predicted force, both in the time and frequency domains. The predicted force was also compared to that of advanced MR damper models, and benchmarked against experimental data, both using a 25N Lord Corporation MR damper as well as a 240 kN MR damper at Lehigh University.

The proposed piece-wise invertible (PWI) damper force model enables parameter estimation by standard gradient descent techniques such as prediction error methods or constrained nonlinear optimization, since the right hand side of the structure's equation of motion can be replaced by an invertible coordinate transformation of the input current and the state vector [18]. This is true even if the structure has a nonlinearity that is a polynomial of the state vector (e.g. the softening cubic nonlinearity  $F(\delta)=K \delta(1- K_{NL} \delta^3)$ ), since the nonlinear terms can be lumped with the right hand side of the equation of motion.

With this hysteretic MR damper force model, more complex structural simulations incorporating MR-MD's are developed to perform the dynamic analysis of a 3 dimensional building. In this MR-MD case

- A proportional plus derivative PD controller calculates the required force needed to reduce the corresponding structural vibration.
- With this force applied to the inverse PWI model, the required current/ voltage is being calculated.
- The current obtained by inverse PWI model is being used in Bouc-Wen model to calculate the MR-MD force.
- Then these forces calculated by Bouc-Wen hysteretic model are fed into the dynamic equation of the motion of the structure plus MR-MD system as control force.

The schematic of this system is given in Figure 2.

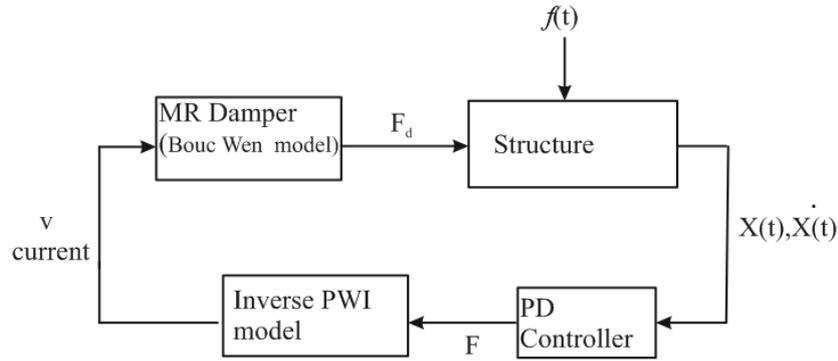


Figure 2: Block diagram of the system incorporating MR Damper.

The force in the Bouc Wen model is given by

$$F = c_0 \dot{x} + k_0(x - x_0) + \alpha z \quad (8)$$

where the evolutionary variable  $z$  is governed by

$$\dot{z} = -\gamma |\dot{x}| z |z|^{n-1} - \beta \dot{x} |z|^n + A \dot{x} \quad (9)$$

By adjusting the parameters of the model  $\gamma, \beta$  and  $\alpha$  one can control the linearity in the unloading and the smoothness of the transition from the pre-yield to the post-yield region. In addition, the force  $f_0$  due to the accumulator can be directly incorporated into this model as an initial deflection  $x_0$  of the linear spring  $k_0$ . Eqs. (8-9) are given for response of the MR damper when the applied voltage, and hence the magnetic field, was held at a constant level. But in this study magnetic field is continuously varied based on the measured response of the system to which it is attached. So the voltage or current is determined in our simulations. For obtaining the current, the following relationships are used (more information about Bouc Wen model and formulations can be found in [6 &7])

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u, \quad c_1 = c_1(u) = c_{1a} + c_{1b} u, \quad c_0(u) = c_{0a} + c_{0b} u \quad (10)$$

where  $u$  represents the current or voltage. In this study, optimal values of the parameters defined in Eqs. (8-10) are determined and given in the following section

