International Conference on Engineering Vibrations
Aberdeen, Scotland, 18 – 21 August 2020

TRANSIENT CHIMERA-LIKE STATES FOR FORCED PENDULA

Dawid Dudkowski[1], Jerzy Wojewoda[1], Krzysztof Czolczynski[1] and Tomasz Kapitaniak[1]

[1] *Division of Dynamics, Lodz University of Technology, Stefanowskiego 1/15, 90-924 Lodz, Poland, tomaszka@p.lodz.pl*

ABSTRACT

Chimera states occur widely in networks of identical oscillators as it has been shown in the recent extensive theoretical and experimental research. In such a state, different groups of oscillators can exhibit co-existing synchronous and irregular behaviors despite homogeneous coupling. Combining both coherent and incoherent types of responses, chimeras arise on the road to better understanding of the variety of phenomena occurring in nature. In our study we consider a star network, in which N identical peripheral end nodes are connected to the central hub node. The study is realized in the form of physical double pendula hanged on an oscillating platform, which have been shown in Fig. 1.

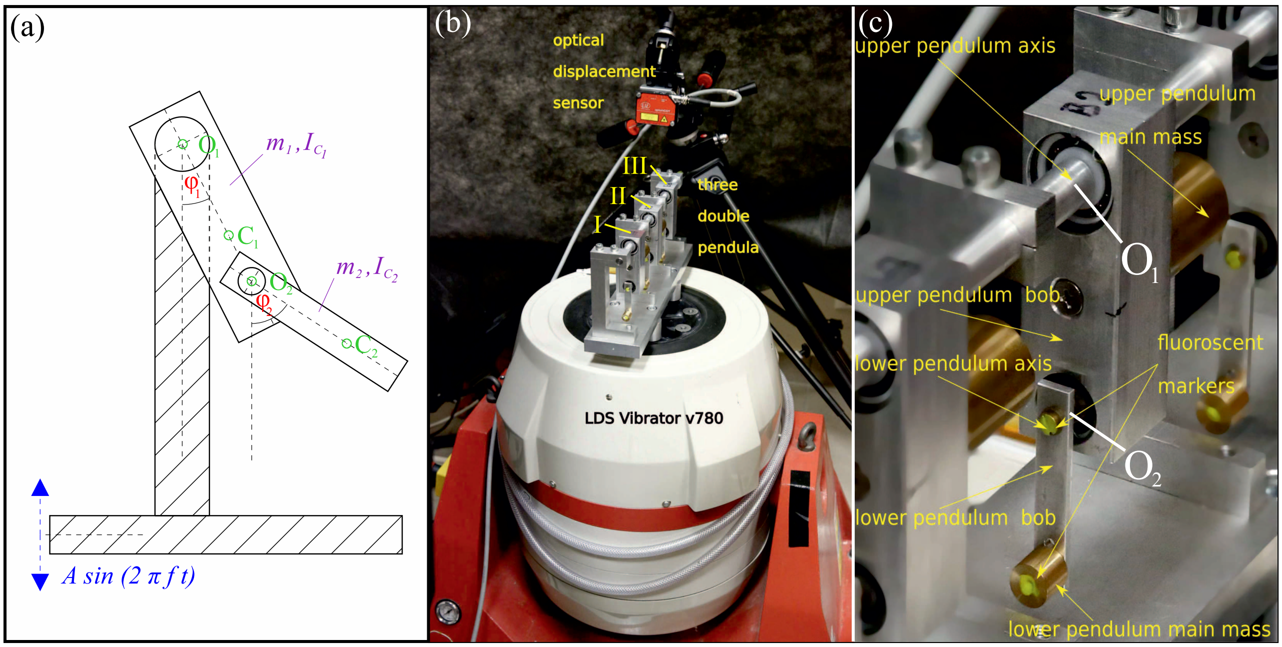

Figure 1: In (a) a scheme of the physical double pendulum used in the experimental research is presented. The experimental rig of three double pendula suspended on an oscillating platform and a zoom on a single double pendulum node are shown in (b) and (c), respectively.

Each double pendulum shown in Fig. 1(a-b) consists of two elements, i.e., the larger pendulum (upper bob) which holds the smaller one (lower bob). The upper bob can rotate or oscillate around the axis O1 connected to the periodically oscillating platform and the lower bob can rotate or oscillate around the axis O2 as shown in Fig. 1(a) (both axis O1, O2 are perpendicular to the figure's plane). The upper and lower bobs have respectively masses m1, m2 [kg] and moments of inertia IC1, IC2 [kgm2] (around the perpendicular axes which crosses bobs mass centers C1, C2). The oscillations of the platform are described by the harmonic function Asin(2ft), where A [m] and f [Hz] are respectively amplitude and frequency.

