

Grazing Induced Bifurcations: Innocent or Dangerous?

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In this lecture I will examine nature of subtle phenomenon such grazing bifurcations occurring in non-smooth systems. I will start with linear oscillators undergoing impacts with secondary elastic supports, which have been studied experimentally and analytically for near-grazing conditions [1]. We discovered a narrow band of chaos close to the grazing condition and this phenomenon was observed experimentally for a range of system parameters. Through stability analysis, we argue that this abrupt onset to chaos is caused by a dangerous bifurcation in which two unstable period-3 orbits, created at "invisible" grazing collide [2].

The experimentally observed bifurcations are explained theoretically using mapping solutions between locally smooth subspaces. Smooth as well as non-smooth bifurcations are observed, and the resulting bifurcations are often as an interplay between them. In order to understand the observed bifurcation scenarios, a global analysis has been undertaken to investigate the influence of stable and unstable orbits which are born in distant bifurcations but become important at the near-grazing conditions [3]. A good degree of correspondence between the experiment and theory fully justifies the adopted modelling approach.

Similar phenomena were observed for a rotor system with bearing clearances, which was analysed numerically [4] and experimentally [5]. To gain further insight into the system dynamics we have used a path following method to unveil complex bifurcation structures often featuring dangerous co-existing attractors.

References

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