#### 4 NUMERICAL EXAMPLE

This MR-MD control strategy is implemented to a 3-dimensional building which is defined in the previous section. The structural behavior of the 3-dimensional building with TMDs and MR-MDs are compared with each other. For this study, a 3-story steel building designed for the SAC project Los Angeles, California region is considered [19]. The building plan is given in Figure 3 (original drawing taken from [19]). The three-story (3-story) structure is 36.58 m by 54.87 m in plans, and 11.89 m in elevation. The bays are 9.15 m on center, in both directions, with four bays in the north-south (N-S) direction and six bays in the east-west (E-W) direction. The building's lateral load-resisting system is comprised of steel perimeter moment-resisting frames (MRFs) with simple framing between the two furthest south E-W frames. The interior bays of the structure contain simple framing with composite floors. The columns are 345 MPa steel. The columns of the MRF are wide-flange. The levels of the 3-story building are numbered with respect to the ground level. The 3rd level is the roof. Typical floor-to-floor heights are 3.96 m. The column bases are modelled as fixed (at the ground level) to the ground. The floors are composite construction (i.e., concrete and steel). The floor system is comprised of 248 MPa steel wide-flange beams acting compositely with the floor slab. In accordance with common practice, the floor system, which provides diaphragm action, is assumed to be rigid in the horizontal plane. The inertial effects of each level are assumed to be carried evenly by the floor diaphragm to each perimeter MRF, hence each frame resists one half of the seismic mass associated with the entire structure. The seismic mass of the structure is due to various components of the structure, including the steel framing, floor slabs, ceiling/flooring, mechanical/electrical, partitions, roofing and a penthouse located on the roof. The seismic mass of the first and second levels is  $9.57 \times 10^5$  kg and the third level is  $1.04 \times 10^6$  kg. The seismic mass of the entire structure is  $2.95 \times 10^6$  kg. The eccentricity of the structure is calculated as 5.22 m in NS direction while it is 0.1 m in EW direction. All the dynamic analysis is performed with simulations developed in MATLAB and verified in Ansys [20].

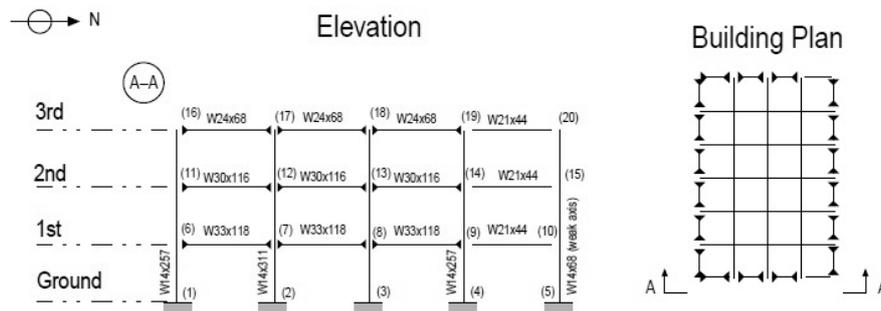


Figure 3. Building plan and elevation.

This structure is idealized with two translational and a single rotational degree of freedom in each story. The 3-dimensional structure is excited under a real earthquake excitation in NS direction. A near field earthquake is selected as *Kobe*: the N-S component recorded at the Kobe Japanese Meteorological Agency (JMA) station during the Hyogo-ken Nanbu earthquake of January 17, 1995.

A single TMD at the third story is placed in the NS direction. The TMD is tuned to the fundamental frequency of the building. The TMD has a mass ratio of 5% and damping ratio of 2%. The numerical values of the mass and stiffness of TMD are  $m_t=100000$  kg,  $k_t = 8.71 \times 10^6$  N/m. For the MR-MD case, the single TMD is replaced with a single MR-MD (MR damper attached to this mass). For the Bouc Wen force model of the MR-MD the parameters are given below

$$c_{0a}=60; c_{0b}=20; k_0=1650; \alpha_a=5; \alpha_b=1; \gamma = 1; \beta=1; A=120; n=2 \quad (11)$$

For the inverse PWI model of the MR-MD the parameters are given below;

$$i_1 = 0; i_2 = 0; M = 0.039; N = - 0.001; \dot{x}_0 = 26.2; f_0 = - 22.533 \quad (12)$$

$$f_1 = 87.853; g_0 = - 1.0776; g_1 = 4.8579; g_2 = - 0.9644$$

PD controller parameters of MR-MD are given below;

$$\text{Proportional (P)}=100, \text{Derivative (N)}= 5, \text{Filter coefficient (N)}:10 \quad (13)$$

The dynamic analysis is performed for 40 seconds with a step size of 0.02 second for the Kobe earthquake N-S component acceleration record. Time displacement curves for the top floor in NS direction are superimposed for uncontrolled structure, structure with TMD and structure with MR-MD in Figure 4. The maximum response reduction percentages of the displacements in NS and EW directions and rotation are given in Table 1 (In the table Rot. represents rotation). Figure 4 and Table 1 show that the maximum story displacement reductions are achieved by the system with an MR-MD. MR-MD performance outperforms the TMD performance. It must be noted that for both the structure with TMD and MR-MD, the rotation reductions are an order of magnitude less than the displacement reductions.