We describe the dynamics of a single unit, uncovering different types of possible behaviors and discuss the properties of three such oscillators. We find that if a single node exhibits transient chaotic behavior in the whole network, the pattern of transient chimera-like state, which persists for a significant amount of time, is created. Our results show, that the phenomenon of transient chimera-like states can arise and its lifetime is not straightforward, highly depending on possible uncertainties occurring in the system. The numerical study is confirmed by the experimental research, exhibiting that the described transient solutions can be observed in simple experiments with mechanical oscillators, which are controlled by elementary dynamical equations. Our finding suggests that transient chimera-like states have the same properties as the classical weak-chimeras and can be indistinguishable in real-world networks.

DRY FRICTION AND CROSS-COUPLING STIFFNESS INDUCED SELF-EXCITED OSCILLATIONS DURING ROTOR/STATOR RUBBING

Jun Jiang[1], Shunzeng Wang[1], Ling Hong[1]
[1]*State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, P. R. China, jun.jiang@mail.xjtu.edu.cn*

ABSTRACT

The dynamics of engineering systems, which are generally governed by nonlinear, sometimes also non-smooth, differential equations, are very complicated. Rotor-to-stator rubbing, which is usually modeled by multiple degree-of-freedom non-smooth nonlinear systems, is one of such examples and of great practical interests, i.e., for the safety of operation of turbomachinery. Actually, two instability mechanisms may appear simultaneously during rotor-to-stator rubbing, namely, dry friction effect and cross-coupling effect, which might induce a self-excited dry friction backward whirl or a self-excited forward whirling motion of the rotor system.

In this work, a piecewise smooth nonlinear rotor/stator rubbing system with cross-coupling stiffness is investigated with the focus on unveiling the characteristics of the two kinds of self-excited oscillations. Especially, the existence boundaries and the onset conditions of the self-excited oscillations are derived on basis of the nonlinear modal analysis by employing the methods of Nonlinear Normal Modes (NNMs), which are global ones and can be well detected in a wide range of rubbing responses. Furthermore, the characteristics of stick-slip oscillations exhibited in the self-excited dry friction backward whirl is also explored from a point of view of non-smooth sliding bifurcations based on the Filippov's convex method. On the other hand, the methodology of spectral-submanifold (SSM)-based reduction is adopted to construct a reduced-order model and predict the response of the self-excited forward whirling motion of the rotor/stator rubbing system. Especially, a modified method for constructing the periodic SSM is proposed in order to release the insuperable difficulty in solving the time-periodic coefficients for the manifold formulation of the periodic SSM. The methodology employed and the results obtained may provide deeper insight into the dynamical characteristics of piecewise rotor/stator rubbing systems and the potential guidance for the design of rotor systems.

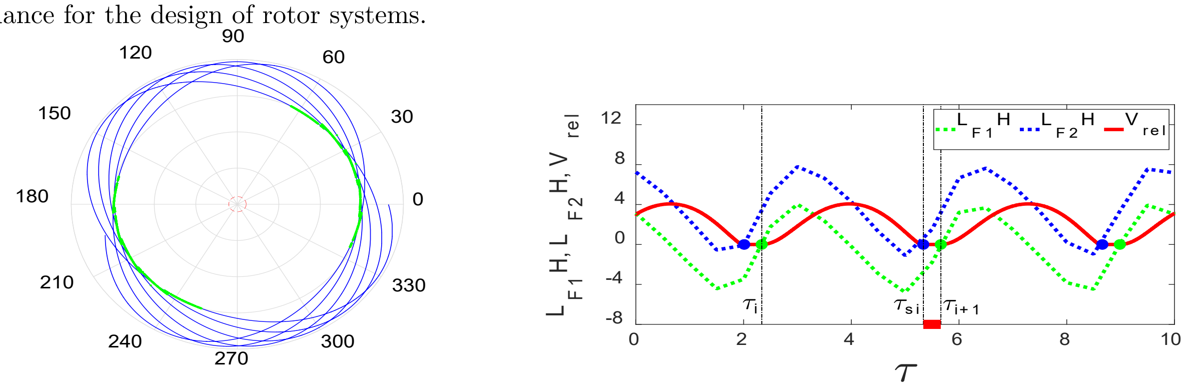

Figure 1: The dry friction backward whirls with stick-slip oscillations. (left) Rotor orbit in pure rolling (green curve) and in slipping (blue curve) where the stator clearance is represented by the red dashed cycle; (right) Time histories of relative velocity and normal components of vector fields.

Keywords: piecewise smooth nonlinear systems, self-excited oscillations, nonlinear normal modes, sliding bifurcations, model reduction

[1] Wang, S. Z., Hong, L., Jiang, J. 2020 Characteristics of stick-slip oscillations in dry friction backward whirl of piecewise smooth rotor/stator rubbing systems. *Mechanical Systems and Signal Processing* **135**, 106387.

[2] Chen, Y. H., Jiang, J. 2013 Determination of nonlinear normal modes of a planar nonlinear system with a constraint condition. *Journal of Sound and Vibration* **332**, 5151-5161.

ORIGAMI DYNAMICS EMPLOYING REDUCED ORDER MODELS

Marcelo A. Savi, Larissa M. Fonseca and Guilherme V. Rodrigues
Center for Nonlinear Mechanics, COPPE - Department of Mechanical Engineering, Universidade Federal do Rio de Janeiro, 21.941.972 - Rio de Janeiro - RJ, Brazil, P.O. Box 68.503

ABSTRACT

There is an increasing need on engineering of compactable or reduced size mechanisms with the ability to complete complex tasks. As a matter of fact, the need for efficient small and weightless devices boost the study of origami structures and developable devices. By combining origami concepts with smart materials, such as shape memory alloys (SMAs), it is possible to develop complex self-foldable structures that can exchange between different configurations through the folding process promoted by the origami pattern. Among the combined SMA-origami applications, it should be highlighted the origami-wheel that has the inherent ability of radius variation, allowing its use on a robot that can drive through different soils, climb or deviate different objects with a reduced number of actuators [2].

A proper description of origami-like elements is essential to capture the structure behavior. The folding process usually involves significant geometric nonlinearity, associated with the pattern chosen and the actuation provided, which promotes additional nonlinear behaviors related to the deformation of the panels. This work presents an analysis of the complex behavior of a waterbomb tessellation, an origami pattern generated by the repetition of the waterbomb pattern. A general analysis considering different kinds of symmetry is carried out, allowing the formulation of reduced-order models. The structure complexity is evaluated considering different degrees of freedom. A comparison between the reduced-order model and the general formulation under symmetric actuation is presented, and the model is validated. Afterward, a dynamical investigation is of concern considering origami-stent and origami wheel considering the reduced-order model. The model is able to describe the system response. Origami wheel is employed to a deformable wheel robot, evaluating its path, with the yaw motion being generated by the origami wheels radius' variation. All these systems present complex responses that need to be deeply investigated for general applications. Figure 1 shows an origami wheel with waterbomb pattern highlighting its unit cell. Three approaches employed for the construction of the reduced-order model is shown: equivalent mechanism, spherical trigonometry and finite element method. Finally, it is presented some dynamical analysis of the origami wheel robot, showing different paths from small parameter variations.

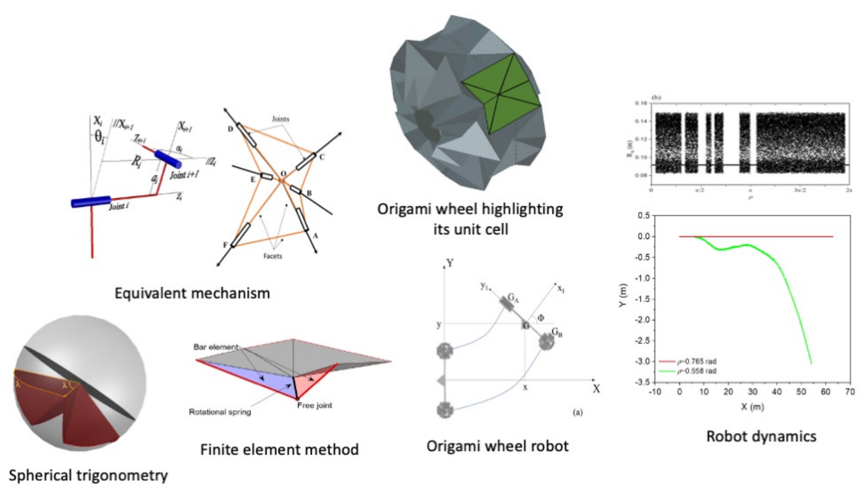

Figure 1: Origami analysis considering three approaches in order to build a reduced-order model employed to analyse the origami wheel robot.

Keywords: Origami, nonlinear dynamics, mechanism, shape memory alloys

[1] Felton, S., Lee, D. Y., Cho, K. J. & Wood, R. J., 2014, "A passive, origami-inspired, continuously variable transmission", IEEE International Conference on Robotics and Automation, pp. 2913–2918.

[2] Lee D. Y. *et al.*, 2013, "The deformable wheel robot using magic-ball origami structure", International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, Portland, Oregon.