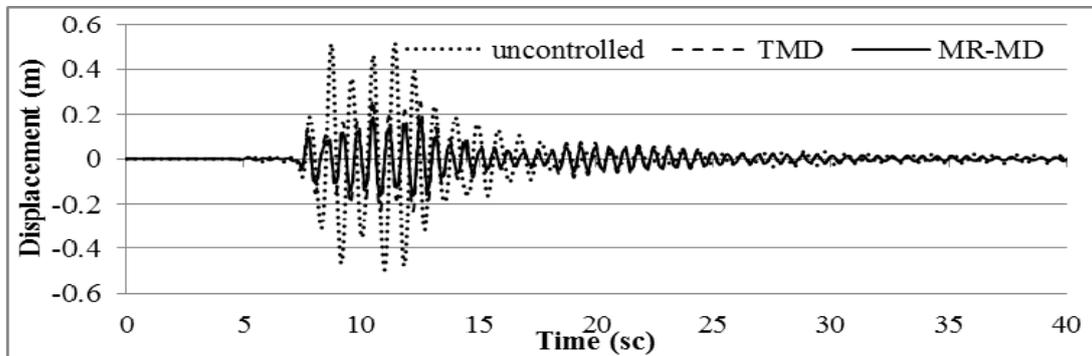


Figure 4: Displacement time histories in NS direction .

Story No & Direction	1-NS	1-EW	1-Rot.	2-NS	2-EW	2-Rot.	3-NS	3-EW	3-Rot.
TMD	52.8	19.8	5.1	49.9	16.1	2.7	47.7	19.9	3.3
MR-MD	64.5	39.8	7.2	62.3	37.0	4.0	60.7	40.0	5.1

Table 1: Uncontrolled response reduction percentages (%)

Maximum absolute accelerations of the floors in NS and EW directions are compared for uncontrolled structure, structure with TMD and structure with MR-MD in Figure 5. The figure shows that maximum absolute acceleration reductions are achieved in the NS direction of the structure with the MR-MD system.

The maximum semi active control force is found to be 0.79 kN. This control force is very low compared with fully active control systems. This control force can be generated with very low power consumptions.

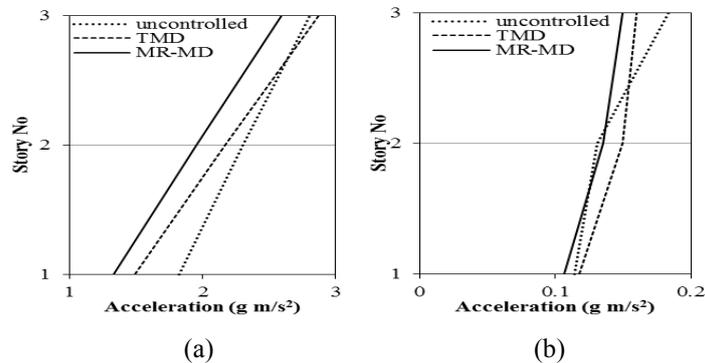


Figure 5: Absolute acceleration responses. ( a:NS direction, b:EW direction)

## 5 CONCLUSIONS

A novel force model for an MR damper is presented and used in this study. This model together with an MR-MD scheme is implemented into a 3 dimensional building subjected to a real earthquake excitation. The response of the system with the MR-MD was compared to the uncontrolled structure and the structure fitted with a traditional TMD. The MR-MD system resulted in a significant response reduction despite using very low power consumption. The performance of the MR-MD outperforms the structure with a perfectly tuned TMD.

These are preliminary results of a work in progress. Future studies will include fitting the structure with MR-MD's in orthogonal directions and subjecting the structure to bidirectional excitation.

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