INVERSE MODELLING OF THE ROD BEHAVIOUR IN THE FLOW USING SURROGATE-BASED APPROACH

Sabine Upnere[1,2]
[1] *Riga Technical University, Kipsalas Street 6B, Riga, Latvia, upnere@protonmail.com*
[2] *Ventspils University of Applied Sciences, Inzenieru Street 101, Ventspils, Latvia*

ABSTRACT

This work is dedicated to flow-induced vibration studies in high-powered cooling systems consisting of many rigid rods between which water flows as a cooling agent. For the safe operation of engineering equipment, it is necessary to continuously monitor whether the system does not exceed its operational limits and does not compromise its structural integrity. At the same time, it is not possible to measure changes in the internal elements (parameters) of the system directly during the operation of the equipment. For complex systems, its behaviour as a function of the input parameters is generally not known. Adequate mathematical models can be used to detect as early as possible changes in the typical behaviour of the system that could indicate its malfunction and structural integrity problems. This paper examines the effects of changes in the mass of the single, flexibly mounted rod inserted into an array of rigid rods.

Computational fluid dynamics simulations were used to establish relationships between dominant frequencies and the mass reduction of the flexibly mounted rod in cross-flow. To model the rod movement, it was described as a mass–spring system. The 13-point orthogonal plan developed by J. Auzins [1], which is a part of a new class of design of experiments for use with orthonormal Legendre polynomial, was used to create the surrogate model of the mass and stiffness as factors and frequency ratio as a response. Applying results of the forward simulation new second-order and 7-term orthonormal Legendre polynomial approximation models were created.

The inverse analysis, which was applied to experimental data, allowed to develop a surrogate model describing the relationship between the frequency ratio, the stiffness and the mass. Of the options examined, the best approximation was achieved by the use of a second-order response surface with the following quality measures: 9.288 % cross-validation error and $R^2 = 0.999$. Pearson's chi-squared test also showed that the model is adequate. Based on inverse model verification, it can be concluded that the resulting surrogate model can predict the behaviour of the analysed model system with high precision when input factor values are changed within 15 % the difference is less than 1 %.

The created approximation models are applicable to the alleviation of uncertainty, monitoring of cooling systems or early detection of damage related to the reduction of the mass of rods or to changes in the stiffness of the supports.

Keywords: inverse model, surrogate model, orthogonal plan, response surface method.

Acknowledgements: This work was partially developed with the financial support of the Latvian Council of Science project "Creation of design of experiments and metamodeling methods for optimization of dynamics of multibody 3D systems interacting with bulk solids and fluids" (lzp-2018/2-0281).

[1] Auzins, J. 2014 High order orthogonal designs of experiments for metamodeling, identification and optimization of mechanical systems. *Proc. 11th World Congress on Computational Mechanics, 5th European Conference on Computational Mechanics, and 6th European Conference on Computational Fluid Dynamics*, 3190–3201.

EVOLUTION OF FRICTION PARAMETERS ALONG A WELL

Shanti Swaroop Kandala[1], Jean Auriol[2], Ulf Jakob F. Aarsnes[3], Roman Shor[4]

[1] *Department of Chemical and Petroleum Engineering, University of Calgary, Calgary, Canada, shantiswaroop.kandal@ucalgary.ca*

[2] *Universitè Paris-Saclay, CNRS, CentraleSupélec, Gif-sur-Yvette, France, jean.auriol@centralesupelec.fr*

[3] *NORCE Norwegian Research Centre AS, Oslo, Norway, ulaa@norceresearch.no*

[4] *Department of Chemical and Petroleum Engineering, University of Calgary, Calgary, Canada, roman.shor@ucalgary.ca*

ABSTRACT

In this work, we consider the evolution of friction coefficients along a drill string in a deviated wellbore using field data. It has been observed in the literature that stick-slip oscillations exist in drill strings even with bit off-bottom and persist with the bit on bottom. In the past, various lumped mass models have failed to explain this phenomenon, as they have only considered bit-rock interaction (BRI) to be the main reason for the stick-slip oscillations. A field validated distributed model of a drill string into which the Coulomb friction is modeled as a source term and where the bottom-hole assembly (BHA) is modeled as a lumped mass is considered for this study [1]. This model is combined with BRI model [3]. Owing the simplicity of this set of models, a soft sensor can be effectively derived by employing some of the recent results in control theory obtained by Aarsnes et al. [2]. In this work, we use topside measurements (i.e., surface measurements) to estimate the evolution of friction coefficients (static and kinetic) using this soft sensor. We assume the torsional motion of the drill string is the dominating dynamics behavior, there is a constant rate of penetration and no distributed axial dynamics. For early time, the observer is run assuming the bit off-bottom and switches to consider BRI once the bit tags bottom. In this work, we analyze the evolution of friction parameters from surface to downhole for an unconventional well drilled in North America.

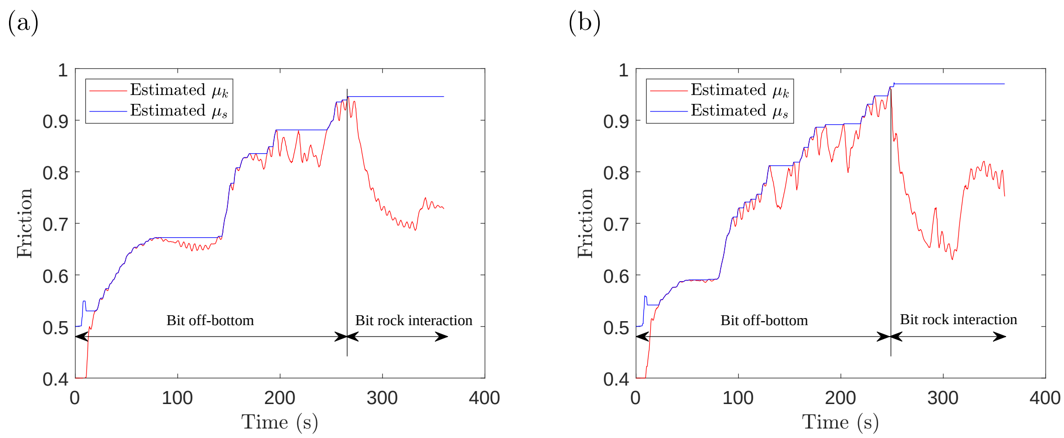

Figure 1: Variation of kinetic and static friction coefficients with time at the BHA for a depth of (a) 9100 ft and (b) 9450 ft.

Figures 1(a) and 1(b) show the evolution of friction coefficients at the BHA with respect to time for two different depths (9100 ft and 9450 ft) for the considered well if a bit-rock model is not taken into account. Once the BRI begins, there is a significant change in the friction coefficients. In literature, few works have separated the estimation of the friction coefficients along drill string from BRI. Therefore, by estimating the static and kinetic friction coefficients along the drill string, followed by an estimation of BRI, this work provides an improved understanding of the nonlinear effects of friction on the drill string which may be useful in many control applications.

Keywords: Drill string, friction parameters, distributed model, torsional dynamics

[1] Aarsnes, U. J. F. and Shor, R. J. 2018 Torsional vibrations with bit off bottom: Modeling, characterization and field data validation. *Journal of Petroleum Science and Engineering* **163**, 712–721.

[2] Aarsnes, U. J. F., Auriol, J., Di Meglio, F. and Shor, R. J. 2019 Estimating friction factors while drilling. *Journal of Petroleum Science and Engineering* **179**, 80–91.

[3] Auriol, J., Aarsnes, U. J. F. and Shor, R. J. 2020 Self-tuning torsional drilling model for real-time applications. *American Control Conference.*

ON MATHEMATICAL FORMULATIONS OF THE DYNAMIC RESPONSE MITIGATION PROBLEMS FOR SEMI-ACTIVE FLUID-BASED ABSORBERS

Cezary Graczykowski[1], Rami Faraj[1]
[1] *Institute of Fundamental Technological Research, Polish Academy of Sciences, Pawinskiego 5B, 02-106 Warsaw, Poland, cezary.graczykowski@ippt.pan.pl*

ABSTRACT

Semi-active fluid-based absorbers, such as hydraulic and pneumatic dampers, are commonly used for impact and vibration protection. In these devices mitigation of the dynamic response is obtained with the use of electromechanical or piezoelectric valves, which control flow of the fluid between two chambers located on the opposite sides of the piston. In the case of impact protection the control of the fluid flow allows to maximize energy dissipation and to obtain optimal force-displacement characteristics providing minimal value of impacting object deceleration. In turn, in the case of vibration protection the flow control enables obtaining minimal value of reaction force through synchronization of the piston motion with external harmonic force. Mentioned above problems require different mathematical formulations depending on the assumed objective functions, limitations of valves operation, available information about external excitation as well as required capabilities of disturbances compensation.

The contribution summarizes recent achievements of the authors in development of the original mathematical formulations of dynamic response mitigation problems and elaboration of the corresponding solution methods. The starting point is presentation of the classical formulation of impact mitigation problem which assumes full knowledge about actual dynamic excitation and lack of valve operation constraints resulting in the possibility of entire impact energy dissipation [1]. The critical analysis of the above approach is the motivation for development of several variational formulations of dynamic response mitigation problems including: i) force-based and kinematics-based optimal control problem taking into account valve constraints, ii) state-dependent path-tracking problem allowing for the compensation of system disturbances and / or changes in system parameters, iii) the multi-objective optimization problem involving maximization of energy dissipation and minimization of generated reaction force. For each mathematical formulation the corresponding solution methods are proposed and briefly described. They include gradient-based methods of variational calculus and direct discretization methods in the first case, dedicated versions of predictive and adaptive control methods (Hybrid Prediction Control [2] and Identification-based Predictive Control [3]) in the second case as well as various versions of Pareto optimization methods in the third case.

Acknowledgements. The financial support of the National Science Centre, Poland, granted under agreement 2018/31/D/ST8/03178, is gratefully acknowledged.

Keywords: semi-active shock-absorber, dynamic response mitigation, impact and vibration protection, optimal control, predictive control, adaptive control.

[1] Holnicki-Szulc, J., Graczykowski, C., Mikułowski, G., Mróz, A., Pawłowski, P. and Wiszowaty R. 2015 Adaptive impact absorption - the concept and potential applications. *International Journal of Protective Structures* 6(2), 357-377.

[2] Faraj, R. and Graczykowski, C. 2019 Hybrid prediction control for self-adaptive fluid-based shock-absorbers. *Journal of Sound and Vibration* 449, 427-446.

[3] Graczykowski, C and Faraj, R. 2020 Identification-based Predictive Control of semi-active shock-absorbers for adaptive dynamic excitation mitigation. *Meccanica*, under review.

PREDICTIVE AND ADAPTIVE CONTROL OF THE FLUID-BASED ABSORBERS UNDER IMPACT AND VIBRATION CONDITIONS

Rami Faraj[1], Cezary Graczykowski[1]
[1] *Institute of Fundamental Technological Research Polish Academy of Sciences, Pawinskiego 5B, 02-106 Warsaw, Poland, rfaraj@ippt.pan.pl*

ABSTRACT

During design and manufacturing of structures and systems engineers focus on meeting the functional, mechanical and legal requirements. For majority of systems appropriately conducted design process ensures stable and robust operation, but for selected applications more attention has to be paid to excitation conditions and dynamical characteristics of the system. In particular, selected systems have to be equipped with additional devices for protection against shock and vibration. The most common protective devices are fluid-based absorbers, which include hydraulic and pneumatic dampers. Nevertheless, other types of excitation mitigation systems are developed and they include also frictional and inertial systems.
As an increasing attention is currently paid to adaptive capabilities of protective systems [1, 2], the contribution addresses the problem of adaptive dynamic response mitigation. Discussed numerical study concerns predictive and adaptive methods introduced by the authors for both the impact absorption and vibration mitigation problems. In particular, the fluid-based absorbers are considered and solutions of different problem formulations are analyzed in detail. The influence of objective functions as well as implications of the constrained operation of the working elements are studied. According to conducted analyses, an efficient reduction of different types of the process disturbances is ensured with the use of predictive approach. Additional introduction of adaptation schemes ensures compensation of the changes in system parameters. For performance assessment the proposed methods are compared with the Adaptive Impact Absorption (AIA) techniques [3] as well as classical control methods.

Acknowledgements. The financial support of the National Science Centre, Poland, granted under agreement 2018/31/D/ST8/03178, is gratefully acknowledged.

Keywords: Adaptive control, Adaptive Impact Absorption, Predictive control, Shock-absorber, Vibration mitigation.

[1] Brzeski, P., Lazarek, M. and Perlikowski, P. 2017 Experimental study of the novel tuned mass damper with inerter which enables changes of inertance. *Journal of Sound and Vibration* 404, 47–57.

[2] Faraj, R. and Graczykowski C. 2019 Hybrid Prediction Control for self-adaptive fluid-based shock-absorbers. *Journal of Sound and Vibration* 449, 427-446.

[3] Holnicki-Szulc, J., Graczykowski, C., Mikułowski, G., Mróz, A., Pawłowski, P. and Wiszowaty R. 2015 Adaptive impact absorption - the concept and potential applications. *International Journal of Protective Structures* 6(2), 357-377.

ON THE ROLE OF PRODUCTS OF INERTIA IN DYNAMICS OF HIGH-SPEED ROTORS

Luboš Smolík[1], Štěpán Dyk[2], Jan Rendl[3]
[1] *University of West Bohemia, Univerzitni 8, 301 00 Plzen, carlist@ntis.zcu.cz*
[2] *University of West Bohemia, Univerzitni 8, 301 00 Plzen, sdyk@ntis.zcu.cz*
[3] *University of West Bohemia, Univerzitni 8, 301 00 Plzen, rendlj@ntis.zcu.cz*

ABSTRACT

Products of inertia are introduced can be introduced to a rotating system during assembly [1] or during balancing [2]. A typical situation which illustrates rise to the products of inertia is an angular misalignment of a disc to a shaft. Childs [1] demonstrated that this misalignment is a source of the dynamic load, which is analogous to the dynamic load induced by rotating unbalance — the products of inertia excite response at the synchronous frequency.

Large turbomachinery, including steam turbines, pumps, and compressors, is typically susceptible to the static and dynamic rotating unbalance and the effects of the products of inertia are usually omitted. This practice is also followed in the simulation of dynamics of small high-speed rotors, e.g. turbocharger rotors. Many recent theoretical works including [3] underestimate the synchronous response of the turbocharger significantly even if the rotating unbalance is set to unrealistically high values. Experimental works [4] demonstrate that the synchronous response of the turbocharger is significant even at speeds which surpass the threshold speed for the oil-induced instability in journal bearings.

This work provides the analysis of nonlinear effects induced by the products of inertia. Figure 1 shows that the products of inertia can excite not only the synchronous response but also higher harmonics or subharmonic components. Moreover, some sources of the products of inertia are identified, and the resulting products of inertia are expressed analytically. These expressions can be used to improve algorithms for balancing.

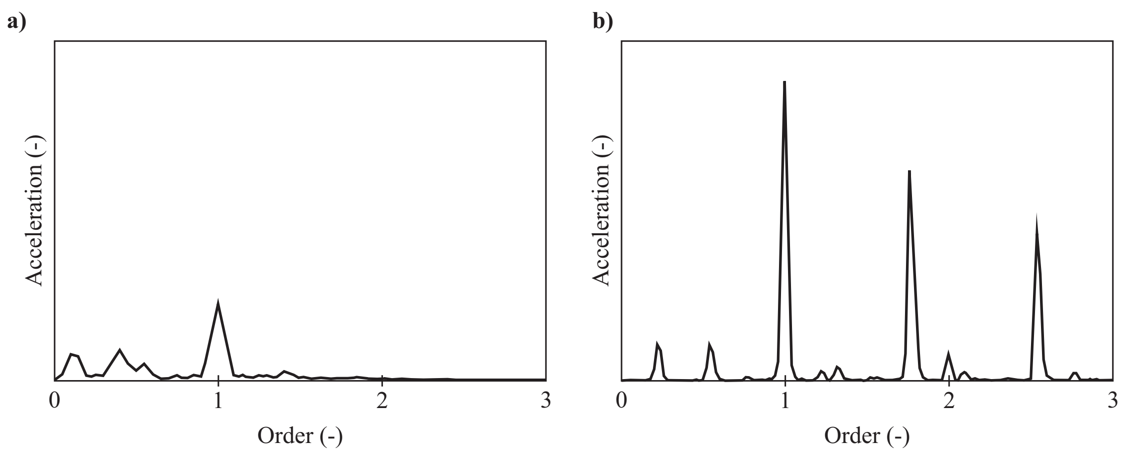

Figure 1: Dimensionless response of a turbocharger excited by rotating unbalance (a) and products of inertia (b)

Keywords: nonlinear dynamics, rotor dynamics, products of inertia

[1] Childs, D. 1993 *Turbomachinery Rotordynamics: Phenomena, Modeling, and Analysis*, ch. 1, pp. 19–22. New York: Wiley.

[2] Vance, J. M. 1966 *Rotordynamics of Turbomachinery*, ch. 5, pp. 176–178. New York: Wiley.

[3] Tian, L., Wang, W. J. and Peng, Z. J. 2013 Nonlinear effects of unbalance in the rotor-floating ring bearing system of turbochargers. *Mechanical Systems and Signal Processing* **34** (1-2), 298-320.

[4] Schweizer, B. and Sievert, M. 2009 Nonlinear oscillations of automotive turbocharger turbines. *Journal of Sound and Vibration* **321** (3-5), 955-975.

International Conference on Engineering Vibrations
Aberdeen, Scotland, 18 – 21 August 2020

DYNAMICS OF HIGH FREQUENCY IMPACT IN JARRING OPERATIONS

Idin Nazzari, Ekaterina Pavlovskaia, Marian Wiercigroch
Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, Aberdeen, AB24 3UE, UK, i.nazzari@abdn.ac.uk

ABSTRACT

A number of attempts have been undertaken to model jarring operations focused on prediction of the jar impact force and force at the stuck point, optimisation of the jar placement, and estimation of loads conveyed to the drill-string as well as surface structures, where only a single impact has been considered [1]. However there have been indications that using lower overpull and high frequency impacts can improve the performance of the jarring process [2], and therefore modelling of repeatable impacts in jarring becomes important. Impact oscillators and drifting oscillators appear to be promising in modelling high frequency impacts in jarring and they have been studied extensively in a wider context [3]. In this work a suitable impact oscillator model of jarring is developed to predict optimum design, operational and environmental parameters. It is assumed that the tool performance could be enhanced by introducing high frequency impacts and maximising the transmitted force within the practical range of parameters in representative real life stuck tool scenarios.

A typical jarring system is modelled to mimic dynamic behaviour of high frequency hammer anvil impacts as shown in Figure 1a. Interactions between the following three elements were considered: the stuck point, the jar tool (consisting of cams, hammer and anvil), and the drill-string above where the overpull is applied. The jar tool excitation and impact cycles were assumed to have three phases: (i) *rest*, when the hammer and anvil are in contact and compressed, (ii) *lift*, when the hammer and anvil separate due to engagement of the cams, and (iii) *impact*, when the cams release, the hammer drops and impacts with the anvil. After the initial impact, the hammer might bounce back and go through multiple impacts before it eventually rests compressed to the anvil due to overpull. At this point the jar is back to the first stage and the cycle can repeat. A periodic truncated ramped function is used to simulate the non-harmonic lifting force on the hammer as shown in Figure 1b.

An evolution of the basins of attraction in a range of excitation frequency is shown for the considered system in Figure 1c and the detailed nonlinear dynamics analysis of this system will be presented during the conference.

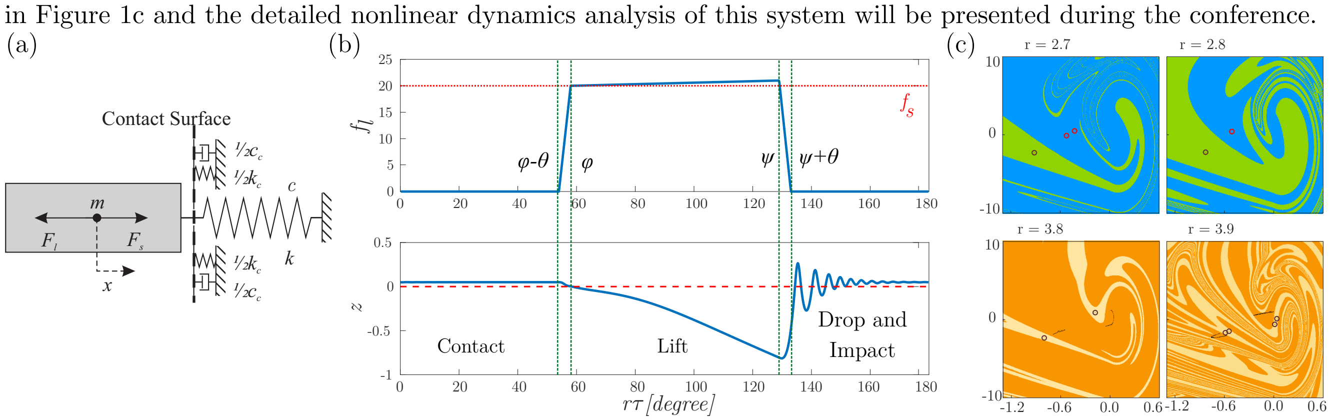

Figure 1: (a) Jarring model, with hammer m impacting the anvil, subject to static force F_s and dynamic force F_l; (b) Typical non-dimensional lift load f_l against cam rotation (top panel) compared to non-dimensional displacement z against cam rotation (bottom panel); (c) Basins of attraction for different values of excitation frequency.

Keywords: Impact, Jarring, Impact Oscillators, Low Dimensional Modelling, Basins of Attraction.

[1] Yin, H., Luo, N., and Li, Q. 2014 Analysis of Hydraulic Jarring Dynamics and Calculation of Impact Force. *Applied Mechanics and Materials*, **557**, 129134.

[2] Joppe, L.C.F., Mackenzie, G. and McGurk, M. 2006 Using High-Frequency Downhole Vibration Technology to Enhance Through-Tubing Fishing and Workover Operations. In *SPE/ICoTA Coiled Tubing Conference & Exhibition*, Proceedings, 2023.

[3] Pavlovskaia, E., Ing, J., Wiercigroch, M. and Banerjee, S. 2010 Complex Dynamics of Bilinear Oscillators Close to Grazing. *International Journal of Bifurcation and Chaos*, **11**, 38013817.